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Measurement of water

HYDRAULICS

BOOKS BY

R. L. DAUGHERTY

HYDRAULIC TURBINES
Third Edition
281 pages, 6 × 9, Illustrated.

CENTRIFUGAL PUMPS
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HYDRAULICS

BY

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THIRD EDITION
THIRD IMPRESSION

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PREFACE TO THE THIRD EDITION

The opportunity of entirely rewriting this book for a new edition has made it possible to bring it up to date and also to make such improvements in the presentation of the subject as experience has suggested. In many places the material has been rearranged and presented in what now seems to be a more logical order. Certain points that caused difficulty have been presented in a different way, which it is hoped will be better and clearer, and various other topics have been explained more fully. For instance, more space has been devoted to certain matters, such as entrance losses to pipes, sudden contraction and expansion, and so on, that are of minor practical importance in themselves, but a knowledge of which leads to a better understanding of hydraulic phenomena. The chapter on pipe friction has been very materially altered and a new and better method of determining pipe friction has been presented, which is valid for any fluid under any condition.

Even more care has been taken than before to avoid special cases, if possible, and especially to refrain from special formulas. It has been the aim in this edition to retain only the most general fundamental formulas in the text and either to eliminate all others or else to place them among the problems for the students to derive. The applications of fundamental principles to specific cases have been illustrated by the solution of numerical rather than algebraic problems. This has the double advantage of making it more difficult for the student to obtain an answer merely by substituting in the final algebraic expression, and also of showing him how a numerical problem may be solved in the simplest and most systematic manner.

The problems formerly found in the body of the text have been materially reduced in number, with the idea that there should be no more of them than the student could reasonably be expected to work. For classroom use, additional problems will be found in the back of the book. At the ends of articles or groups of articles will be found one or more examples to illustrate the

application of what has immediately preceded. These examples may be considered as the minimum that any student should be expected to solve for himself. At the end of each chapter will be found a small number of problems of wider scope, covering material in the whole chapter. Each example or problem is intended to be essentially different from every other. Answers are given to all of the examples or problems found in the body of the text.

In dealing with fundamental principles primarily, it has not been thought advisable to present all the empirical formulas that exist, where they practically parallel other common formulas and do not add anything new in principle. It was for this reason that the Fteley and Stearns weir formula, for example, as well as others, was not given in previous editions. In harmony with this policy the Bazin formula for open channels has been omitted in this new edition, because, no matter how excellent it may be, it is not in common use in this country; it does not cover any field that is not covered by other formulas; and it is not different in principle. It is believed that such parallel presentations merely confuse the student.

For the practicing engineer, problems may be solved quite readily by the use of various aids, such as pipe-flow diagrams, but it is not thought best to present information in this way in an elementary text, as the beginner is apt to learn how to read diagrams rather than how to acquire the proper grasp of the fundamentals. For the same reason tables of values often are not given where a simple algebraic expression suffices, as a table often obscures the factors involved and their relations.

New material has been inserted in various chapters, and there has been added a new chapter on non-uniform flow in open channels.

R. L. D.

California Institute of Technology,
Pasadena, Calif.
June, 1925.

PREFACE TO THE SECOND EDITION

In this edition it has been possible to add new material which the author has found necessary in his present courses. This consists principally of graphical methods of solving certain practical problems, the determination of the economic size of pipe, and problems of flow through compound pipes, branching pipes, pipes with laterals, and through rotating channels. The treatment of various topics has also been extended in many cases, and certain other portions have been rewritten where experience has indicated that this was desirable for greater effectiveness.

The author has been aided in this revision by the helpful suggestions of many individuals and in particular of Prof. F. G. Switzer of Cornell University.

R. L. D.

RENSSELAER POLYTECHNIC INSTITUTE
TROY, N. Y.,
May, 1919.

PREFACE TO FIRST EDITION

This book has been prepared as a text for students who are required to cover a wide field in hydraulics in a limited amount of time. Therefore the treatment has been made as brief and concise as is consistent with clearness. Attention has been given mostly to matters which are of fundamental importance and but little space has been devoted to those things which are of small practical value, except where necessary to illustrate basic principles. As a step in saving the student's time a liberal use has been made of diagrams, curves, and half-tones. These not only save words but often give a clearer idea at a glance than can be obtained in any other way.

The treatment throughout has been made as consistent as is possible. The solution of all problems involving the flow of water is made to depend upon applications of Bernoulli's theorem, which is the key to a rational treatment of the subject. The student is not told in the very beginning that $V = \sqrt{2gh}$ and then compelled to unlearn it later. Experience in the classroom has shown that many students will persistently apply that formula whether it fits the case or not. By deriving it at a later time by an application of Bernoulli's theorem, they will more readily see that it is a very special case and thus realize more fully its limitations.

An effort has been made to avoid special cases so far as is possible. The treatment in the text and the equations are for the most part entirely general. Special cases are given only when necessary to illustrate the application of some general principle, or where a special case makes some proposition clearer, and when the general treatment is too complex. But the attention of the reader is called to the fact that the equations there given are not universally applicable.

Classroom experience has shown that very few students obtain a true physical conception of the subject of hydraulics. To most of them, even some of the best, it is very largely an abstract subject. This is partly due to the fact that, with their limited experience and observation, they have actually seen but few of

the things with which the book deals and hence they can form no adequate mental picture of the physical facts. In order to overcome this, so far as possible, a large number of illustrations from photographs have been employed. As a further step in implanting a true physical idea in the mind of the student, a great deal of care has been exercised in the arrangement and presentation of the subject and a constant attempt has been made to connect one part with another. In many cases the problems have been taken from actual practice and have also been arranged so as to be instructive in themselves.

In considering turbines and centrifugal pumps the first essential is to convey a fair idea as to the general appearance, construction, and arrangement of such machines and possibly some simple features of their operation, since it is useless to plunge directly into a mass of equations which are no more than mathematical gymnastics to most students. The second step should be the presentation of the principles of operation together with a general idea as to actual characteristics. These facts could then be explained by as much theory as one had time to go into. In this text but very little theory has been given and that of the simplest kind, though it is believed that what is given is both general and rational. By the aid of this theory the nature of the characteristics of these machines can be accounted for. After that one is ready to take up certain very useful and practical commercial factors by the aid of which one can classify turbines or pumps, can compare one type with another, and can make an intelligent selection of the best type for certain conditions.

The simple theory of hydraulic machinery that has been given here covers about all that is really useful in a text of this scope. The design of turbines and pumps is too empirical, and requires too much judgment and experience backed up by a good supply of test data, to be expressed by a few equations. Any brief treatment of this phase of the subject would be false and misleading, hence it has been omitted. For any more extended treatment of these subjects the reader is referred to other publications of the author.

The main idea underlying the entire text has been to present fundamental principles. After this ground has once been covered, those who desire to specialize in hydraulics are prepared to

study certain topics more intensively. The devotion of considerable space to an account of experiments and test data is unwarranted here, though the student should not lose sight of the fact that the study of such is desirable when important work is undertaken. However, a sufficient amount of information on experimental coefficients and empirical factors has been given so that a correct idea may be formed both as to the range of values and the considerations that enter into the choice of a suitable value for a given case.

Very naturally some very important topics in practical hydraulics have been omitted altogether or treated very briefly and superficially because they did not involve fundamental principles and hence were not within the scope of this text, or else were of such a nature as to belong to advanced treatises. The final apology which the author makes for this work is that it has been prepared primarily to meet the needs of his own classes.

The author wishes to acknowledge his indebtedness to the various parties whose names are attached to certain of the illustrations for their kindness in furnishing the same. He is also indebted to E. H. Wood, Professor of Mechanics of Engineering in Sibley School of Engineering, Cornell University, and to D. R. Francis, Instructor in Sibley School of Engineering, Cornell University, for valuable assistance in the criticism of the manuscript and the reading of the proof. R. L. D.

Cornell University
Ithaca, N. Y.,
April, 1916.

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NOTATION

A = area in square feet (in turbines and pumps it is the total area of the streams measured normal to the absolute velocity of the water).
a = area in square feet of all the streams in turbines or pumps measured normal to the relative velocity of the water.
b = barometer in feet of water.
C = coefficient in Chezy's formula.
c = coefficient of discharge.
c_c = coefficient of contraction.
c_v = coefficient of velocity.
D = diameter of turbine runner or pump impeller in inches.
d = diameter of pipe in feet.
d'' = diameter of pipe in inches.
e = efficiency.
e_h = hydraulic efficiency.
e_m = mechanical efficiency.
e_v = volumetric efficiency.
F = force in pounds.
f = friction factor in pipes.
G = any weight in pounds.
g = acceleration of gravity in feet per second per second.
H = total effective head in feet, $= p + z + V^2/2g$.
H' = any loss of head in feet.
h = head in feet.
I = moment of inertia.
K = any constant.
k = any coefficient of loss.
l = any length in feet.
m = hydraulic mean depth (or hydraulic radius) in feet
N = roughness factor in Kutter's formula.
= revolutions per minute.
N_s = specific speed, $N \times \sqrt{\mathrm{B.hp.}}/\mathrm{h}^{5/4}$.
n = any abstract number.
= exponent of V.
P = power in foot-pounds per second.
= total pressure on area in pounds.
p = intensity of pressure in feet of water.
p' = intensity of pressure in pounds per square foot.
p'' = intensity of pressure in pounds per square inch.
Q = total quantity of water in cubic feet.
q = rate of discharge in cubic feet per second.
r = radius to any point in feet.

s = slope of hydraulic gradient = H'/l.
= specific gravity compared to water.
T = torque or moment of a force in foot-pounds.
t = time in seconds.
= pipe thickness in inches.
U = absolute viscosity in centipoises.
u = linear velocity of a point in feet per second.
V = absolute velocity of water (or relative to earth) in feet per second.
v = velocity of water relative to some moving point in feet per second.
W = pounds of water per second, = wq.
w = density in pounds per cubic foot.
z = any vertical distance in feet; in measuring "head" it is a vertical elevation *above* any arbitrary datum plane.
α = angle between V and u measured between their positive directions.
β = angle between v and u measured between their positive directions.
ϕ = ratio of peripheral speed of turbine runner or pump impeller to $\sqrt{2gh}$.
ϕ_e = value of ϕ for which the maximum efficiency is obtained.
ω = angular velocity in radians per second = $2\pi N/60 = u/r$.
μ = absolute viscosity in pounds per foot-second.

Values of quantities at specific points will be indicated by subscripts. In the use of subscripts (1) and (2) the water is always assumed to flow from (1) to (2).

ABBREVIATIONS

g.p.m. = gallons per minute.
sec. ft. = cubic feet per second.
r.p.m. = revolutions per minute.
hp. = horsepower.
b.hp. = brake horsepower = d.hp.
w.hp. = water horsepower.

HYDRAULICS

CHAPTER I

INTRODUCTION

1. Definition of Subject.—*Hydromechanics* is the science of the mechanics of fluids. It may be subdivided into three branches: *Hydrostatics* is the study of the mechanics of fluids at rest; *hydrokinetics* deals with the flow of fluids; while *hydrodynamics* is concerned with the forces exerted by or upon fluids in motion.

Hydraulics is practical hydromechanics, that is, it is the study of the applications of hydromechanics to engineering problems.[1] While it might deal with any fluid it is generally restricted to liquids and especially to water.

By idealizing conditions and ignoring phenomena that are known to exist, it is possible to study hydromechanics as a subject in pure mathematics. But naturally the results of such studies, though interesting, are often of little practical value. The determination of actual results by rigorous mathematics is often impossible because of the fact that the exact nature of certain hydraulic phenomena are either unknown or if known are so complex that it is not feasible to express them as mathematical functions. We must, therefore, resort to a combination of rigid mathematics, empirical expressions, and experimental coefficients. The science that results, based partly upon pure reasoning and partly upon experimental evidence, is called hydraulics.

It is seen that hydraulics is not an exact science. In its actual applications much depends upon the judgment and the experience of the engineer. In many cases it is necessary to compute or estimate results for which satisfactory experimental data are lacking. And in applying any experimental factors or empirical formulas it is well to have some familiarity with the work upon

[1] The derivation of the word "hydraulics" means "flow of water in a pipe" but usage has given the word a much broader significance.

which they were based in order to judge as to their application to the case in hand.

2. Distinction between a Solid and a Fluid.—The distinction between a solid and a fluid is ordinarily quite clear but there are plastic solids which flow under the proper circumstances and even metals may flow under high pressures. On the other hand, there are certain very viscous liquids which do not flow readily, and it is easy to confuse them with the plastic solids. The definition of a fluid as a substance which flows must be extended therefore. The distinction is that any fluid, no matter how viscous, will yield in time to the slightest stress. But a solid, no matter how plastic, requires a certain magnitude of stress to be exerted before it will flow.

Also when the shape of a solid is altered by external forces the tangential stresses between adjacent particles tend to restore the body to its original figure. With a fluid these tangential stresses, which are proportional to the viscosity, can act only while the change is taking place. When motion ceases the tangential stresses disappear and the fluid does not tend to regain its original shape.

3. Distinction between a Gas and a Liquid.—A fluid may be either a gas or a liquid. A gas is quite compressible and when all external pressure is removed it tends to expand indefinitely. A gas is, therefore, in equilibrium only when it is completely enclosed. A liquid, on the other hand, is relatively incompressible, and if all pressure, except that of its own vapor, be removed the cohesion between adjacent particles holds them together so that the liquid does not expand indefinitely. Therefore, a liquid may have a free surface, that is, a surface from which all pressure is removed, except that of its own vapor.

The volume of a gas is greatly affected by changes in either pressure or temperature or both. It is usually necessary, therefore, to take account of changes in volume and temperature when dealing with gases. Since the mechanics of gases is largely one of heat phenomena it is called thermodynamics.

The volume of a liquid is affected to a very small extent by changes in pressure or temperature and for most purposes the changes in volume or temperature may be ignored.

4. Compressibility of Water.—Water is usually said to be incompressible, and as compared with gases it is relatively so. But it is much more compressible than many solids such as steel

or even wood where the elastic limit is not passed. Its bulk or volume modulus of elasticity, the ratio of the change of pressure per unit area to the change of volume per unit of volume, is

$$E_v = 294{,}000 \text{ lb. per square inch.}$$

This value holds only for pressures below 1,000 lb. per square inch and for temperatures near the freezing point. For higher temperatures it increases slightly. Thus at 77°F. it is about 327,000

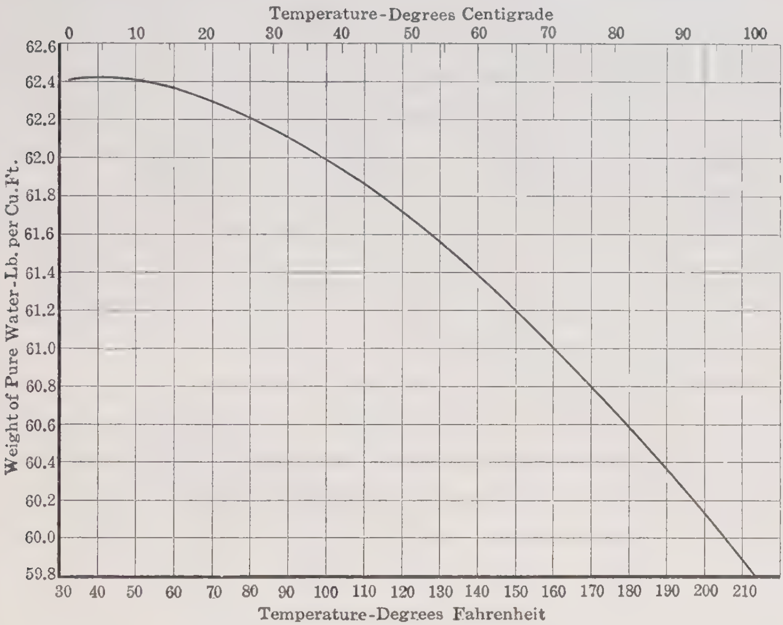


Fig. 1.—Density of pure water.

lb. per square inch and at 212°F. it is 360,000 lb. per square inch. Also for higher pressures than the above the modulus is materially larger. Thus at a pressure of 65,000 lb. per square inch Hite found a value of $E_v = 650{,}000$ lb. per square inch.

It may be seen that increasing the pressure from atmospheric to 1,000 lb. per square inch will reduce the volume of a body of water by $985 \div 294{,}000$ or about one-third of 1.0 per cent. Therefore, it is seen that the usual assumption regarding water as incompressible is justified.

5. Density of Water.—The density of water varies somewhat with the temperature as well as with the pressure. In Fig. 1 can

be seen the values of the density of pure water at atmospheric pressure for the range of temperature from freezing to boiling. The presence of impurities increases these values somewhat. Thus ocean water may ordinarily be taken as weighing 64.0 lb. per cubic foot. In the computations in this book it will be sufficient to take

$$w = 62.4 \text{ lb. per cubic foot.}$$

6. Accuracy of Computations.—No computed result can be more accurate than the data upon which it is based and it is therefore not only useless but also misleading to carry out results to more significant figures than the data warrant. It should be noted that the number of significant figures has no relation to the location of the decimal point. Thus 347,000, 34.7, and 0.0000347 are all values given to three significant figures and are of the same degree of accuracy. It is incorrect in such a figure as the first to preserve any more figures such as 347,129, if three-figure work is all that is warranted. And if it is warranted, it is likewise incorrect in such a value as the last to abbreviate it to 0.00003, for that is equivalent to saying that its value is 0.0000300 to three significant figures.

There are some quantities that may be known with a high degree of accuracy, but in hydraulic work most experimental factors are uncertain in the third significant figure and there are some coefficients or values which are uncertain even in the second significant figure. Thus slide rule work is all that is usually justified.

Suppose, for example, that the product is desired of two quantities whose values are 34.7 and 125. Multiplying these two numbers together 4,337.5 is obtained, but the answer that should be given is 4,340. If the values were known to be 34.700 and 125.00 then the exact product would be permissible. But if the values are experimental they may range, for example, from 34.6 to 34.8 and 124 to 126, respectively. The products of the minimum and maximum values in each case are 4,290 and 4,380, thus showing that our result of 4,340 is uncertain in the third significant figure as should be expected when the given data are not correct in the third figure.

7. Notation.—The use of a systematic and consistent notation is highly desirable and familiarity with the notation will save time and trouble. A table of the notation employed in this book is given on page xv.

So far as possible an attempt has been made to employ the same notation that the majority of other writers use in this and in related subjects. This will result in a few cases of the same letter being used for different quantities but in such instances the quantities are so unlike that it is believed no real confusion can result. Unfortunately, the necessity of avoiding real conflicts in notation prevents the use of certain letters that most naturally suggest themselves for several different quantities, the quantities not being sufficiently removed from each other to permit the duplication.

8. Units.—The standard system of units employed in this book is based upon the foot, pound, and second. With few exceptions all formulas are to be used with such units. There are some few exceptions that commercial practice makes necessary or desirable. For instance, the diameter of a pipe is customarily given in inches rather than in feet. Any exceptions to the general rule will be clearly indicated.

It should be noted that the units of the answer in any computation can be determined from the units involved in the separate items. Thus the product of velocity and area is the product of $(ft./sec.) \times sq.\ ft. = cu.\ ft./sec.$ The familiar quantity $v^2/2g$ is $(ft./sec.)^2/(ft./sec.^2) = ft.$ The product of the depth of water by the density of water is $ft. \times lb.\ per\ cu.\ ft. = lb.\ per\ sq.\ ft.$, etc.

It will frequently be necessary to use the value of g, the acceleration of gravity. Its units are *feet per second per second*, often written $ft./sec.^2$ The value of g varies with latitude and elevation. Its value for any locality may be computed by the following formula according to Pierce,

$$g = 32.0894(1 + 0.0052375 \sin^2 l)(1 - 0.0000000957e),$$

where l is the latitude in degrees and e is the altitude in feet. For ordinary purposes g may be taken as 32.2 ft. per second.^{2}

9. PROBLEMS

1. If a body of water is subjected to a pressure of 65,000 lb. per square inch, how much less will its volume be than in a perfect vacuum?

Ans. 10 per cent.

2. What pressure will be required to reduce the volume of a body of water by 0.1 of 1.0 per cent if the temperature is 32°F. and the initial pressure 10 lb. per square inch.

Ans. 304 lb. per square inch.

3. If the temperature is 77°F. what would be the result in problem 2?

Ans. 337 lb. per square inch.

4. A cubic foot of ocean water at the surface and at ordinary temperature weighs 64.0 lb. At the surface it is under a pressure of 14.7 lb. per square inch. What will be the weight of a cubic foot at a depth such that the pressure is 2,000 lb. per square inch? Assume $E_v = 310{,}000$ lb. per square inch. (Density is inversely proportional to volume.)

Ans. 64.4 lb.

5. The radiator of an automobile holds 2.0 cu. ft. of water. It is filled with water at 50°F. After the engine has been running the temperature of the water is 180°F. Assuming no loss by evaporation or otherwise and neglecting expansion of radiator, how much water will have run out the overflow?

Ans. 3.6 lb.

6. If *cubic feet* of water are multiplied by the density of water in *pounds per cubic foot* and by *feet*, in what units will the answer be?

7. If *torque*, which is the product of a force in pounds and a distance in feet, is multiplied by *angular velocity* in radians per second, what units will be involved in the answer?

8. If *pounds* per second are multiplied by *feet per second* and divided by g, in what units is the answer?

9. If a force in *pounds* is multiplied by velocity in *feet per second* in what units is the answer?

CHAPTER II

INTENSITY OF PRESSURE

10. Definition of Intensity of Pressure.—By intensity of pressure is meant pressure per unit area. It may be expressed in various units such as pounds per square inch, pounds per square foot, or, as will be seen later, in feet of water, inches of mercury, etc.

If P represents the total pressure on some finite area, A, while dP represents the total pressure on an infinitesimal portion of area, dA, the intensity of pressure is

$$p' = \frac{dP}{dA}. \qquad (1)$$

If the pressure is uniformly distributed over the area in question the intensity of pressure would then be $p' = P/A$. If the pressure is not uniformly distributed the latter expression will give the average value only.

The word "pressure" is usually used for "intensity of pressure" though the latter term should be employed where there is any possibility of misunderstanding. The word "pressure" is also used to designate the resultant force exerted on an area. In order to distinguish clearly this usage from intensity it would be well to employ the term "resultant pressure" or "total pressure."

11. Variation of Pressure in a Liquid.—Consider a slender prism of the liquid in Fig. 2 as a free body in equilibrium. The forces acting upon it will be the pressures on its various faces and the pull of gravity. If the intensity of pressure at M be denoted by p'_1, the total pressure on the end at M will be p'_1dA, where dA is the cross-section area. In similar manner the total pressure on the end of N will be p'_2dA. The weight of the volume of liquid is evidently $wdAl$. Since the prism of water is in equilibrium, the algebraic sum of the components in any direction of all the forces acting on it will be zero. If the forces be resolved along the axis MN, the three forces mentioned will be the

only ones that will appear since the forces acting on the sides of the prism are all normal to the axis. Hence

$$p'_1dA - p'_2dA + wdAl \cos \alpha = 0.$$

Since $l \cos \alpha = z_2 - z_1$ it follows that

$$p'_2 - p'_1 = w(z_2 - z_1). \qquad (2)$$

This equation shows that the difference in the intensity of pressure at two different points varies directly as the difference in the depths of the two points. Also if point M be taken at the level where p'_1 is zero, and if z be the elevation of such level above any other point, then in general

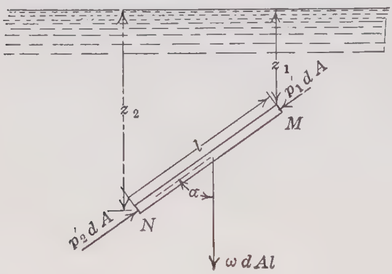


FIG. 2.

$$p' = wz. \qquad (3)$$

From this equation it can be seen that the intensity of pressure varies directly as the depth of the point in question below the level where p' is zero.

The results of Eqs. (2) and (3) are strictly true only for an incompressible fluid in which the density, w, is constant at all depths. For practical purposes water is an incompressible fluid and hence these equations may be applied. But, owing to the high degree of compressibility of gases, they should not be used for a gas except where there are relatively small differences in pressure.

12. Surface of Equal Pressure.—It may be seen from Eq. (3) that all points in a connected body of water at rest are under the same intensity of pressure if they are at the same depth. This indicates that a surface of equal pressure is a horizontal plane. Strictly speaking it is a surface everywhere normal to the direction of gravity and it is, therefore, approximately a spherical surface concentric with the earth. But a limited portion of such a spherical surface is practically a plane area.

A *free surface* is strictly one on which there is no pressure. Usually, however, the surface of a liquid exposed only to the pressure of the atmosphere is said to be a free surface.

13. Pressure the Same in All Directions.—In a solid, owing to the existence of tangential stresses between adjacent particles,

the stresses at a given point may be different in different directions. But in a fluid at rest no tangential stresses can exist and the only forces exerted between adjacent surfaces are normal to the surfaces. Therefore, the intensity of pressure at a given point is the same in every direction.

This can be proved by reference to Fig. 3, where we have a small triangular element of volume whose thickness perpendicular to the plane of the paper is constant and equal to dz. Let α be any angle, p' the intensity of pressure in any direction, and p'_x the intensity of pressure on a vertical plane. The following forces act upon this volume of fluid: The pressure on the vertical face is $p'_x dydz$, the pressure on the slanting face is $p'dldz$, then there are the pressures on the horizontal face and on the two faces parallel to the plane of the paper, and the weight of the volume. Their values are not required. Since this volume is a fluid body at rest there are no other forces besides these normal to the surfaces, and, since it is a condition of equilibrium, the sum of the components in any direction is equal to zero. Writing such an equation for components in a horizontal direction we have only:

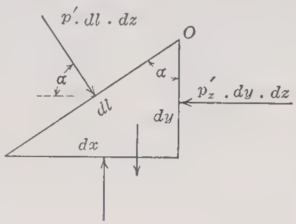


Fig. 3.

$$p'dldz \cos \alpha - p'_x dydz = 0.$$

Since $dy = dl \cos \alpha$, it is seen that:

$$p' = p'_x.$$

This result is independent of the angle α, and, therefore, it follows that the intensity of pressure is the same upon any plane passing through O.

14. Pressure Expressed in Height of Liquid.—In Fig. 4 imagine a body of liquid upon whose surface there is no pressure. Then by Eq. (3) the intensity of pressure at any depth z is $p' = wz$. For a given liquid w is constant and thus there is a definite relation between p' and z. That is, any pressure per unit area is equivalent to a corresponding height of liquid. In hydraulic work it is often more convenient to express intensity of pressure in terms of height of a column of water rather than in pressure per unit area.

Even if the surface of the liquid in Fig. 4 is under some pressure, the relation stated is still true; for this pressure on the surface could be expressed in terms of height of the liquid and such value added to z. The resulting value of p' would thus be increased by the amount of this surface pressure.

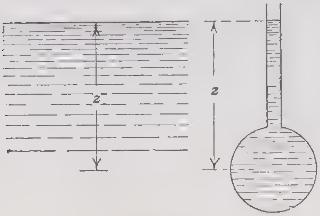


Fig. 4.

The intensity of pressure expressed in terms of the height of a column of liquid will be denoted by the letter p. It will thus be seen that

$$p' = wp. \tag{4}$$

This equation is true for any consistent system of units. Thus if w is density in pounds per cubic foot, p must be expressed in feet, and p' will then be in pounds per square foot. For pure water at ordinary temperatures the relation is $p' = 62.4p$. It is quite common to express intensity of pressure in pounds per square inch, but p is rarely found expressed in other units besides feet of water. Since $p' = 144p'' = 62.4p$, then

$$p'' = 0.4333p \text{ and } p = 2.308p''.$$

EXAMPLES

10. Neglecting the pressure of the atmosphere upon the surface, what is the pressure in pounds per square inch at a depth of 3,000 ft. in fresh water? At a depth of 3,000 ft. in the ocean?

Ans. 1,300, 1,333.

11. A certain pump for a hydraulic press delivers water at a pressure of 5,000 lb. per square inch. To what height of pure water would that be equivalent? To what height of liquid having a density of 100 lb. per cubic feet?

Ans. 11,540 ft., 7,200 ft.

12. The specific gravity of mercury is 13.57, that is, its density is 13.57 times that of pure water. How many feet of mercury are equivalent to a pressure of 100 lb. per square inch? How many feet of water are equivalent to a pressure of 10 ft. of mercury?

Ans. 17.05 ft., 135.7 ft.

15. Barometer.—If a tube such as that in Fig. 5 has its lower end immersed in a liquid and the air is exhausted from the tube, the liquid will rise in the latter. If the air is completely exhausted there will be zero pressure on the surface of the liquid in the tube, and the liquid will have reached its maximum height.

This device is called the barometer and is used for measuring the pressure of the atmosphere.

By Art. 12 it may be seen that the intensities of pressure at o (within the tube) and at a (at the surface of the liquid outside) are the same. That is $p_o = p_a$. And, since the pressure on the surface of the liquid in the tube is zero, the intensity of pressure at o is by Eq. (3)

$$p'_o = wy.$$

And by Eq. (4) $p'_o = wp_o$. Thus the pressure of the air in terms of height of the liquid column is

$$p_a = y. \tag{5}$$

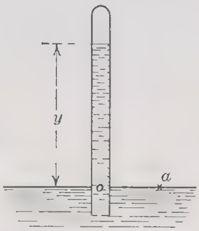


Fig. 5.—Barometer.

The liquid employed is usually mercury because its density is sufficiently great to enable a reasonably short tube to be used, and because its vapor pressure is negligible at ordinary temperatures. If water were employed the height of the tube would be inconvenient, and also its vapor pressure at ordinary temperatures is appreciable so that instead of having a perfect vacuum at the top of the tube there would be a space filled with water vapor. The height attained by the liquid would consequently be less than what would otherwise be the case. The diameter of the tube should be at least 0.5 in. in order to eliminate errors due to capillarity.

The pressure of the atmosphere is different in different localities, depending upon elevation, and at a given point it varies from time to time according to the temperature and other factors.

In round numbers the pressure of the atmosphere may be taken as 14.7 lb. per square inch, 30 in. of mercury, and 34 ft. of water. (These values are not exact equivalents.)

EXAMPLES

13. If the pressure of water vapor at 80°F. is 0.505 lb. per square inch what would be the height of the water barometer if the atmospheric pressure were 14.7 lb. per square inch? (Use correct density of water for this temperature.)

Ans. 32.9 ft.

14. Assuming the density of air to be 0.0807 lb. per cubic foot, what would be the height of the air surrounding the earth and producing a pressure of 14.7 lb. per square inch, if air were incompressible?

Ans. 26,250 ft.

16. Vacuum.—Pressures less than that of the atmosphere are usually called vacuums, a perfect vacuum meaning an entire absence of all pressure. Vacuum is usually measured from the pressure of the atmosphere as a base and is commonly, though not necessarily, measured in inches of mercury. If the atmospheric pressure is 30 in. of mercury, a perfect vacuum would then be a vacuum of 30 in. And a vacuum of 10 in. of mercury would mean that there was a real pressure of 20 in. of mercury.

EXAMPLE

15. The barometer reads 30 in. of mercury and within a certain vessel there is a vacuum of 22 in. of mercury. What is the real pressure within that vessel in pounds per square inch? What is the excess external pressure on the walls of the vessel in pounds per square inch?

Ans. 3.92, 10.78.

17. Absolute and Relative or Gage Pressures.—If the pressure is measured above absolute zero pressure it is called absolute pressure. If it is measured from the atmospheric pressure as a base it is called relative or gage pressure, since it is only relative pressure that a gage measures. Thus Fig. 6 shows a compound gage which will measure pressures either above or below that of the atmosphere. When the gage is open to the atmosphere the hand points to zero. If the gage is connected to any vessel in which there is a pressure above that of the surrounding air the hand will turn in a clockwise direction from zero. If the pressure is a vacuum the hand will move in the opposite direction. Thus the gage measures only the difference between the pressure on the inside of the gage tube and that of the air surrounding the gage.

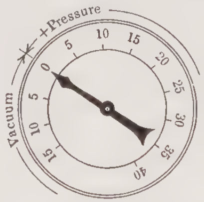


Fig. 6.—Compound gage.

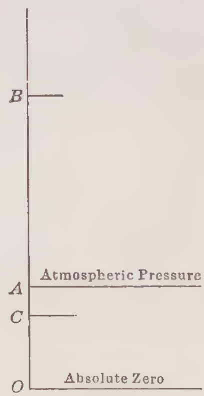


Fig. 7.

In Fig. 7 let O indicate entire absence of all pressure or absolute zero, and the ordinate OA represent the pressure of the atmos-

phere. Then suppose there is any pressure such as at B. The gage will read the value AB and this is the gage pressure. The absolute pressure is OB. Also if there is a vacuum of AC, the gage pressure is $-AC$, the minus sign merely indicating a value below atmospheric just as a plus sign indicates a pressure above that of the atmosphere. But the absolute pressure is OC.

When dealing with absolute pressure all values are positive. In the case of gage pressures only values above that of the atmospheric pressure are positive, but the minus sign for pressures below that of the atmosphere serves only to indicate a vacuum. There may still be a real pressure between adjacent particles of water. A true negative pressure would mean that the water was in a state of tension, and as water can sustain only a very slight tensile stress it is impossible to have a pressure below absolute zero. Absolute zero is the point where the stress in the liquid would change from compression to tension.

In most problems in hydraulics absolute pressures are not of special interest. The important factor is usually the difference between the pressure inside a vessel and that outside, for example, and in general this would be the gage pressure. And in many other cases the atmospheric pressure acts alike at all points, and balances out. Hence gage pressures only are usually used.

EXAMPLES

16. A gage reads 20 lb. per square inch when the gage itself is surrounded by the atmosphere. If the air surrounding the gage be exhausted to a vacuum of 20 in. of mercury while the real pressure of the fluid on the inside of the gage tube remains the same, what will be the reading of the gage?

Ans. 29.7 lb. per square inch.

17. If the barometer reads 30 in. of mercury and a vacuum gage reads 5 in. of mercury, what is the absolute pressure?

Ans. 25 in.

18. Instruments for Measuring Pressure. *Gage.*—The familiar pressure or vacuum gages have already been referred to and the combination of the two is shown in Fig. 6. In this type of instrument a curved tube is caused to change its curvature by changes of pressure within the interior of the tube. The moving end of the tube then rotates a hand by means of some intermediate links. It is usually assumed that the pressure indicated by the gage is that existing at the center of the gage. Thus the location of the center of the gage should always be

taken into consideration. For instance, referring to Fig. 8, the pressure at A is the gage reading plus the distance z. If the gage reads pounds per square inch, as is customary,

$$p_A = 2.308p'' + z.$$

Piezometer Tube.—A piezometer tube is a simple device for measuring moderate pressures. It consists of a tube in which the liquid can freely rise, without overflowing, until equilibrium is established. To prevent error due to capillarity the diameter of the tube should be at least 0.5 in. The height of the surface of liquid in the tube will give the pressure desired directly. It should be noted that if the water, whose pressure is desired, is flowing, the true pressure can be obtained only by having the axis of the tube at the point of connection perpendicular to the stream flow, and, furthermore, the interior opening should be smooth and free from all projections. If the end of the pipe projects into the stream, as in the case of the fourth tube in Fig. 99, the pressure read will be too low.

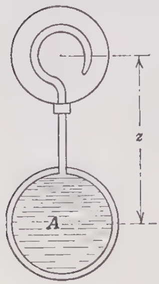


Fig. 8.

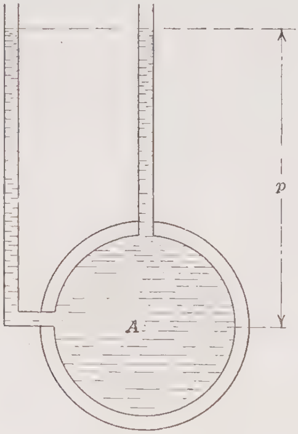


Fig. 9.—Piezometer.

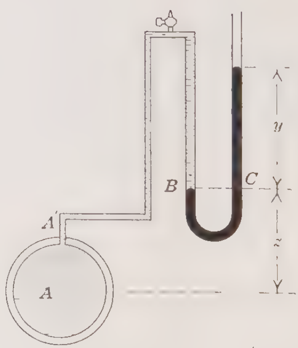


Fig. 10.

Mercury U-tube.—For high pressure the water piezometer is not suited and some modification must be adopted. The mercury U-tube shown in Fig. 10 may then be used. If s is the specific gravity of the mercury (or whatever liquid may be employed)

the pressure at the point C is sy. This is also the pressure at B, but the pressure at A is greater than this by the amount z, if the tube from A' to B is filled with water. If it were filled with air, then, neglecting the slight weight of the air within the tube, the pressures at B and at the surface of the water at A' would be equal. In practice it would be difficult to insure the absence of air, and if the tube were partially filled with air and partially with water it would be troublesome to make correction, and accuracy would be impossible unless it were known just what proportion

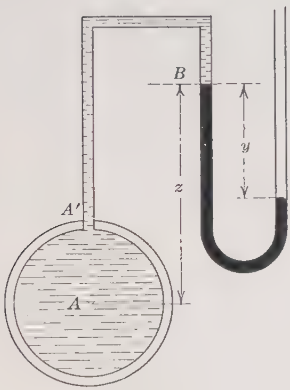


FIG. 11.

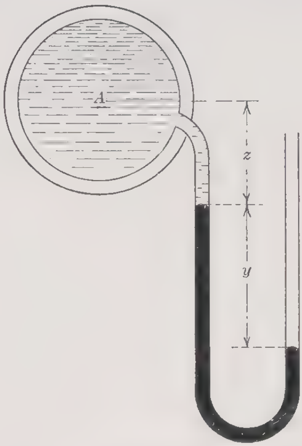


FIG. 12.

of the tube was filled with water and what with air. It is therefore desirable to provide some means of permitting all the air to escape and its place to be taken by water. If the connecting tube in Fig. 10 is filled with water the pressure at A is

$$p_A = z + sy.$$

In measuring a vacuum y must be interpreted as a negative quantity in Fig. 11 so that, if the tube is filled with water,

$$p_A = z - sy.$$

If this connecting tube from A' to B were filled with air then the correction for the height above A' would be negligible, but it is difficult to insure this being filled with air and error will be introduced if it is not. Thus the arrangement in Fig. 12 is much

better, as that permits no air to collect in the tube and introduce errors in the readings. In this case z is negative so that

$$p_A = -z - sy.$$

Differential Gage.—The differential gage is used for measuring differences of pressure only. One form of this is shown in Fig. 13 (a). Assuming the entire connecting tubing to be filled with water except that portion of the U that is filled with the

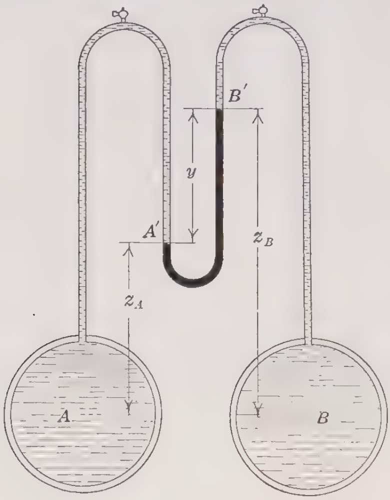


FIG. 13 (a).

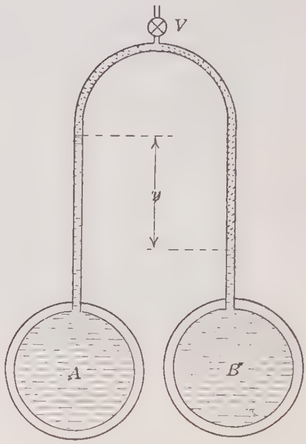


FIG. 13 (b).

denser liquid, such as mercury for instance, the pressure at A' will exceed that at B' by the amount sy. That is

$$p_A' - p_B' = sy.$$

But

$$p_A' = p_A - z_A$$

and

$$p_B' = p_B - z_B.$$

Substituting these values

$$\begin{aligned} p_A - p_B &= sy + z_A - z_B \\ &= sy - y = (s - 1)y. \end{aligned}$$

In the differential manometer the left-hand column of mercury, or whatever it is, has a column of water of height y resting upon it that is not balanced by a corresponding amount on the right-hand column, hence the pressure difference is not sy alone.

It is obvious that, if A and B are not at the same elevation, the pressures computed by the equation above must be corrected for the difference in level.

The differential manometer, when filled with a heavy liquid, such as mercury, is suitable for measuring large differences in pressure. For small pressure differences a liquid lighter than water may be used, and in this case the manometer is arranged as shown in Fig. 13 (*b*). Naturally the liquid must be something that will not mix with the water. By the same method of analysis as the preceding it may be shown that

$$p_A - p_B = (1 - s')y$$

where s' is a specific gravity whose value is less than 1. As the density of the liquid approaches that of water the value of $(1 - s')$ approaches zero, and very high values of y are obtained for small pressure differences. Thus the instrument becomes very sensitive.

For conditions intermediate between these two extremes, it is often very satisfactory to use air instead of another liquid and to pump the air into the manometer in Fig. 13 (*b*) through valve V until the pressure is such as to bring the tops of the two water columns to the desired elevations. Any change in this air pressure raises and lowers both water columns by the same amount so that the difference between them is constant. In this case the value of s' may be considered as zero, since the density of air is relatively negligible, and the difference in pressure is given by y direct.

EXAMPLES

20. Two vessels are connected to a differential manometer using mercury (specific gravity = 13.57), the connecting piping all being filled with water. When the mercury reading is 36 in., what is the pressure difference in feet of water? If another type of manometer were used in which was a liquid whose specific gravity was 0.8, what would be the pressure difference in feet of water corresponding to a differential reading of 36 in.? What would it be if $s' = 0.95$?

Ans. 37.7 ft., 0.6 ft., 0.15 ft.

21. The pressure in A of Fig. 13 is 20 lb. per square inch, while that in B is 18 lb. per square inch. The elevation of B is 10 ft. above A. What will be the differential reading in inches of mercury?

Ans. $y = -5.12$ in.

19. The Hydraulic Press.—The most important device operating upon the principle of equal transmission of intensity of pres-

sure in all directions is the hydraulic press. If in Fig. 14 a force P_1 be applied to the small piston whose area is A_1 the intensity of pressure throughout the whole volume of liquid will be increased by the amount $p' = P_1/A_1$. Then the total additional force exerted upon the face of the large piston will be $p'A_2 = (P_1/A_1)A_2 = P_1(A_2/A_1)$. It is thus seen that a small force exerted on the smaller piston is enabled to oppose a much greater load on the large piston. If G_1 and G_2 denote the weights of the pistons while z is the difference in elevation of their faces, the result is, for equilibrium

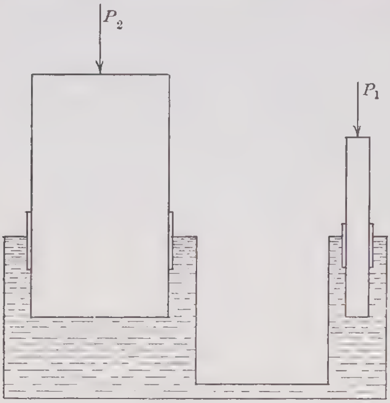


Fig. 14.—Hydraulic press.

$$\frac{P_1 + G_1}{A_1} = \frac{P_2 + G_2}{A_2} \pm wz.$$

Since the volume of liquid in the vessel must remain constant, it follows that the distance moved by the larger piston must be much less than that moved through by the smaller piston.

EXAMPLE

22. In Fig. 15 the diameter of the small piston is ¾ in. and that of the large one is 20 in. The big plunger weighs 1,000 lb. and sustains an external load of 6,000 lb. The liquid used is water. What total force P applied to the small piston will secure equilibrium? When the small piston has descended 10 ft. how far will the plunger have been raised?

Ans. 4.12 lb., 0.169 in.

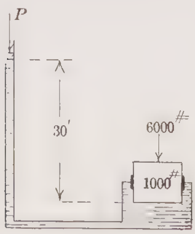


Fig. 15.

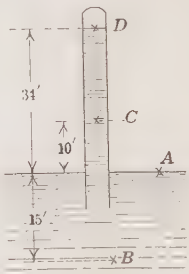


Fig. 16.

20. PROBLEMS

23. In Fig. 16 what are the values of absolute pressure at A, B, C, and D, assuming the liquid to be water? What are the values of gage pressure?

What is the value of the vacuum at C? (Give answers in feet of water, pounds per square inch, and inches of mercury in each case.)

24. In Fig. 17 the cylinder is 2 ft. in diameter and the weight of the piston and load is 4,000 lb. What will be the gage reading in pounds per square inch? If the mercury manometer in Fig. 17 reads 35 in., how far is the top of the lower mercury column below the piston? If the manometer remained

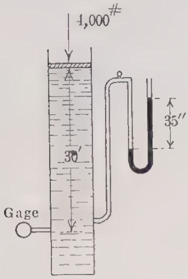


FIG. 17.

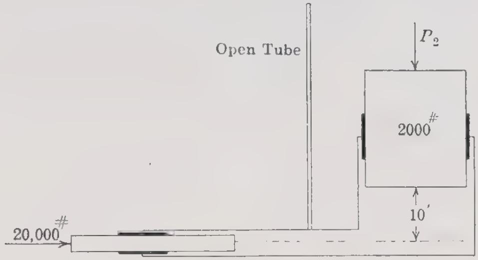


FIG. 18.

at this same place but the connection were made to the tank at a different level, would the mercury reading change?

Ans. 21.9 lb. per square inch, 19.2 ft.

25. The small piston in Fig. 18 has a diameter of 3 in. Neglecting friction, when a force P_1 of 20,000 lb. is applied to it, what will be the force P_2 that can be exerted by the plunger with a diameter of 24 in.? To what height would water rise in the piezometer tube shown?

Ans. 1,276,000 lb., 6,520 ft.

CHAPTER III

HYDROSTATIC PRESSURE ON AREAS

21. Total Pressure on Plane Area.—Since fluids at rest are being dealt with, no tangential forces can be exerted and hence all pressures are normal to the surfaces in question. If the pressure were uniformly distributed over an area, the total or resultant pressure would be the product of the area, and the intensity of pressure and the point of application of the force would be the center of gravity of the area. In general the intensity of pressure is not uniform, hence further analysis is necessary.

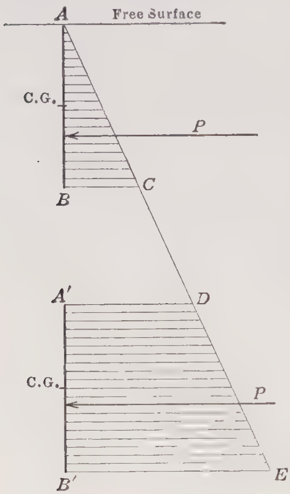


Fig. 19.

In Fig. 19 consider a vertical plane whose upper edge lies in the free surface. Let this plane be perpendicular to the plane of the paper so that AB is merely its trace. The intensity of pressure will vary from zero at A to BC at B. It will thus be seen that the total pressure P will be the summation of the products of the elementary areas and the intensities of pressure upon them. It is also apparent that the resultant of this system of parallel forces must be applied at a point below the center of gravity of the area, since the center of gravity of an area is the point of application of the resultant of a system of uniform parallel forces. If the plane be immersed to $A'B'$ the intensity of pressure varies from $A'D'$ at A' to $B'E$ at B'. Since the proportionate change of intensity of pressure from A' to B' is less than before, it is clear that the center of pressure will approach nearer the center of gravity.

In Fig. 20 let MN be the trace of a plane making any angle θ with the horizontal. The view to the right is the projection of this area upon a vertical plane which is also normal to the plane

containing MN. Let z be the depth of any point and y be the distance of any point from OX, the axis of intersection of the plane, produced if necessary, and the free surface. The coordinates of the center of gravity of the area may be denoted by $\bar{z}$ and $\bar{y}$ respectively. The coordinates of the point of application of the resultant force P are z' and y'.

Consider an element of area dA so chosen that the intensity of pressure may be uniform over it. Such an elementary area may be represented by a horizontal strip across the plane. If x denotes the width of the plane at any point, then

$$dA = x\,dy,$$

and, since z is constant, the intensity of pressure is constant. Thus

$$p' = wz,$$
$$dP = p'dA = wz\,x\,dy.$$

Since

$$z = y \sin \theta,$$

then

$$P = w\int zx\,dy = w \sin \theta \int xy\,dy. \tag{6}$$

If x can be expressed as an algebraic function of y, the above may be integrated, and the value of the total pressure in any given case may be found. Suppose, however, that the area is irregular in outline, as shown in Fig. 20, so that, while simultaneous values

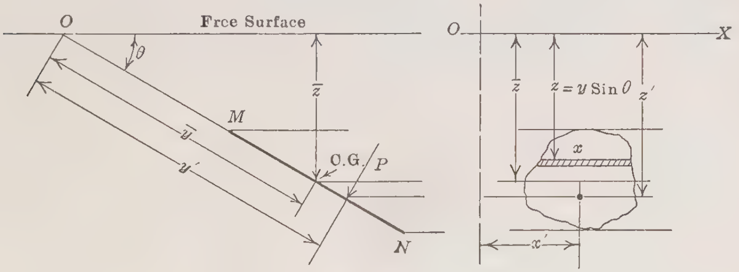


FIG. 20.

of x and y may be found by measurement, the relationship between them cannot be expressed in any simple way. Or suppose there is an algebraic equation which applies, but the resulting expression is such that it cannot readily be integrated. In any practical case, a numerical value of the integral can be found by the following graphical means.

Multiply simultaneous values of x and y and plot the product xy against values of y, as is shown in Fig. 21. The area under this curve is the value of the integral.[1]

In case the area is a simple geometrical figure, as it usually is, so that its area and the location of its center of gravity can readily be computed, a simpler procedure than that just outlined

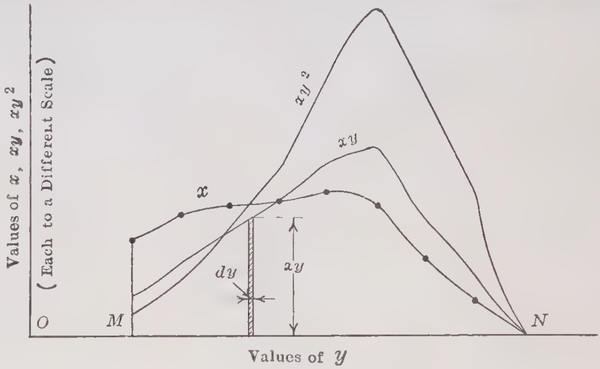


FIG. 21.

may be followed. Since $x\,dy$ is the elementary area dA, Eq. (6) may be written

$$P = w \sin \theta \int y\,dA$$

but $\int y\,dA$ is known to be $\bar{y}A$.
Hence

$$P = w \sin\theta\, \bar{y}A.$$

But $\bar{y} \sin \theta = \bar{z}$, which is the depth of the center of gravity of the area below the free surface. Therefore

$$P = w\bar{z}A. \tag{7}$$

Thus the total pressure on any plane area may be found by multiplying the area by the depth of its *center of gravity* and by the density of the liquid. The value of this quantity is independent of the angle of inclination of the plane.

[1] An element of area of this figure may be seen to have a length xy and a width dy. Hence the summation of all such elementary areas must be the value of $\int (xy)dy$.

In general if u and v are coordinates of an area and u is plotted to such a scale that 1 in. = a units, while the scale for v is 1 in. = b units, then the scale for the area is 1 sq. in. = ab units.

It is not necessary actually to plot the area and measure it by a planimeter, as the value of the area can be computed by various methods, such as Simpson's rule, or Durand's rule.

EXAMPLES

26. A rectangular plane area is 5 by 6 ft., the 5 ft. side being horizontal and the 6 ft. side vertical. Determine the magnitude of the resultant pressure when the top edge is: (*a*) in the water surface; (*b*) 1 ft. below the surface; (*c*) 100 ft. below the water surface.

Ans. (*a*) 5,620 lb., (*b*) 7,500 lb., (*c*) 193,000 lb.

27. Assume an area which is at an angle of 30 deg. with the horizontal and the top point of which is at a vertical distance of 2 ft. below the water surface. Let the following be values of the horizontal widths in feet taken at intervals of 6 in. along the plane: 0, 2, 3, 3.4, 3.5, 3.6, 3.6, 3.6, 3.5, 3.4. Determine the magnitude of the total pressure on the area.

Solution: The initial value of y is 4 ft. and is multiplied by the first width which happens to be 0 in this case, though a finite value might just as well appear, depending upon the shape of the area. Doing the same for each width the following values of xy are obtained: 0, 9, 15, 18.7, 21, 23.4, 25.2, 27, 28, 28.9. These values may be plotted against corresponding values of y and the area under the curve will be the value of $\int xy\,dy$ or $\bar{y}A$. Note that the area just obtained is the value of the *moment* of the plane area with which our problem is concerned with respect to the axis O of Fig. 20.

Solving by Simpson's rule for the sake of illustration though, unlike Durand's, it may be used only with an odd number of ordinates and hence adds one more complication here:

$$(0 + 28) + 4(9 + 18.7 + 23.4 + 27) + 2(15 + 21 + 25.2) = 462.8.$$

This is seen to be obtained by multiplying even-numbered ordinates by 4, and odd ones by 2, except the first and the last. This sum should then be multiplied by one-third of the interval, which is ⅙ ft. The result is 77.1. To this must be added the trapezoidal area left over, whose width is ½ ft. and whose two bases are 28 and 28.9. This area is 14.2, giving us a total area whose value is 91.3 cu. ft., since these are the units here involved. Hence the total pressure is

$$P = 62.4 \times 0.500 \times 91.3 = 2{,}850 \text{ lb.}$$

28. Find the area and the location of the center of gravity of the plane figure in the preceding problem.

Ans. 14.1 sq. ft., $\bar{y} = 6.48$ ft.

22. Depth of the Center of Pressure.—The point of application of the resultant force on an area is called the center of pressure. The line of action of a force is usually located by taking moments. In this case it is convenient to take OX in Fig. 20 as the axis of moments. On any element of area dA the total pressure is

$$dP = w \sin\theta\, xy\, dy$$

and its moment is

$$ydP = w \sin\theta\, xy^2 dy.$$

The moment of the resultant force being the sum of the moments of its components, if y' is the distance to the center of pressure,

$$y'P = w \sin \theta \int xy^2 dy.$$

As in the preceding article, this integral may be evaluated in a numerical case by finding the value of an area whose ordinates are values of xy^2. The analytical solution, on the other hand, gives the following: Since $xdy = dA$ and the integral of y^2dA is the moment of inertia of the area A about the axis O

$$y'P = w \sin\theta \, I_o.$$

This may be divided by the value of P in the preceding article with the result that

$$y' = \frac{w \sin\theta \, I_o}{w \sin\theta \, \bar{y} \, A} = \frac{I_o}{\bar{y}A}. \tag{8}$$

That is, the distance of the *center of pressure* from the axis, where the plane, or plane produced, intersects the water surface, is obtained by dividing the moment of inertia by the static moment of the area A about the same axis.[1]

This may also be expressed in another form, by noting that $I_0 = \bar{y}^2A + I_g$, where I_g is the moment of inertia of an area about its own gravity axis, and that I_g in turn is equal to $k_g{}^2A$, where k_g is the radius of gyration about the gravity axis, hence

$$y' = \frac{\bar{y}^2A + k_g{}^2A}{\bar{y}A} = \bar{y} + \frac{k_g{}^2}{\bar{y}}. \tag{9}$$

From these equations it may be seen that the location of the center of pressure is independent of the angle θ, that is, the plane area may be rotated about the axis OX without affecting the location of the center of pressure. However, this will not hold for $\theta = zero$ since the value of P would also be zero.

[1] In Fig. 21 the area under the curve of values of x is equivalent to the actual area A of the plane surface in question, though it may be altered slightly in appearance, since one side of the actual area becomes here the straight line MN. If OM here represents the same distance as in Fig. 20, then the distance from the vertical axis through O to the center of gravity of the area under the x curve is the value of $\bar{y}$ for the area A. And the distance from this axis to the center of gravity of the area under the xy curve is the value of y' for the area A. Since each area is the moment of the area of lower degree, it follows that the value of y' may be obtained by dividing the area under the xy^2 curve by that under the xy curve, while the value of $\bar{y}$ may be obtained by dividing the area under the xy curve by that under the x curve, paying due regard to the scale values used in plotting.

From Eq. (9) it may also be seen that the center of pressure is always below the center of gravity. Also as the depth of immersion is increased for a given value of θ, the distance $\bar{y}$ increases. But as k_g remains constant in value it may be seen that the last term in Eq. (9) becomes relatively small, hence y' approaches $\bar{y}$ in value. The same thing would be true if the depth of the center of gravity $\bar{z}$ remained constant while the plane was rotated so as to approach a horizontal direction. (This is entirely different from rotation about the axis OX. since $\bar{y}$ no longer remains constant.)

EXAMPLES

29. Determine the depth of the center of pressure for the three cases in problem 26.

Ans. 4.00, 4.75, 103.03 ft.

30. Determine the center of pressure for the case in problem 27, using Simpson's rule.

Ans. $y' = 6.68$ ft.

23. Lateral Location of Center of Pressure.—For most practical problems the depth of the center of pressure is all that requires solution, since the areas dealt with are usually such that a straight line can be drawn through the centers of all horizontal lines. In such cases the center of pressure is seen to lie on this line. But in case this is not so, it would be necessary to compute x' as in Fig. 20, x' being measured from any axis parallel to trace MN.

Again moments, as in the preceding article are employed. If x is the distance to the center of gravity of an element from the axis in question, the moment of dP is

$$xdP = wxy \sin\theta \, dA.$$

Hence the value of x' is

$$x = \frac{\int xdP}{\int dP} = \frac{w \sin\theta \int xydA}{w \sin\theta \int ydA}$$

$$= \frac{\int xydA}{yA}. \tag{10}$$

This equation differs from Eq. (7) simply because $\int xydA$ is given instead of $\int y^2dA$. The latter quantity is more frequently met with; it is given a name, symbolized by the letter I, and values of I for different areas can usually be found in tables.

The former expression is called "product of inertia"; is symbolized by the letter J, but owing to the infrequent use that is made of it values of J cannot usually be obtained save by integration. Lacking the knowledge of the value of J for any area, simply proceed to evaluate $\int xydA$ just as $\int y^2dA$ should be evaluated in case the value of I for the area in question was not known.

It will be found that reduction formulas can be used here as with moments of inertia. If J indicates the product of inertia with respect to the intersection of any two axes, while a and b are the coordinates of the center of gravity of an area about which the product of inertia is J_g, it will be found that

$$J = J_g + Aab.$$

In using Eq. (10) it must be noted that y is to be measured as in Fig. 20, while x may be measured from any axis in the plane of the figure and perpendicular to OX.

EXAMPLES

31. Given a right triangle with height, h, and base, b, with its vertex in the water surface and its plane vertical. Find the value of y' and then determine x': (a) by inspection; (b) by calculus.

Ans. $y' = \frac{3}{4}h$; $x' = \frac{3}{8}b$.

32. Find the center of pressure on an area which is a quadrant of a circle. It is placed in a vertical plane and one edge lies in the water surface.

Ans. $\bar{y} = 4r/3\pi$; $y' = 3\pi r/16$; $x' = 3r/8$.

24. Resultant Thrust on Plane Areas.—So far, the total pressures on one side of a plane area alone have been dealt with. Of course, when the area is completely immersed in a fluid as shown in some of the previous illustrations, the total pressure on one side is balanced by that on the other and the net effect is zero. But when the two sides are not subjected to the same pressure, there is a resultant thrust whose value is desired.

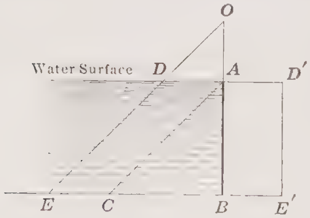


Fig. 22.

So far the surface of the liquid has been considered as being free from all pressure. Thus, in Fig. 22 the intensity of pressure should be considered as varying from zero at A to BC at B. But in reality there is some pressure, in general, from the atmosphere

acting upon the water surface equivalent to a height of about 34 ft. of water, and thus the true free surface might really be at point O, the distance AO being equal to the height of the water barometer. The absolute intensity of pressure upon the left-hand side of the plane AB, therefore varies from AD to BE. But in practical applications the difference between the pressure on the left-hand side and that on the right-hand side is desired. But the pressure on the right-hand side is that due to the atmosphere and its intensity is uniform from A to B being equal to AD'. But $AD' = AD = CE$. Hence atmospheric pressure is added alike to both sides, and it is useless to consider it. Therefore, atmospheric pressure is neglected altogether, and the water surface is treated as a true free surface in most calculations.

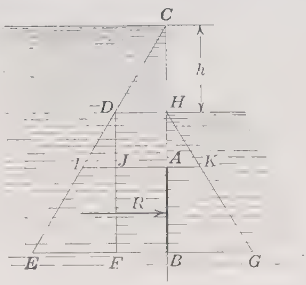


Fig. 23.

Suppose there is an area, such as AB in Fig. 23, with a fluid pressure on both sides but of different intensities. Of course, the magnitudes of the total pressures on both sides of the area could be computed and the difference would be the resultant desired. But it would also be necessary to find the centers of pressure on both sides and then locate the line of action of the resultant of these two forces. The following analysis will indicate a much easier solution.

At A the intensities of pressure on the two sides are AI and AK. If IJ be laid off equal to AK the net difference in the intensity of pressure will be AJ. In similar manner at B the net intensity of pressure is BF. And it is readily seen that, since CDE and HKG make the same angle with the vertical, the values of HD, AJ, and BF are equal. Thus the resultant intensity of pressure on the area AB, is uniform and equal to HD in value. But HD is the intensity of pressure at the depth h. Hence the resultant thrust on any area with both sides completely covered by the same liquid is

$$R = whA \tag{11}$$

where h is the difference in level of the two liquids. And since the net intensity of pressure is uniform, the resultant thrust will act through the center of gravity of the plane area.

EXAMPLES

33. Suppose that a rectangular area is 2 ft. wide by 3 ft. high and that its upper edge lies in a water surface. What twisting moment will be necessary in a shaft through A (Fig. 22), perpendicular to the plane of the paper, to withstand the water pressure? It will be assumed that the gate receives no support save what the shaft affords, and that atmospheric pressure acts alike on the water surface and the right-hand side of the gate.

Ans. 1,123 ft.-lb.

34. Suppose that the right-hand side of the gate in problem 33 is under a vacuum of 30 in. of mercury, and that the barometer reading is 30 in. of mercury. What twisting moment would be required?

Ans. 20,200 ft.-lb.

35. Suppose that the barometer reads 30 in. of mercury and that the right-hand side of the gate in problem 33 is under a vacuum of 20 in. of mercury. What twisting moment would be required?

Ans. 13,800 ft.-lb.

36. Suppose that the barometer reads 30 in. of mercury, that the right-hand side of the gate in problem 33 is under atmospheric pressure, while the surface of the water is under a gage pressure of 50 lb. per square inch. What twisting moment would be required?

Ans. 65,900 ft.-lb.

37. Suppose in Fig. 23 that AB is a circular gate of 3 ft. diameter, that $BC = 10$ ft. and $BH = 4$ ft. Find: (*a*) magnitude and line of action of total pressure on left-hand side only; (*b*) magnitude and line of action of total pressure on right-hand side only; (*c*) resultant thrust on gate.

Ans. (*a*) 3,747 lb., 1.566 ft. below top of gate; (*b*) 1,102 lb., 1.725 ft. below top of gate; (*c*) 2,645 lb., 1.500 ft. below top of gate.

25. Horizontal Pressure on Curved Surface.—On any curved or irregular area in general, such as that whose trace is AB in Fig. 24, the pressures upon different elements are different in direction and an algebraic or calculus summation is impossible. Hence Eq. (7) can be applied only to a plane area. But the component of pressure in certain directions may be found. Thus if each dP is multiplied by $\cos\ \theta$, θ being a variable angle which each elementary force makes with the horizontal, the total horizontal force would be

$$P_x = \int dP \cos \theta. \tag{12}$$

In general it will be tedious to integrate the latter and often practically impossible. Hence the following procedure may be employed.

Project the irregular area in question upon a vertical plane, the trace of the latter being $A'B'$. The projecting elements are AA', BB', etc. It is seen that these projecting elements, which

are all horizontal, enclose a volume whose ends are the vertical plane $A'B'$ and the irregular area whose trace is AB. This volume of liquid is in equilibrium under the action of the following forces. Upon the vertical plane at the left there is a force P', gravity G' acts upon the volume and is vertical, the pressures on the projecting elements are all normal to these elements, hence normal to P'. Then there are the pressures upon the area in question at the right-hand end, the horizontal component of pressure being represented by P_x and the vertical component by P_y. Since a condition of equilibrium exists, the sum of all the

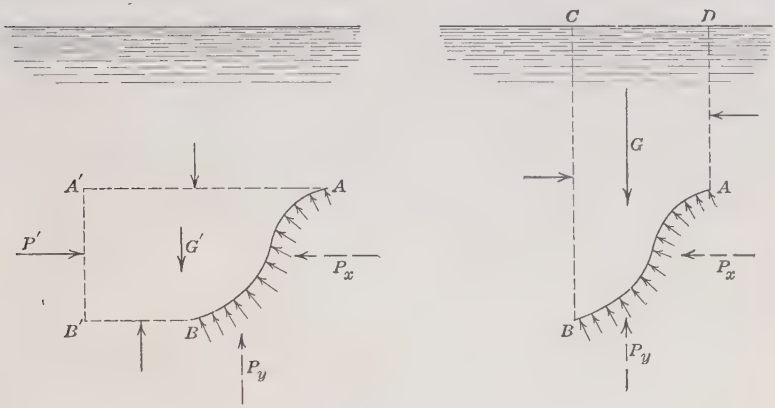


FIG. 24.

forces in any direction must be equal to zero. But in a horizontal direction the only forces are P' and P_x.
Hence

$$P_x = P'. \tag{13}$$

That is the component, in any given horizontal direction, of the pressure upon any area whatever is equal to the pressure upon the projection of the area upon a vertical plane which is perpendicular to the given horizontal direction. The lines of action must also be the same.

26. Vertical Pressure on Curved Surface.—The vertical component of pressure on an irregular surface can be found by a method similar to that for the horizontal pressure. Thus in Fig. 24 if a volume of liquid is taken, of which the area in question forms the base and vertical elements such as AD and BC form the sides, it is found that the following forces are acting. Considering CD a free surface the pressure on the upper face is zero.

The pressure on the lower face is composed of the two components P_x and P_y. Gravity, G, is the only other vertical force, the pressures on the sides all being horizontal. Summing up the vertical forces and equating to zero,

$$P_y = G. \tag{14}$$

Hence the vertical component of pressure on any area whatever is equal to the weight of that volume of liquid which would extend vertically from the area to the free surface.

27. Component of Pressure in Any Direction.—In general, the component of pressure in any direction aside from horizontal and vertical cannot be found, since the weight of the volume of liquid, such as $AA'B'B$ in Fig. 24 would have to enter the equation. But if the depth of immersion is great so that the pressures on AB and $A'B'$ are great compared with the weight G' the latter may be neglected. Hence in such cases only, the component of pressure in any direction may be taken as the pressure upon an area projected in that direction upon a plane which is perpendicular to the given direction.

Of course with a plane area the component of pressure in any direction may be found by multiplying P by the proper function of some angle. Or it may be convenient to find it by the methods of Arts. 25 and 26. Also for a plane area, since $P \cos \theta = (w\bar{z}A) \cos \theta$, it may be seen that the component of pressure is the same as the pressure upon an area of value $A \cos \theta$, provided the center of gravity of such area be the same depth as the center of gravity of the given plane.

28. Resultant Pressure on Curved Surface.—In general there is no single resultant pressure on an irregular surface, for a system of non-parallel and non-coplanar forces does not usually reduce to anything simpler than two single forces. Thus in general P_x and P_y are not in the same plane and hence cannot be combined. But in some special cases of symmetrical surfaces, these two components will lie in the same plane and hence can be combined into a single force.

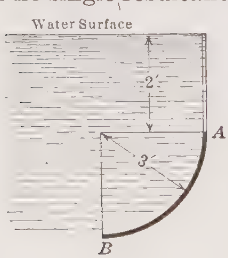


Fig. 25.

EXAMPLE

38. In Fig. 25 is shown a quadrant of a circular cylinder, AB, whose length perpendicular to the plane of the paper is 4 ft. (*a*) Find the horizontal com-

ponent of pressure. (b) Find the vertical component of pressure. (c) Find the magnitude and direction of the resultant water pressure. (d) What locates its line of action?

Ans. (a) 3,262 lb., (b) 2,622 lb., (c) $z' = 3.7$ ft., (d) $x' = 1.38$ ft. from B.

29. Pipes under Pressure.—If the internal pressure in a cylindrical pipe is great enough to be considered in determining the thickness of pipe wall necessary, it will be large enough so that the weight of the water may be disregarded. Hence according to Art. 27 we may compute the resultant pressure in any direction. Suppose that in Fig. 26, we pass a plane XY through a diameter of the pipe as shown. The total pressure on one-half of the pipe in any direction, such as that normal to XY, will evidently be $p' \times 2r \times l$, l being any length of pipe. This follows directly from Art. 27 or may be seen from the fact that the thrust of the water on the wall of the pipe normal to XY must be balanced by the thrust of the water on the plane XY.

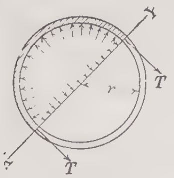


FIG. 26.

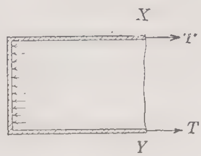


FIG. 27.

This pressure will tend to rupture the pipe across the plane XY and is resisted by the tensions in the walls of the pipe, such as T. Evidently $2T = 2p'rl$. If the thickness of the pipe wall be denoted by t, and the stress induced in it by S_h, then $T = S_htl$. Hence

$$S_ht = p'r. \tag{15}$$

From Eq. (15) the thickness of wall necessary may be computed for any allowable unit tensile stress. However, it is well to note that p' should be the maximum intensity of pressure that may occur and in case of water hammer these intensities are much greater than the static pressures alone. Also it may often be found that Eq. (15) gives entirely too thin a wall to stand ordinary handling and to allow for a certain amount of corrosion. In practice p' is, therefore, increased to allow for possible water hammer and the thickness determined by Eq. (15) is then increased to a value necessary for these other reasons. The tension in the case shown is called *hoop tension*.

Referring to Fig. 27 it may be seen that a cylindrical pipe may also be ruptured by forces parallel to the axis. Thus the pressure on the blank end is balanced by the tension in any section such as XY. The total pressure, assuming it to be of uniform intensity, is $p' \times \pi r^2$. And the tension across a section XY is $T = S_l \times 2\pi rt$. Hence, equating these two,

$$2S_l t = p'r. \tag{16}$$

This stress is called *longitudinal tension* and it may be seen that it is one-half the hoop tension.

For cylinders with thin walls these formulas will hold, since they assume uniform intensity of stress across the metal. But with thick walls they do not hold. In the case of hoop tension in a cylinder with thick walls it is usually assumed that the intensity of stress is a maximum at the inner face and decreases to zero at the outside of the wall. Also the elasticity of the material enters into the hypothesis. John Sharp[1] gives the following empirical formula for hoop tension in a cast-iron cylinder with thick walls

$$S \log_e \frac{r_2}{r} = p' \tag{17}$$

where r_2 = external radius and r = internal radius. For wrought-iron and steel cylinders he gives the empirical expression

$$S\left[\left(\frac{r_2}{r} - 1\right) + \log_e \frac{r_2}{r}\right] = 2p'. \tag{18}$$

Equation (15) with S understood as compressive stress would also hold for external pressure provided the pipe remained truly cylindrical. But actually it may become slightly distorted from the cylindrical form and then there is a possibility of sudden collapse. A large thin tube which can stand a high internal pressure can withstand only a small external pressure. All formulas for determining the strength of pipes against external pressure are purely empirical. So far no satisfactory expression has been deduced, and sufficient data are lacking.

30. Buoyant Force of the Water and Flotation.—Considering the body $EHDK$ immersed in a fluid in Fig. 28, it is seen that it is acted upon by gravity and the pressures from the surrounding fluid at least. In addition there may be other forces applied. On the upper surface of the body the vertical component of the pressure, P_y, will be equal to the weight of the volume of fluid $AEHDC$. In similar manner the vertical component of the

[1] "Some Considerations Regarding Cast-iron and Steel Pipe."

pressure on the under surface, P'_y, will be equal to the weight of the volume of fluid $AEKDC$. It is evident that P'_y is greater than P_y and that the total vertical force exerted by the fluid is upward and is equal in magnitude to

$$P'_y - P_y = \text{weight of volume } AEKDC - \text{weight of volume } AEHDC.$$

But the difference between these two volumes is the volume of the body $EHDK$. Hence for any body immersed in a fluid such as water the buoyant force of the water is equal to the weight of the water displaced.

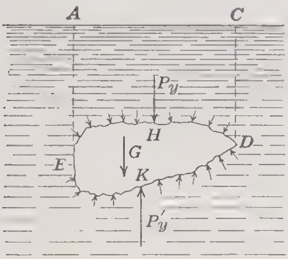


FIG. 28.

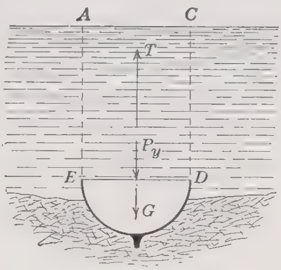


FIG. 29.

If the body remains in equilibrium in the position shown in Fig. 28, when no other forces are acting, it is seen that $G = P'_y - P_y$. Hence the body must be of the same density as the fluid in which it is immersed. If it is lighter than the fluid, a downward force will have to be applied whose value is $B - G$, B being the buoyant force of the fluid. If the body is denser than the fluid, it will have to be supported by a force whose value is $G - B$. But if the body rests on the bottom of a body of fluid (Fig. 29) in such a way that the fluid does not have access to the under side, there will be no buoyant effect, for then $P'_y =$ zero. Thus in the case of a ship, for example, sunk in the mud at the bottom of a body of water, the pull T necessary to raise the ship is not only the weight of the ship but also the weight of the entire volume of water resting on top of it. Thus in Fig. 29, $T = G + P_y$.

If no external forces are applied to a body which is lighter than the fluid, it will float on the surface, such portion of its volume being immersed as is necessary to displace an amount of fluid equal in weight to the weight of the body.

If the body is slightly heavier than the fluid, it will sink. If it is less compressible than the fluid and there is sufficient depth, it will sink until such a depth is reached that the density of the fluid is equal to its own density. If it is more compressible than the fluid its own density will be increased more rapidly than that of the water and it will sink to the bottom.

EXAMPLES

39. A body whose volume is 2 cu. ft. weighs 200 lb. What will be the force necessary to sustain it when it is immersed in fresh water? In ocean water?

Ans. 75.2 lb., 72 lb.

40. In Fig. 30 the cube A is 12 in. along each edge and weighs 100 lb. It is attached to the square prism B which is 6 by 6 in. by 8 ft. and weighs 30 lb. per cubic foot. What length of B will project above the water surface?

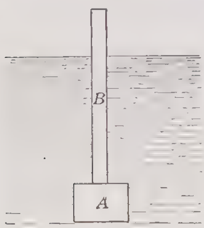


FIG. 30.

Ans. 1.76 ft.

41. A balloon weighs 250 lb. and has a volume of 10,000 cu. ft. When it is filled with hydrogen which weighs 0.0056 lb. per cubic foot what load will it support in air which weighs 0.08 lb. per cubic foot?

Ans. 494 lb.

42. The specific gravity of a solid is 0.8. What portion of its volume will be above the surface of the water upon which it floats?

43. A body weighs 50 lb. and has a volume of 4 cu. ft. What vertical force is necessary to sink it beneath the surface of the water?

31. Metacenter.—For a body floating on the surface of the water, such as in Fig. 31, there are only the two vertical forces, its weight G and the buoyant force of the water B. The latter acts through the center of gravity of the water displaced. This point is called the center of buoyancy. If the body is in equilibrium, these two forces must be in the same straight line. Suppose that by some external agency the body is rolled or displaced through some angle θ. The center of gravity is naturally unchanged in its position in the section but the center of buoyancy, in general, will change. Thus G and B constitute a couple. In Fig. 31 (b) this is a righting couple since it tends to restore the body to the upright position.

It may be seen that the line of action of B cuts the axis at point M. This point is called a *metacenter*. As the angle θ varies, the amount of this couple will vary and the point M will also change its location. The position which M approaches as θ

approaches zero is the *true metacenter*. It may be seen that if the couple is a righting couple the point M must always be above C the center of gravity. It is necessary in ship design to insure that M will be above the center of gravity for all angles of heel. Thus not only is it necessary to locate the true metacenter but

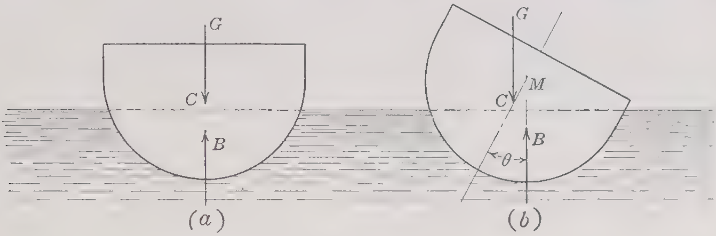


FIG. 31.

also to compute the moment of the righting couple for all values of θ which are likely to be encountered. Further consideration of this topic properly belongs to the subject of ship design.

32. PROBLEMS

44. Suppose a cylinder 1 ft. in diameter and 3 ft. high is filled with water to the top. Find the total pressure on the bottom by computing the weight of the water and then by using Eq. (7). Then assume the top of the cylinder to be covered except for a small pipe which extends vertically for some distance. Assume the diameter of this pipe to be so small that a pint of water poured into it fills it with water to a height of 20 ft. above the top of the cylinder. Find the total pressure on the bottom.

Ans. 147 lb., 1,128 lb.

45. Suppose in Fig. 20 that we have a rectangular area 5 by 6 ft., that $AB = 6$ ft., the 5-ft. edge being normal to the plane of the paper, and that $\bar{y} = 4$ ft. Find the magnitude of the total pressure and the location of the center of pressure when θ has values of 90, 60, 30, and 10 deg.

Ans. (*a*) $P = 7{,}488$ lb., $y' = 4.75$ ft.; (*b*) $P = 6{,}490$ lb.; (*c*) $P = 3{,}744$ lb.; (*d*) $P = 1{,}302$ lb.

46. Suppose that in problem 45 $\bar{y}$ was variable but that $\bar{z} = 4$ ft. Solve with values of θ of 90, 60, 30, and 0 deg.

Ans. (*a*) $P = 7{,}488$ lb., $y' = 4.75$ ft.; (*b*) $y' = 5.265$ ft.; (*c*) $y' = 8.375$ ft.; (*d*) $y' =$ infinity, $z' = 4$ ft.

47. Find the depth of the center of pressure on a vertical triangular area whose altitude is h and whose base is b if: (*a*) its vertex lies in the water surface, and its base is horizontal; (*b*) its base lies in the water surface.

Ans. (*a*) $y' = \frac{3}{4}h$; (*b*) $y' = \frac{1}{2}h$.

CHAPTER IV

APPLICATIONS OF HYDROSTATICS

33. The Gravity Dam.—One of the most important of the many applications of hydrostatics is the design of dams, of which there are several types. The gravity dam is one which depends for its stability upon its weight. A typical cross-section of such a dam is shown in Fig. 32. If the face AB is curved it will be necessary to compute the two components of the water pressure, H being equal to the pressure on a plane whose trace is $A'B$

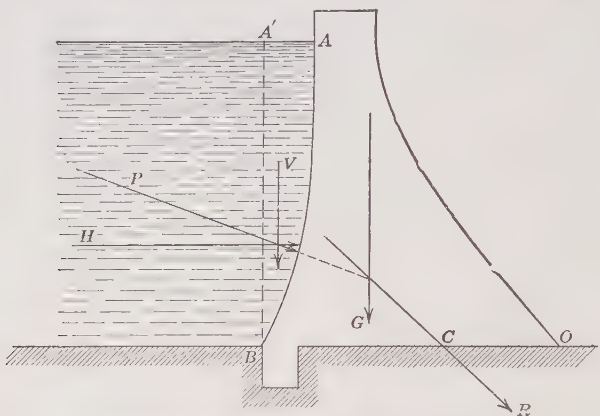


FIG. 32.—Cross-section of gravity dam.

while V is the weight of the volume of water represented by ABA'. In all computations it is customary to consider a length of dam (perpendicular to the plane of the figure) of 1 ft. Evidently the stability of a gravity dam is independent of the total length of the dam.

The total water pressure P combined with the weight of the section G gives a resultant pressure on the base whose value is R. This pressure is distributed all over the base BO but may be considered to have a single point of application C. If R be resolved into two components at the point C, evidently the value of the horizontal component must be equal to H while that

of the vertical component will equal $G + V$. By taking moments of all the forces about O it will be easy to locate the point C.

If the dam rests solidly upon impervious rock and there is no leakage of water along any plane, or if a cutoff wall at B runs down deep enough to stop percolation, and the base of the dam is well drained, the above forces are all that act upon the structure, excepting of course the support of the earth which is equal and opposite to R. But if water does have access to the under side of the dam there will be exerted upon BO a vertical upward pressure due to this. How much this may amount to depends upon conditions. Thus if water saturates the foundation but does not have an opportunity of escaping past O the whole base of the dam will be subjected to a water pressure equal to BA' in intensity. But if the water can escape past O there will be a flow of water under the dam and consequently the pressure must decrease from BA' at B to a very much smaller value at O. It is often reasonable to assume the pressure as zero at O. But in any event the admission of water to the base of the dam tends to decrease the safety of the structure.

It may be seen that the horizontal thrust of the water H is opposed solely by the friction between the dam and the foundation upon which it rests. If the coefficient of friction here be denoted by μ then it is clear that if the dam is safe against sliding the value of H must be less than $\mu(G + V)$. The factor of safety against sliding is the ratio of the latter quantity to H. Any leakage of water under the base of the dam decreases the pressure between the dam and the material upon which it rests and thus tends to decrease the frictional resistance. The frictional resistance can be increased by sinking portions of the dam into trenches such as in the case of the cutoff wall at B.

If it were possible for the dam to act as a rigid body under all circumstances it could then fail by overturning about O as an axis. It is seen that with O as a center of moments, H tends to overturn the dam but is resisted by G and V. If water pressure acted upon the base it would also tend to overturn the dam. The factor of safety against overturning is the ratio of the moment of $G + V$ to the moment of H, and the water pressure on the base, if any is allowed for. However, before a masonry dam of any size would overturn, the material along the base near O would be crushed due to the high intensity of pressure it would be under. Thus although the point C might be to the left of O in Fig. 32 so that

the structure is safe against overturning, the base would still not be safe against crushing. Hence, the second consideration of the stability of the dam is not as to whether it will or will not overturn, but is concerning the distribution of stresses along the base BO.

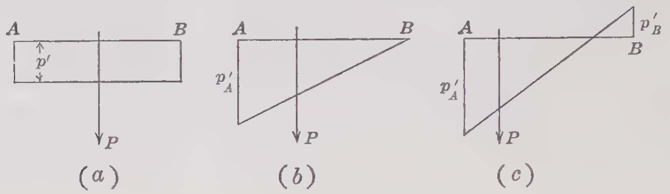
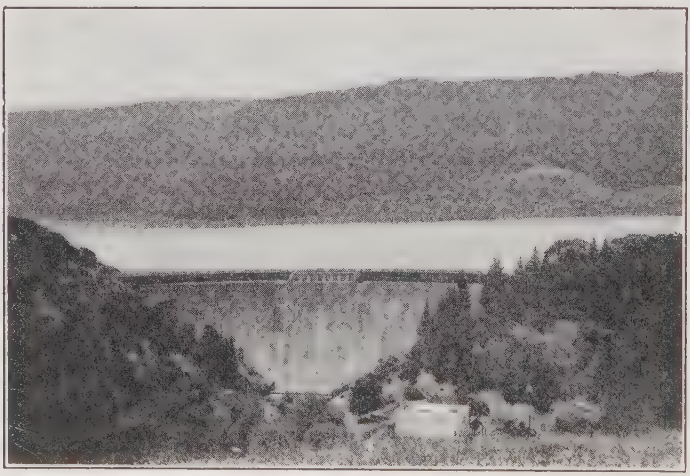


Fig. 33.

Referring to Fig. 33, a uniform intensity of stress p' distributed over an area represented by AB gives a resultant pressure P applied midway between A and B. If, however, the stress varies uniformly from p'_A at A to zero at B, the resultant P will pass through a point one-third the distance from A to B. If the total



From a photograph by the author.

Fig. 34.—Concrete dam at Crystal Springs Lake, California. 145 feet high.

pressure P has the same value in both cases, it is clear that the intensity of pressure at A is greater in the latter case than in the former. And if P is applied at a point less than one-third the distance from A to B, the intensity of stress will be still greater at A, and at B the intensity of stress p'_B will be opposite

in sign to that at A. It is thus clear that it is desirable to have the resultant pressure pass as nearly through the midpoint as possible. And if tensile stresses are to be avoided the resultant pressure must be kept within the middle third. As masonry is not supposed to endure tensile stresses, it is customary so to design the dam that the resultant pressure falls within the middle third of any section.

It is not only necessary to undertake such an analysis of the dam as a whole, but also to investigate the stability of *all* portions of the dam with respect to *any* horizontal plane. In all such studies the maximum height of water should be assumed. But also the pressures should be determined when the reservoir is empty as the inner face of the dam might then be subjected to excessive vertical stresses.

34. The Framed Dam.—Contrasted with the gravity dam is the framed dam shown in Fig. 35, which depends for its stability

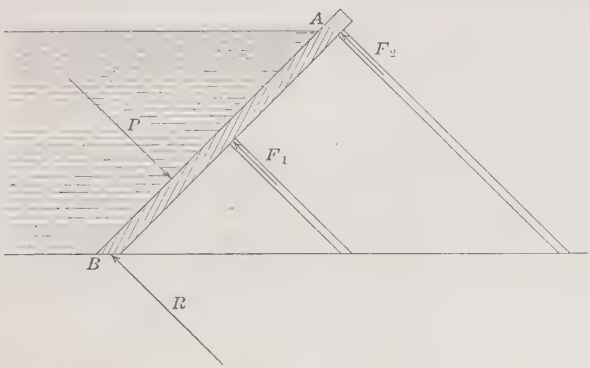


FIG. 35.—Framed dam.

upon the strength of its members. It consists of a watertight deck AB, supported by struts, trusswork, or buttresses at certain intervals along the length of the dam (perpendicular to the plane of the figure). The deck is always inclined so that the weight of the water upon it may hold the structure down and increase the factor of safety against sliding.

35. The Arch Dam.—In the case of a short high dam in a situation where firm support can be had from the walls on either side the arch dam is desirable. It is designed to withstand the water pressure by pure arch action and to transmit the pressures to the abutments at either end. The material in an arch dam is

usually much less than in a pure gravity dam, but any arch dam acts to some extent as a gravity dam. Its analysis is not within the scope of this text.

36. The Earth Dam.—Under favorable circumstances the earth dam is a very economical type. A typical section of such a dam may be seen in Fig. 36. The slopes on both the upstream and downstream faces are less than the angle of repose of the material used. In order to make such a dam watertight it is provided with an impermeable core which may be a thin vertical wall of concrete or other material, or, as in Fig. 36, it may be

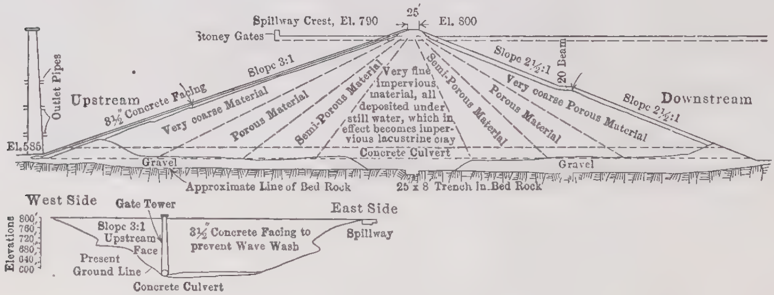


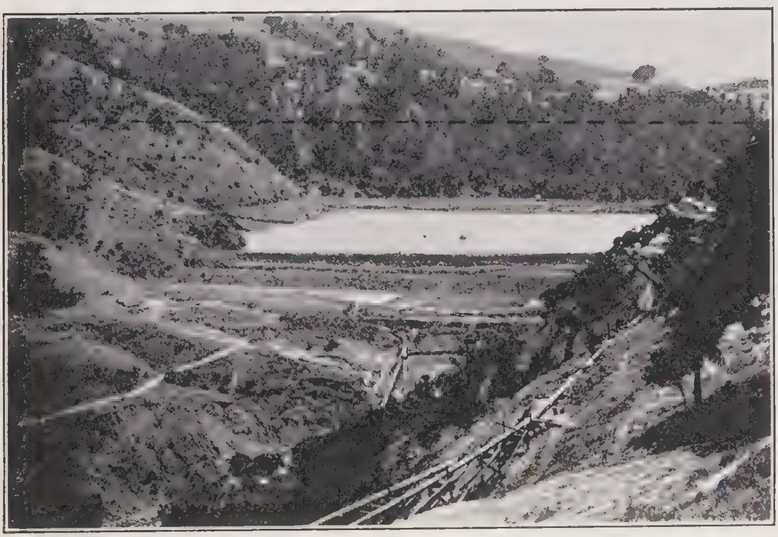
FIG. 36.—Section of Calaveras earth dam.

obtained by depositing fine earth under water. Figure 38 shows the pool of water in the center of the dam where such a core is being formed. There is little mathematical analysis to be made for such a dam. The main problems are those of construction and careful selection of the materials employed.

37. Additional Notes on Dams.—In most cases there are times when there is an excessive quantity of water that must be disposed of, usually by allowing it to flow over a spillway that is provided for that purpose. The spillway may be located at a different place from the dam so that no water ever overtops the latter. Again the spillway may occupy a portion of the crest of the dam as in Fig. 34 where the spillway can be seen in the middle. In other cases, such as in Figs. 36, 37, and 38, the spillways are located at one end of the dam and consist of rectangular canals through which the flood waters are discharged. But in Fig. 39 it may be seen that the entire crest of the dam is used for a spillway. This dam also shows the curved face that is provided to minimize the scouring effect of the waterfall upon the bed of the stream at the toe of the dam. For it must be recognized that

From a photograph by the author.

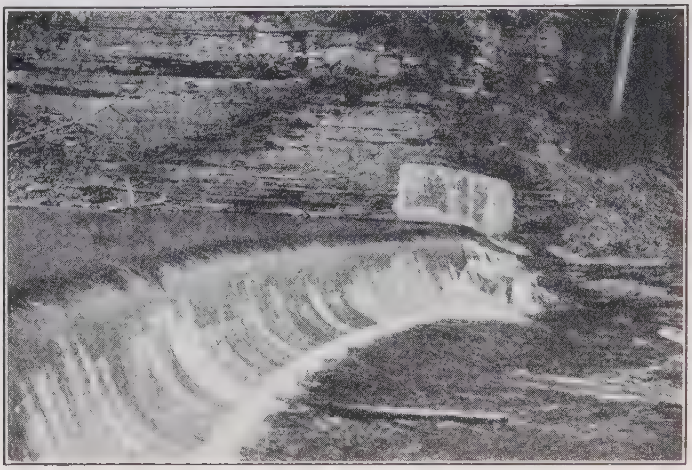
Fig. 37.—Upstream face of San Andreas earth dam. 90 ft. high.



From a photograph by the author.

Fig. 38.—Incompleted Calaveras earth dam. Ultimate crest will be at dotted line making it the highest earth dam in the world.

water in falling over a dam acquires kinetic energy that must be expended in some way, and unless suitable provision is made for this it may be expended in undermining the dam itself.



From a photograph by the author.

FIG. 39.—Low dam at Ithaca, N. Y.

38. Flashboards.—In storing water by means of a dam it is desirable to keep the water level as high as possible without flooding any lands upstream. If, therefore, the crest of the dam were located at the elevation allowable under normal conditions it would be excessively high in times of flood. In order to overcome this difficulty movable devices are employed called flashboards, movable crests, and various other names (Fig. 40). These are all schemes for increasing the height of the dam by equipment which can be removed when necessary. In some cases they work automatically, being either washed away when the water reaches a certain stage or caused to drop to a horizontal position. Other types require removal by hand in such emergencies. After the flood is past and the drier season comes on they may be replaced again. Some of these are entirely auto-

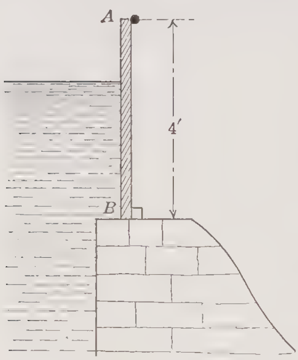


FIG. 40.—Flashboard.

matic in their action as in the case of the Stickney automatic crest outlined in Fig. 41. We have here two planes AB and BC rigidly connected and rotating about B. The water pressure on AB together with the weight of the shutters and the additional weight added at C tend to rotate the device in one direction, but that is opposed by the pressure of the water on BC. By a

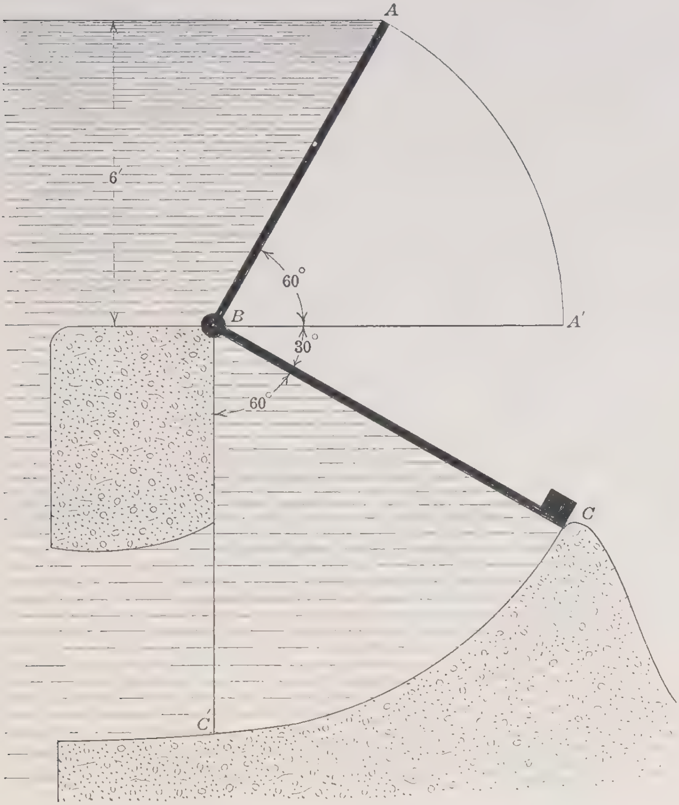


FIG. 41.—Automatic dam crest.

suitable adjustment of area and weights it is possible to keep this crest in the position shown until the water reaches the level of A. Then the pressure on AB may be sufficient to cause it to drop to the position $A'BC'$. Hence the crest of the dam will then be reduced to the height of B, and the flood water will pour over the shutter BA' and hold it down. But when the excess waters have

passed and the water level drops to B, or thereabouts, the pressure on BC', no longer opposed by that on BA', will raise the crest to the initial position.

39. PROBLEMS

48. Find the magnitude and point of application of the resultant pressure on the 2 ft. circular gate shown in Fig. 42.

Ans. 758 lb., 4.52 ft.

49. The gate AB in Fig. 43 rotates about an axis through B. If the width is 4 ft., what torque applied to the shaft through B is required to keep the gate shut?

Ans. 19,677 ft.-lb.

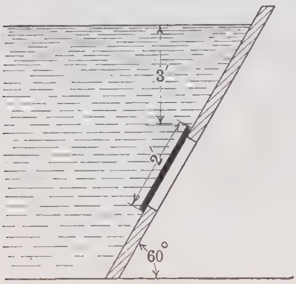


FIG. 42.

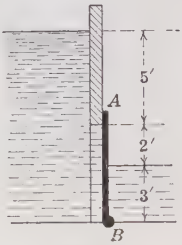


FIG. 43.

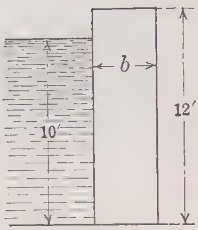


FIG. 44.

50. What value of b in Fig. 44 is necessary to keep the masonry wall from sliding. Masonry weighs 150 lb. per cubic foot and the coefficient of friction equals 0.4. Will it also be safe from overturning? If it has a factor of safety against sliding of 2, where will the resultant of the water pressure and its weight cut the base?

Ans. 4.34 ft., 3.67 ft. from toe.

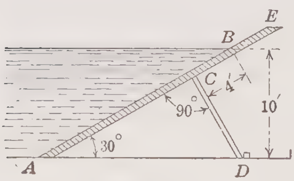


FIG. 45.

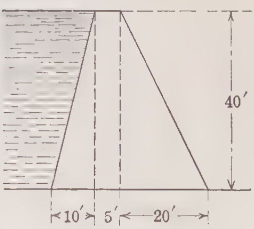


FIG. 46.

51. In the framed dam shown in Fig. 45, the struts CD are placed 5 ft. apart along the dam (perpendicular to the plane of the figure). What will be the load on each strut? What will be the value of the reaction at A? If the length BE is 4 ft. and the depth of the water flowing over the crest at E is 3 ft. what will be the load on the strut?

Ans. 13,030 lb., 5 × 3,630 lb., 39,400 lb.

52. Assume the weight of the dam in Fig. 46 to be 150 lb. per cubic foot, that there is no seepage of water under its base, and that the coefficient of friction between the dam and the material upon which it rests is 0.6. For 1 ft. length compute: (*a*) Horizontal component of water pressure. (*b*) Vertical component of water pressure. (*c*) Weight of dam. (*d*) Is it safe against sliding? (*e*) Is it safe against overturning? (*f*) Where does the resultant of the water pressure and the weight of the dam cut the base?

Ans. (*f*) 15.55 ft. from toe.

CHAPTER V

HYDROKINETICS

40. Actual and Ideal Conditions.—From the standpoint of pure mechanics the subject of hydrokinetics is rather unsatisfactory. This is due to the fact that so many assumptions are necessary, many of which are known not to be true. Thus in the greater portion of the work all particles of water in any cross-section of a flowing stream are assumed to move in parallel

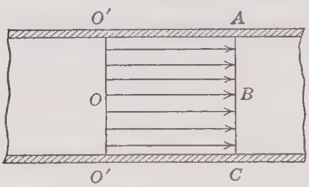


FIG. 47.

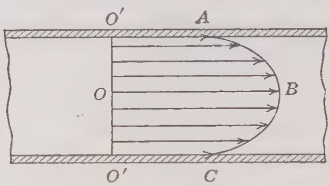


FIG. 48.

paths and with equal velocities. This is shown in Fig. 47, a particle of water at point O moving along the axis of a pipe with a velocity OB. But every other particle of water across section $O'OO'$ is assumed to move with the same velocity, giving us the velocity curve ABC, in this case a straight line. But it is well known that in a pipe the actual velocity curve is similar to ABC in Fig. 48, the velocity of a particle of water at O in the center of the pipe being OB while that of a particle near the wall of the pipe is $O'A$. Experiment shows that in general OB is about twice the value of $O'A$ and that the mean velocity of all the particles is about 0.84 OB. It is this mean velocity that is actually used in our computations. Hence our results are based upon a velocity which is possessed by only a few of the particles of water, the greater portion of them moving with either higher or lower velocities. Usually there is no attempt made to deal with the actual velocity curve ABC of Fig. 48 because there is

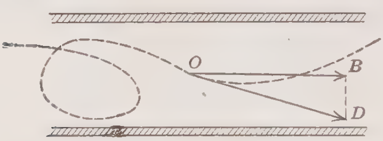
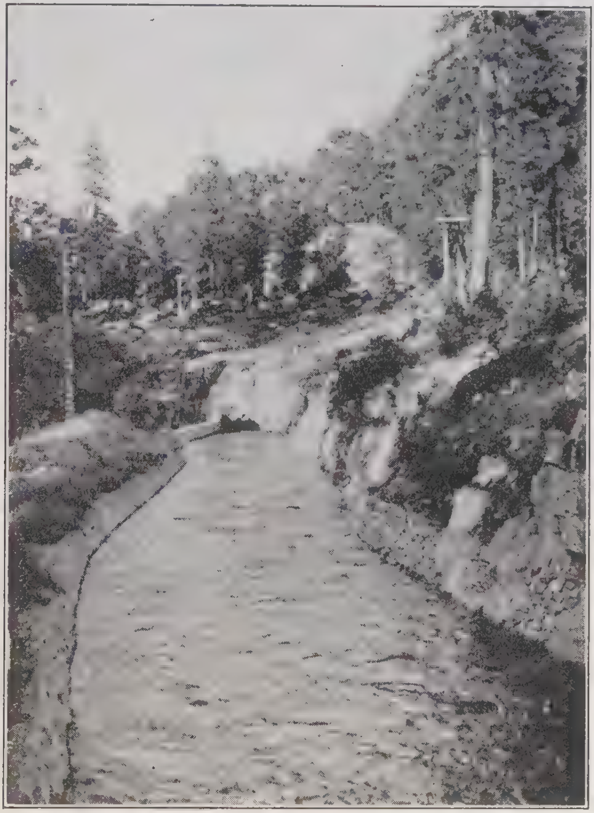


FIG. 49.

no assurance as to its exact nature in every case and, if there were, the equations would be too complicated for practical use.

But in Fig. 48 all particles of water have been assumed to be moving in straight lines parallel to the axis of the pipe, which is known to be very seldom the case. In fact the path of a given particle is very irregular as is shown in Fig. 49 and at the instant



From a photograph by the author.

Fig. 50.—Showing vortices on surface of canal.

in question a particle at point O may be moving with some velocity OD. But in most practical problems the important quantity is OB, which is the axial component of the true velocity. Thus not only do the equations ordinarily deal with a mean velocity, but they deal with a component of the true velocity. Instead of water flowing in parallel threads the true phenomena has been very aptly compared to the motion of a cloud of feathers

blown along by the wind. Water tends to travel in vortices as may often be observed upon the surface of an open stream such as the canal shown in Fig. 50. In this particular scene the water was flowing with a moderate velocity (about 3 miles an hour) over a reasonably smooth bed but the surface was covered with little vortices.

Since actual conditions depart so widely from the ideal conditions assumed by our imperfect theory one can expect the theory to provide little more than a framework upon which may be hung the results of experimental investigation.[1]

The mean velocity at any section (strictly the mean axial component of velocity) is obtained by dividing the total rate of discharge by the total area of the section. That is

$$V = \frac{q}{A}. \tag{20}$$

EXAMPLES

53. Experiment indicates that the velocity curve ABC of Fig. 48 is approximately a semi-ellipse and that OB is about twice $O'A$. Assuming this to be so, find the ratio between the mean velocity and the maximum velocity. (The total rate of discharge is $\int VdA$ and the value of this integral is the volume of the solid $O'ABCO'$. Dividing the solid by the area of the base, πr^2, we should have the mean ordinate or in this case the mean velocity. The volume of an ellipsoid is two-thirds that of the circumscribing cylinder.)
Ans. 0.833.

[1] In the light of what has just been said about the imperfections of the theory and because of what will appear later in regard to the difficulty of estimating the correct values of various empirical factors, the impression is sometimes created that the reliability of computed results in hydraulics is much less than in other branches of engineering. This, however, is not the fact. The probable variation in many of our values is a matter of a few per cent only in some cases and in other cases the range may be such that maximum probable values are twice the minimum. But even then this is much less than the possible variation implied in structural design, for instance, where a "factor of safety" of from 6 to 10 may be used. Thus a hydraulic engineer may specify a pipe line which is 5.5 ft. in diameter when one 5.0 ft. in diameter would have had the required capacity. The factor of safety, if it may be so called, is here only 1.1 based on diameter or 1.21 based on area, which is more logical.

In a certain instance the computed friction loss in a line containing a number of pipe fittings, there being nine different items to be considered, was 25.2 ft. Later the loss was measured in a test and found to be 25.6 ft. This is closer than one would have any reason to expect in every case, but illustrates the fact that computed results may be very close to the truth.

54. Suppose that a stream is divided into five areas that have values of 2.5, 2.5, 2.0, 2.0, and 1.0 sq. ft. while the velocities in each area may be considered as 3, 3, 4, 4, and 5 ft. per second, respectively. What is the total rate of discharge? What is the mean velocity?

Ans. 36 cu. ft. per second; 3.60 ft. per second.

41. Critical Velocity.—The path followed, or assumed to be followed, by a single particle of fluid is called a stream line. It has been found that for very low velocities the stream lines are really straight parallel lines, as pictured in Fig. 48 except that the velocity next to the pipe wall, $O'A$, is zero.[1] But as the velocity increases the flow suddenly becomes turbulent as in Figs. 49 and 50. The velocity at which this change occurs is called the critical velocity. Flow at velocities below the critical is spoken of as "stream-line" or "viscous" flow, while at velocities above the critical it is called "turbulent" or "sinuous" flow. The value of the critical velocity is a function of the viscosity of the fluid, which in turn is a function of the temperature, and also the size of the tube or pipe; the larger the latter the lower the critical velocity. For ordinary size pipes, with which the engineer has to deal, the critical velocity, in the case of water, is so low that its value is of no interest.[2]

If the friction loss in a pipe is plotted as a function of velocity and logarithmic scales are used for both quantities, a diagram such as Fig. 51 is obtained. The slope of the line OA is 1 while that of BD varies from 1.72 to 2, according to circumstances which will be discussed later. The region ABC is one of unstable flow and values may be found anywhere within the triangular area. This region indicates the velocity at which the flow passes from the stream line to the turbulent flow. The slopes indicate that below the critical velocity the loss of head varies directly as the first power of the velocity, while above the critical it varies as V^n where n may range between 1.72 and 2. The value of the critical velocity is less as the surface of the pipe is rougher, and

[1] For this case, noting that O' and A now coincide, the curve ABC is a parabola and the mean velocity is 0.5 the maximum velocity.

[2] In the case of water at 68°F. in a ⅛-in. pipe (actually 0.27 in.) the lower critical velocity is 0.452 ft. per second. In a 6-in. pipe it is only 0.02 ft. per second, thus being too small to be of practical importance. In a case where the author pumped various liquids through a 6-in. line, however, the critical velocity varied from 0.019 ft. per second for gasoline to 30.7 ft. per second for a viscous fuel oil. Thus in the case of very viscous liquids the flow is nearly always below the critical velocity.

may be materially lowered by some disturbing factor, but in stream line flow itself the friction loss is independent of the character of the surface, which is to be expected, since the velocity of the fluid in contact with the surface is *zero*.[1]

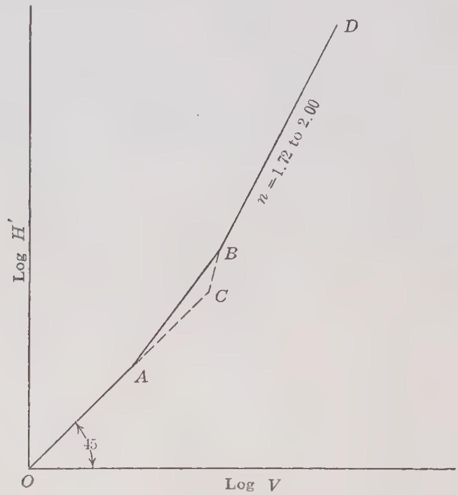


FIG. 51.—Relation between velocity and loss of head.

42. Steady Flow.—By steady flow is meant that at any point in a stream all conditions remain constant with respect to time. This does not mean that the conditions at any one point are necessarily like those at some other point.

Unsteady flow is met with in cases where change is taking place as a function of time. Thus suppose a pipe line is flowing full of water and a valve is closed suddenly at its lower end. The velocity of the water is brought to zero, and in so doing

[1] The friction loss in a length l may be represented for the case of viscous flow by the equation

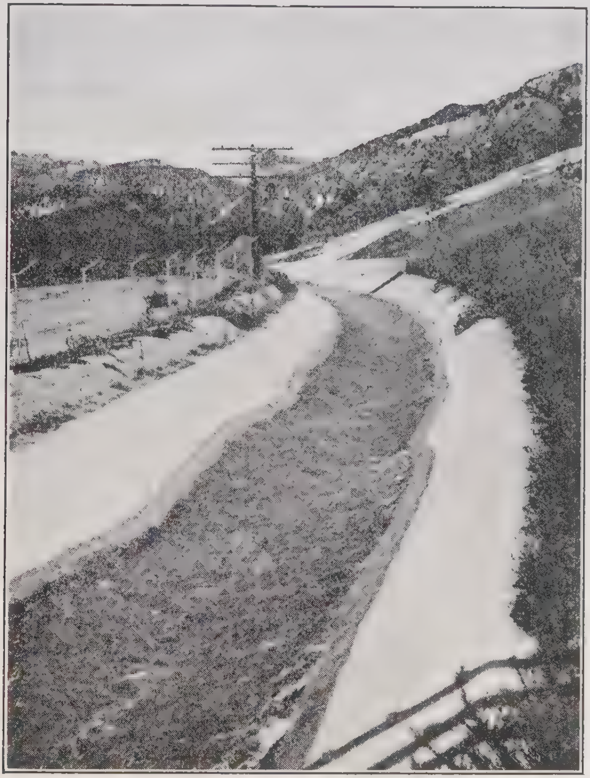
$$\triangle p'' = 0.000668 \times U \times l \times V/(d'')^2$$

where $\triangle p''$ is the drop in pressure in pounds per square inch and U is the value of the viscosity relative to water at 68 deg. (see Appendix I). The value of the lower critical velocity at A, for commercial pipes, is given by

$$V = 0.122 \times U/(d'' \times s)$$

where s is the specific gravity relative to water at 68°F. The higher critical velocity (at B) is roughly three times as great. Increasing velocities tend to follow the line AC while decreasing velocities tend to give the line BA, but small factors may cause an abrupt shift from one to the other.

there are certain pulsations of pressure, which if violent enough, are recognized as water hammer. While such changes are in progress the flow is unsteady. Again suppose that a gate is opened so as to admit water into an open canal originally empty. As the canal fills with water the level at any point steadily rises and also the velocity, in general, is changing at all points. While



From a photograph by the author.

Fig. 52.—The Los Angeles Aqueduct.

such changes are under way the flow is unsteady. But when equilibrium is finally established, the water level at any point and the velocity of flow across any section no longer vary from time to time and there is then steady flow.

In the strictest sense of the word steady flow is seldom met with in ordinary engineering work as it would be found only with velocities below the critical velocity. For with all veloci-

ties above the critical there exists continual fluctuations of flow at any point due to the irregular motion of the individual particles. It is for this reason that manometers or pressure gages attached to pipes, in which water is flowing, continually pulsate. Another evidence may be seen in Fig. 52, the dark band on either side of the water being where the latter has wet the concrete by wave action.

For all practical purposes these slight fluctuations at individual points are disregarded. If the average conditions over the entire section are reasonably constant with respect to time, the flow is considered as steady. While problems of unsteady flow are often problems of great practical value, especially in connection with the speed regulation of water power plants, they are rather difficult of mathematical treatment. Fortunately they are not as common as the more simple problems of steady flow. For the most part this text will be devoted to the latter.

43. Rate of Discharge.—The volume of water flowing across any section per unit time is called the *rate of discharge.* It must not be confused with velocity, since it is the product of the cross-section area of the stream and the velocity of flow across the section. It may be expressed in various units such as cubic feet per minute, gallons per day, etc., depending upon the custom in that particular class of work. In the foot-pound-second system of units such as are employed in this text it would naturally be in cubic feet per second. This is often called "second foot" for brevity and written as "sec. ft."[1]

44. Equation of Continuity.—In Fig. 54 it is apparent that the volume of water between any two sections such as (1) and (2) must remain constant if the flow is steady. Hence it follows that the rate at which water flows in at (1) must be equal to the rate at which it flows out at (2), otherwise there would be a change in the volume contained between the two sections. Thus it may be said that for steady flow $q_1 = q_2$.

If the flow is unsteady this is not necessarily so. For suppose that the closure of a gate above (1) shut off the flow of water at (1), water would still be found flowing for a time past (2) though at the expense of the volume stored between the two sections. Hence in case of unsteady flow, where the volume in any distance is changing, the equation of continuity no longer applies.

[1] In irrigation work in India the term "cusec" has gained acceptance for this rate of discharge.

The equation of continuity states that for steady flow

$$q = A_1V_1 = A_2V_2 = \ . \ . \ . \ . \ . \ = AV = \text{constant.} \quad (21)$$

This equation justifies the use of the term "rate of discharge" for the rate of flow across any section even though it be in the middle of a length of pipe or at some point in a river. For at some ultimate point the pipe or stream actually discharges in the usual sense of the word. And the rate of discharge at this point is equal, if the flow be steady, to the rate of volume flow at all sections throughout the stream.

It may also be noted that, in general

$$W = wq = wAV = \text{constant}$$

which is the rate of discharge on a weight rather than a volume basis.

EXAMPLES

55. In Fig. 53 the portion of pipe between A and C is the frustum of a right circular cone with vertex at O. If the rate of discharge is 10 cu. ft. per second, find the velocities at A, B, and C. In general if x denotes the distance from O, prove that Vx^2 = constant and find the value of the constant in terms of q and the cone angle α.

Ans. 51, 12.75, 3.18 ft. per second.

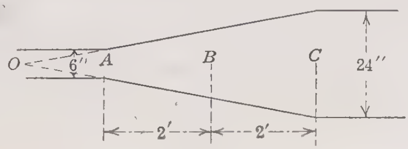


FIG. 53.

56. The canal shown in Fig. 50 is 14.5 ft. wide and 4.2 ft. deep. If the velocity of the water is 3.5 miles per hour, find the rate of discharge. This same water flows through a penstock, shown in Fig. 206, which is 52 in. in diameter and then is discharged through four nozzles, the jets from which are approximately 7 in. in diameter. What are the velocities?

Ans. 313 cu. ft. per second, 21.2 and 292 ft. per second.

45. General Equation for Steady Flow.—In the case of steady flow we may derive a very useful equation commonly known as Bernoulli's theorem in honor of Daniel Bernoulli who proposed it in 1738. We shall make use of the principle of *work and kinetic energy*, and the following conditions will be assumed:

(*a*) Flow is steady.

(*b*) Fluid is incompressible.

(*c*) Velocity across any cross-section is uniform.

In Fig. 54 let M and N be any two cross-sections of a filament of a stream in steady flow. Suppose that during an infinitesimal time interval particles passing M and N move to M' and N' respectively. The pressure, elevation, velocity, and cross-section area between M and M' will be denoted by p_1, z_1, V_1, and A_1 respectively, while between sections N and N' these will be p_2, z_2, V_2 and A_2 respectively. Since the flow is steady and the fluid is incompressible, the volumes of water passing M and N during any time interval must be equal so that $A_1 ds_1 = A_2 ds_2$.

From the principle of work and kinetic energy, the net work done on the volume between M and N while it moves to the position between M' and N' is equal to the corresponding change in its kinetic energy. The net work done on the volume under consideration is the sum of three parts: (1) The work done by pressures normal to the external surface of the filament; (2) the work done by gravity; (3) the work done by frictional forces.

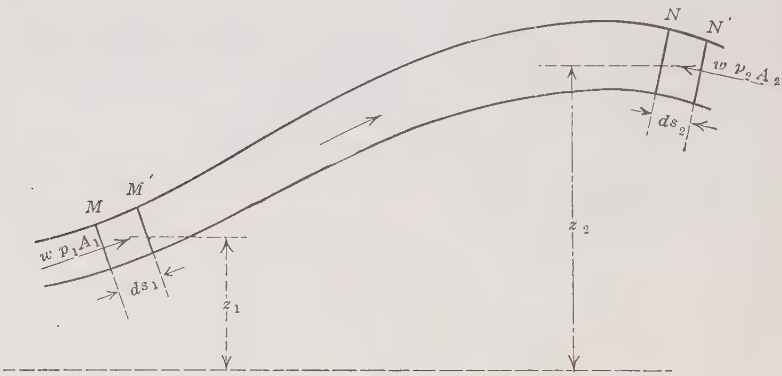


FIG. 54.

In the first item it is necessary to consider only the work done by the pressures on the end cross-sections, since the side pressures do no work. These forces at M and N are wp_1A_1 and wp_2A_2, respectively, and the displacements of their application points in the directions in which they act are ds_1 and $-ds_2$, respectively. Hence the net work done by these forces is $wp_1A_1ds_1 - wp_2A_2ds_2$.

Since the location of the center of gravity of the portion of the filament between M' and N remains unchanged, the net work done by gravity during the time interval is equal to that due to

the change of elevation of the volume of water A_1ds_1 from z_1 to z_2. The net work done by gravity is thus $wA_1ds_1(z_1 - z_2)$.

The work done by friction will be neglected for the present.

Since the kinetic energy of the portion between M' and N remains unchanged if the flow is steady, the whole change of kinetic energy is the difference between the kinetic energies of the parts between N and N' and between M and M'; that is

$$wA_2ds_2\frac{V_2^2}{2g} - wA_1ds_1\frac{V_1^2}{2g}.$$

Combining all the work and energy terms in an equation and noting that $A_1ds_1 = A_2ds_2$

$$wA_1ds_1(p_1 - p_2 + z_1 - z_2) = wA_1ds_1\frac{V_2^2 - V_1^2}{2g}. \qquad (22)$$

Dividing both terms by wA_1ds_1 and rearranging

$$p_1 + z_1 + \frac{V_1^2}{2g} = p_2 + z_2 + \frac{V_2^2}{2g}. \qquad (23)$$

This is Bernoulli's theorem, but, since all real fluids are viscous, it is impossible for flow to take place without fluid friction and hence a term should always be included to account for the energy converted into heat, and hence lost.

Methods of computing this loss of energy will be presented in subsequent chapters. For the present it will be represented as H', and the general equation of energy may then be written as follows:

$$p_1 + z_1 + \frac{V_1^2}{2g} - H' = p_2 + z_2 + \frac{V_2^2}{2g} \qquad (24)$$

where H' represents the energy lost *between* the two sections considered.

It is seen that at any section of the stream there are three quantities that are involved—the pressure, the elevation, and the velocity, which appears as $V^2/2g$. It is often convenient to represent the sum of these three items by a single symbol, such that in general

$$H = p + z + \frac{V^2}{2g}. \qquad (25)$$

Using this briefer notation Eq. (24) may be rewritten as

$$H_1 - H' = H_2. \qquad (26)$$

Since H' is always positive, it follows that H_2 must always be less than H_1. Thus the value of H must continuously decrease

in the direction of flow, unless energy is received from an external source, such as a pump.[1]

46. Head and Energy.—In the preceding article every term which appears in Eq. (22) represents energy or its equivalent, work. The quantity wA_1ds_1, which was eliminated to give Eq. (23), is seen to represent a weight. Thus each item in Eq. (23) represents energy per unit weight of water.

A flowing stream carries across any section, therefore, an amount of energy *per unit weight* which may be represented by the term H, as defined by Eq. (25). This may be called the total energy per unit weight of water, or briefly the *total head.* In our system of units it represents *foot-pounds per pound* of water and thus, while a measure of energy, is expressed in linear units. In like manner each energy term in Eq. (25) may be represented by a linear dimension.

The term p indicates intensity of pressure expressed in feet of water. It is called *pressure head.*

The elevation of a point *above* any arbitrary datum plane is indicated by z, the value of which will be negative if the point is at a lower elevation than the datum. It is a measure of the potential energy of a weight of water due to its elevation and is called *elevation head.*

The term $V^2/2g$ may also be seen to reduce to a linear quantity when the units involved in V and g are analyzed. The linear distance equivalent to $V^2/2g$ is the height through which a body might fall in a vacuum from rest and acquire the given velocity. In many cases it is a purely artificial quantity in that there is no actual height which one can see that gives any indication of its value. If m signifies mass and G weight, then $m = G/g$, and kinetic energy $mV^2/2$ may be represented as $GV^2/2g$. Thus $V^2/2g$ represents kinetic energy per unit weight of water and is called *velocity head.*

The three forms of energy just described are mutually convertible into each other or into mechanical work. Thus either pressure, elevation, or velocity (or any two of them) may increase in the direction of flow, but there must be a compensating decrease in at least one of the other heads. Also, due to friction losses, the sum of the three must continuously decrease in the direction of flow.

[1] In this case an additional factor should be inserted in Eq. (26) to express the gain of energy, as will appear in a later chapter.

It is obvious that, since all other items of Eq. (24) are in linear units, the quantity H' must also be expressed as a linear dimension. It represents the loss of energy per unit weight between two points in a stream and is called *lost head.*

This loss of energy is due, first, to the conversion of useful energy into the kinetic energy of eddies and turbulence, which is a different thing from the kinetic energy of translation represented by $V^2/2g$. Kinetic energy of turbulence cannot be converted back into any of the three forms first described nor into mechanical work. Ultimately it degenerates into heat due to friction between adjacent particles and between particles and the boundaries of the stream. This heat energy is not available for any useful purpose and eventually may be lost from the system by radiation. So far as the result is concerned, it is immaterial whether this kinetic energy of turbulence and this heat energy are retained or whether they are all radiated.

With this point of view of lost head the following might be written

$$H_1 = H_2 + H'$$

and it might be stated that at section (2) the total head or energy was equal to that at (1), but that it was divided into two portions; H_2 representing the available and H' the unavailable energy. The only way energy can really be lost from the system between (1) and (2) is by radiation of heat energy.

EXAMPLES

57. Assuming a body of water at rest in Fig. 55, so that there is no loss of head, what are the values of the pressure head at A, B, C, and D? What are the values of the elevation head? What are the values of the effective head at these four points?

58. In Fig. 56 the point A is 30 ft. higher than B. Assuming the pipe to be of uniform diameter, in which direction will the water be flowing if the pressure at A is 20 lb. per square inch and that at B is 40 lb. per square inch? What is the head lost between the two points?

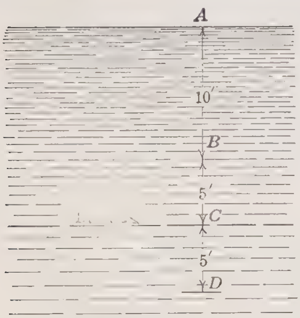


FIG. 55.

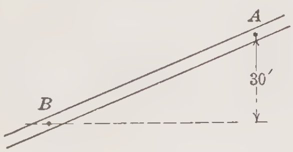


FIG. 56.

What would the pressure be at A if the flow were to be in the opposite direction, the rate of discharge and pressure at B remaining the same?

Ans. $H' = 16.2$ ft., $p_A = 78.4$ ft.

59. In Fig. 57 suppose 8 cu. ft. of water per second to be flowing from A to C. Assume a loss of head from A to B to be equal to $0.001V^2_B$ and an equal loss to occur between B and C. If the pressure head at B is 2 ft., how high will the water stand in piezometer tubes at A and C?

Ans. 139.8 ft., 122.95 ft.

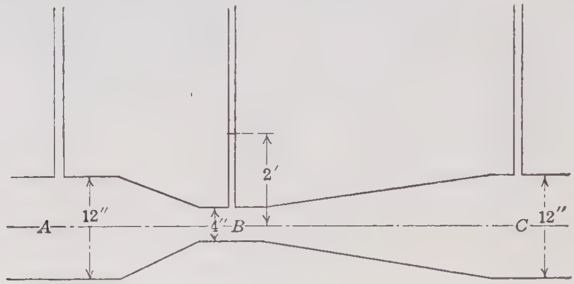


Fig. 57.

47. Head and Power.—Since head may be said to represent energy per pound of water and, since power is the time rate of expenditure of energy, then head must also represent power per pound of water per second. Thus, if W lb. of water per second are available and the total head is H, the power, in foot-pounds per second is

$$P = WH. \tag{27}$$

To show the application of the preceding, suppose that the surface of a lake is 500 ft. above the site for a power house and that the lake is capable of furnishing 200 cu. ft. of water per second. It is desired to find the power available.[1] This is merely $P = 62.4 \times 200 \times 500 = 6{,}240{,}000$ ft.-lb. per second. Also a jet discharging 50 lb. of water per second with a velocity of 120 ft. per second will have what available horsepower? The total head is $120^2/2g = 231$ ft. The horsepower $= 50 \times 231/550 = 21$.

[1] The familiar expression "power = force × velocity," cannot be used here for neither of these factors has any physical significance in this case. Neither can this relation be used in the case of a jet, because the value of the force exerted by the jet depends upon what happens when it strikes an object. And the velocity of the point of application of the force, which is the velocity of the object being acted upon by the water, is unknown in this instance. The available power of the jet is independent of what becomes of it.

EXAMPLES

60. A jet of water free from all pressure is 7 in. in diameter and has a velocity of 250 ft. per second. What is the horsepower?

Ans. 7,390 hp.

61. A pipe line draws water from a lake and delivers it to a power house at a point 500 ft. below the level of the surface of the lake. The water is delivered at a velocity of 170 ft. per second by a jet 6 in. in diameter, and free from all pressure (save that of the atmosphere). What horsepower has been lost in the pipe line?

Ans. 18.9 hp.

48. Hydraulic Gradient.—If a piezometer tube be erected at B in Fig. 58, the water will rise in it to some height BB' equal to the pressure head existing at that point. If the lower end of the pipe were closed so that no flow could occur, the height of this column would evidently be BM. The drop from

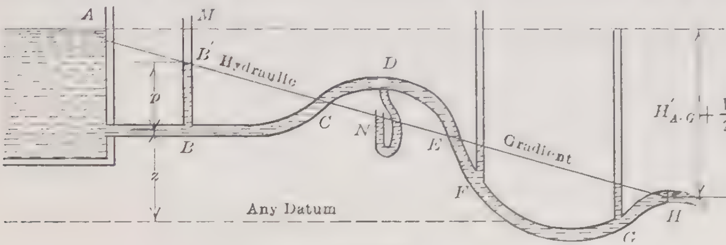


FIG. 58.

M to B' that is found when flow takes place is due to two factors, one of these being that a portion of the pressure has been converted into the velocity head which the water has at B and the other that there has been a loss of head through friction between A and B.

If a series of water piezometers were erected along the pipe line, the water would rise in them to various levels. The line drawn through the summits of such an imaginary series of water columns is called the *hydraulic grade line* or the *hydraulic gradient.* It is seen that this line is an indication of the pressure variation along the pipe. Thus at any point the vertical distance from the pipe line to the hydraulic gradient is the pressure at that point. Since at C this distance is zero it follows that at C the pressure is atmospheric. And at D the line is below the pipe indicating that at the point in question the pressure is below that of the atmosphere and is equal to $-DN$. The advantage of the construction of the hydraulic gradient is that

it gives a very clear picture of the pressure variation along a pipe line. Also in practical applications the profile of a proposed pipe line should be drawn to scale. Then by computing a few points only the hydraulic gradient can be drawn and from it the pressures at all points can be readily measured.

The vertical distance from the hydraulic gradient to the level of the water surface at A represents $H' + V^2/2g$. Hence the position of the hydraulic gradient is independent of the position of the pipe line. Thus it is not always necessary to compute pressures at various points in order to plot the gradient. Instead values of $H' + V^2/2g$ may be laid off below the proper horizontal line, and this procedure is often more convenient. It is usually necessary to locate only a very few points, and often only two, the terminal points, are sufficient. For example if Fig. 58 represents the profile of a pipe of uniform diameter drawn to scale, the hydraulic gradient can readily be drawn as follows. At the intake to the pipe there will be a drop below the level of the surface of the water which should be laid off equal to the sum of $V^2/2g$ plus the entrance loss.[1] At H the pressure is known to be zero gage pressure and hence the gradient must pass through the end of the pipe. In the case shown the hydraulic gradient is practically a straight line and hence may be drawn at once from these two points. The location of other points, as B', may be computed if desired. In the case of a long pipe line the velocity may be such that the drop in the gradient at the entrance is very small and hence the error is very slight if the gradient is drawn as a straight line from the surface of the water above the intake to the lower end of the pipe.

The hydraulic gradient is not necessarily a straight line. For a pipe of uniform diameter it will be a straight line only if the pipe itself is straight. If the pipe is of uniform diameter the drop in the hydraulic gradient along its length is then a measure of the loss of head and this will be proportional to the horizontal distances in the figure only when the latter in turn are proportional to the actual lengths of pipe. But for ordinary amounts of curvature the hydraulic gradient will deviate but very little from a straight line. Of course if there are losses of head aside from those due to ordinary pipe friction there will be abrupt drops in it, and any variations in velocity head due to changes in diameter affect the hydraulic gradient.

[1] This and other details are explained more fully in chap. VII.

It may be seen that if the velocity head is constant the drop in the hydraulic gradient between any two points is the measure of the loss of head between those two points. And the slope of the gradient is a measure of the rate of loss. Thus in Fig. 59 the rate of loss is much less in the larger pipe than in the smaller one. If the velocity changes, the hydraulic gradient might

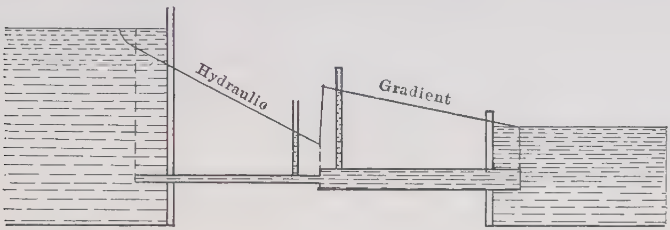


FIG. 59.

actually rise in the direction of flow as may be seen in both Figs. 59 and 60. Additional illustrations of the hydraulic gradient for other cases are to be seen in Chaps. VII and VIII.

It is sometimes instructive to represent not only the variation of pressure head but also the variation of total head. If any

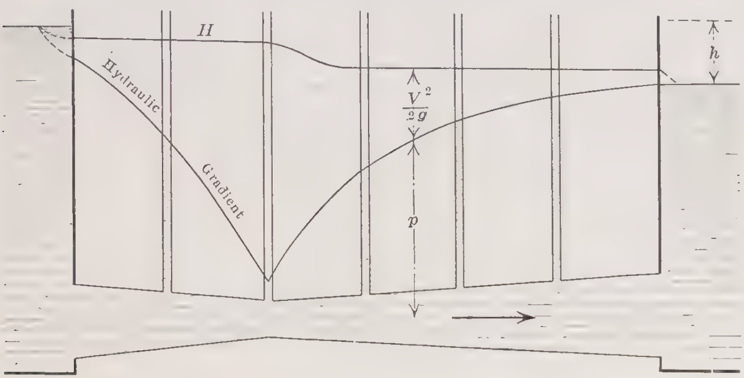


FIG. 60.

arbitrary datum plane is assumed, the vertical distance from it to any point in the pipe represents the elevation head for that point. And the vertical distance from this point to the hydraulic gradient represents the pressure head. Hence the vertical distance from the datum plane to the hydraulic grade line represents the sum of pressure head plus elevation head. If to this is added

the velocity head a curve is obtained, as shown in Fig. 60, the ordinates of which represent total head or energy. And, as in the case of the hydraulic gradient, the location of this total head curve is independent of the position of the pipe and may be plotted by laying off values of loss of head below a horizontal line. The particular one shown, plotted from experiments made by the author, shows that the chief loss of head occurs in the diverging portion and just beyond the section of minimum diameter. The total loss of head between the two tanks is h and both entrance and discharge losses are here represented.

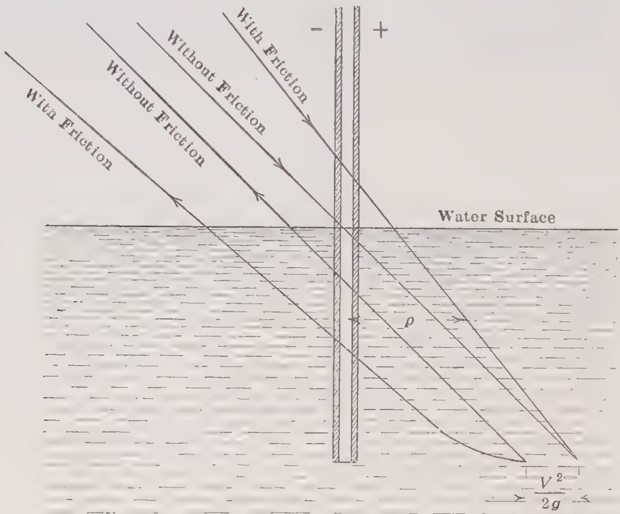


FIG. 61.—Pressure gradients in vertical pipe with flow in either direction.

As shown in Fig. 60, for example, the hydraulic gradient or pressure gradient, as it might be called, may either fall or rise, but the curve for H, which might be called the total energy gradient, must fall continuously in the direction of flow, unless energy is received from some external source.

It may be observed that the hydraulic gradient in all cases represents what would be the free surface, if one could exist and maintain the same conditions of flow.

In Fig. 61 are shown pressure gradients for different conditions of flow in a vertical pipe. Pressures above the atmospheric are laid off horizontally to the right and pressures below the atmospheric to the left of the center line of the pipe. It may be

observed that at any point in the pipe the pressure is greater when the flow is down than when it is up, and that the pressure in the former case is increased by friction while in the latter it is decreased.

EXAMPLES

62. Let Fig. 62 represent the profile of a pipe line of uniform diameter which takes water from a reservoir at *A*, carries it across a valley at *C*, and discharges freely into the air at *E*. The lengths *AB*, *BC*, *CD*, and *DE* are each equal to 1,000 ft. *AB* and *DE* are both horizontal and at the same level. The illustration in the text is drawn to scale. If both the horizontal and vertical scales are 1 in. = 1,000 ft. so that h = 1,000 ft., compute the loss of head in *AB* and *DE* by proportion, assuming the hydraulic gradient to be drawn from the water surface vertically above *A* and the loss of head

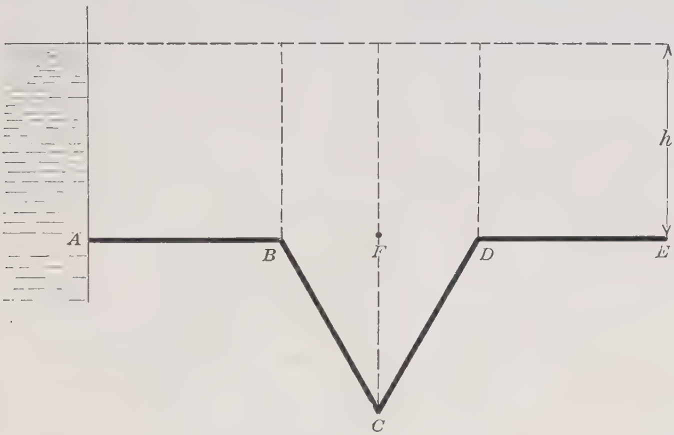


FIG. 62.

is directly proportional to the length of the pipe. Draw the hydraulic gradient and scale off values of the pressure at *B*, *C*, and *D*.

Ans. Pressures = 750, 1,367, and 250 ft.

63. In the preceding problem the vertical and horizontal scales are the same so that all dimensions appear in their true proportions. Thus the length of *BC* is 1 in. and the angle *FBC* is 60 deg., which is there the true slope of the pipe. In most cases the horizontal distances are so much greater than the vertical dimensions that different scales are used. Thus assume that for Fig. 62 the horizontal scale is 1 in. = 1,000 ft., but that the vertical scale is 1 in. = 100 ft. Then h = 100 ft., which would have been insignificant on the drawing with the former scale. With these scales, the distance *BF* represents $0.5 \times 1{,}000 = 500$ ft., but the distance $CF = 0.866 \times 100 = 86.6$ ft. Hence the distance *BC*, which represents 1,000 ft. of pipe, cannot be measured by either scale, so that only horizontal or vertical distances can be scaled off of such a drawing. If these scales are used, and $h =$

100 ft., repeat the procedure called for in the example above. What is the true slope of BC in this case?

Ans. 9 deg. 50 min.

49. Further Considerations Regarding Energy Equation.—In the ideal case of water flowing in parallel straight lines with equal velocities and without friction, Bernoulli's theorem may be applied between any two points whatsoever, because the total head or energy is constant throughout the stream. But in the actual case where there are losses of energy and where the conditions of flow are as described in Art. 40, the general equation of energy should be written between two points in the *same* stream line, so that a particle of water may be assumed to flow from one of these points to the other.[1] It is no more permissible to apply the equation between two points in adjacent stream lines than between two separate streams in different channels.

In practical applications of the general equation entire streams are usually considered, rather than stream lines. Hence the average velocity and the average head for the entire section are dealt with. But the average head for some section of the stream is equated to the average head for some other section of the *same* stream.

In considering an entire stream, rather than a single stream line, the kinetic energy per unit time is assumed to be $WV^2/2g$, where V is the average velocity. This is not strictly true for, if the velocity varies from point to point over the section, the kinetic energy is the sum of the kinetic energies of all the individual particles. Considering an elementary area dA, the flow through it will be $wV'dA$, where V' is the actual velocity at the point in question. The kinetic energy of the elementary stream would be $wV'^3dA/2g$. Hence the total kinetic energy for the entire stream is

$$\frac{w}{2g}\int V'^3dA. \tag{28}$$

If the velocity is constant this will become $wV^3A/2g = WV^2/2g$ since the true velocity at every point is the average velocity

[1] Only in the case of velocities below the critical are the actual stream lines known. In cases of turbulent flow, stream lines followed by a single particle are very irregular and are in general unknown. However, it is permissible within limits to make assumptions as to certain stream-line paths to assist one in a theoretical analysis, as results of practical utility are thereby often obtained.

for the section. But in reality the velocity does vary to some extent over the section and hence Eq. (28) gives the true kinetic energy. If the law of variation of V' throughout the section is known this integral can be evaluated, but in any event it can be shown that the kinetic energy so obtained is greater than that computed by using the average velocity. Thus making the assumption that the velocity in the center of a circular pipe is twice that near the walls and that the velocity curve is a semi-ellipse it will be found that the true kinetic energy is 1.06 times that based upon the mean velocity. Fortunately the difference is not great in important cases met with in practice. Thus in the case of a jet of water from a good nozzle, where there is little variation in velocity, the difference may be a matter of about 1 per cent only. A correct application of Eq. (24) would require some factor to be inserted before the velocity head, based upon the average velocity, to give a correct value. But if the velocity curves at sections (1) and (2) are similar and the velocities nearly the same in value the error in one may nearly balance that in the other. Hence it is not customary to allow for this discrepancy between the true kinetic energy and that computed by using the average velocity.

It has been found that the pressure head across any horizontal diameter of a section of a stream flowing in a straight channel is constant, or at least the variation is so slight that it is difficult to detect any consistent departure from this rule.[1] In any vertical line the pressure merely varies with the elevation. Thus the sum of elevation plus pressure head is constant for all points across the cross-section. But, since it is known that the velocity in the center of the channel is higher than that at the boundary wall, this would lead to the conclusion that the total energy in the center is higher than that at the wall. This is the fact. Since Bernoulli's theorem should not be applied in actual cases to adjacent stream lines, there is no warrant for claiming that the energies should be equal. Also, if it is assumed that particles did maintain themselves in actual parallel stream lines, then, if all particles started from some source with the same stock of energy, those nearer the boundary would have lost more by friction than those in the center.

[1] In flowing around a bend centrifugal action sets up an inequality of pressure, but this does not concern us here.

However, no such parallel stream lines can exist for any distance with turbulent flow, and it is known that particles actually flow from the boundary to the center of the stream and back again. It would thus appear that a particle gains energy as it flows towards the center. This is entirely possible. Particles may be conceived to transfer energy back and forth among each other, so that a particle may either gain or lose, as sinuous flow and vortex motion causes it to flow back and forth in cross-currents. But the stream, as a whole, continually loses energy.

In using the general equation of energy, it is apparent that only the *difference* in elevation of the two sections is involved. Hence the datum plane may be chosen at any arbitrary height. But when it is desired to compute the energy or power available, it is necessary that the datum plane be chosen at a definite location, which is that at which the energy or power may be desired to be utilized. In like manner one may be concerned only with the difference in the pressures at two sections, and hence it is immaterial whether gage or absolute pressures be considered, since the atmospheric pressure would appear on both sides of the equation and cancel out. Thus the solution of most problems is independent of the barometer pressure.[1] As in the case of elevation, however, it is necessary to consider the base from which pressure is measured when energy or power is involved. Since the pressure cannot ultimately be less than atmospheric, the latter is a suitable base for most cases and hence again justifies the use of gage pressures.

EXAMPLE

64. Assume that in a rectangular stream the velocity of the water is uniform from side to side at any depth but that it varies from the top to the bottom inversely as the depth. If the velocity at the top is twice that at

[1] An exception to this is the limiting condition when pressures below the atmospheric approach the vapor pressure of the water, which must be expressed in absolute units. Usually the physical properties of water are not functions of pressure to any extent, as are those of gases and vapors; and hence in hydraulics it is not necessary to consider absolute pressure as is done in thermodynamics. But at this minimum pressure the water tends to become a vapor and hence carries attention over into the other field. Another exception to the statement above is when water flows down a penstock to a high head plant. The barometer pressure at the two ends of the pipe line would then be appreciably different, but this difference is negligible compared with the actual pressures involved.

the bottom find the ratio between the true kinetic energy passing a section per unit time and that based upon the mean velocity.

Ans. 1.11.

50. Applications of General Equation.—For the solution of problems in hydrokinetics we have two fundamental equations, the equation of continuity (21) and the general equation for steady flow (24), usually known as Bernoulli's theorem. In most cases the following procedure may be employed:

1. Choose a datum plane through any convenient point.

2. Note at what sections the velocity is known or assumed. If at any point the cross-section is great as compared with its value elsewhere, the velocity will be so small that the velocity head may be disregarded.

3. Note at what points the pressure is known or assumed. In a body of water at rest with a free surface the pressure is known at every point. The pressure in a jet is the same as that in the medium surrounding the jet.

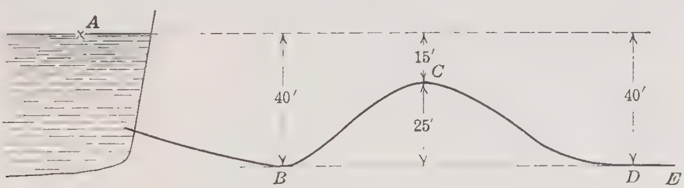


Fig. 63.

4. Note if there is any point where the three items of pressure, elevation, and velocity are known.

5. Note if there is any point where there is only one unknown quantity.

It is generally possible to write Eq. (24) between two points such that they fulfil conditions (4) and (5) respectively. Then the equation may be solved for the one unknown. If it is necessary to have two unknowns then Eq. (24) must be solved simultaneously with Eq. (21). The procedure is best shown by applications such as the following:

In Fig. 63 water flows from reservoir A through pipe BCD, which is 6 in. in diameter. The diameter of the stream discharging freely into the air at E is 3 in. Assume that the loss of head in friction in any length of pipe may be represented as $H' = kV^2/2g$, where k depends upon the length of the pipe and other factors, which will be explained later. Suppose that the roughness of the pipe and the lengths between the various

points are such that the values of k from the reservoir to B, from B to C, from C to D, and from D to E are 2, 4, 4, and 1 respectively or the total loss in the entire pipe is $11V^2/2g$. Let it be required to find the pressure at C when flow takes place.

At C there is both an unknown pressure and an unknown velocity, hence Eq. (24) cannot be applied immediately as one equation is capable of determining only one unknown. Let the procedure outlined then be followed. The location of a datum plane is immaterial in the solution of the problem but it is usually convenient to take it through the lowest point in the figure and thus avoid negative values of z. Therefore let a datum plane through E be assumed. In the reservoir it is found that the velocity is negligible because of the large area as compared with the area of the pipe. At a point A on the surface of the water the pressure is found to be atmospheric, which is also the case with the stream at E. Thus whatever the pressure of the atmosphere may be its effects can easily be shown to balance out, and therefore it is neglected altogether. Hence at A it is found that everything is known, while at E the velocity head is the only unknown.

Apply Eq. (24), or its equivalent Eq. (26), between points A and E. It is found that

$$H_A = 0 + 40 + 0$$

$$H_E = 0 + 0 + \frac{V'^2}{2g}$$

$$H'_{A-E} = 11\frac{V^2}{2g}.$$

Now V' is the velocity of the jet at E while V is the velocity in the pipe but one may be replaced in term of the other by Eq. (21). (It is seldom necessary to compute areas for this. It is both easier and more accurate to use the ratios of the areas, which means the ratios of the diameters squared.) Now $V' = AV/A'$, where A' is the area of the jet. But $A/A' = (6/3)^2 = 2^2 = 4$. Hence $V' = 4V$ and $V'^2 = 16V^2$. Replacing V' by V and substituting in Eq. (26)

$$40 - \frac{11V^2}{2g} = \frac{16V^2}{2g}.$$

Thus $V^2/2g = 40/27 = 1.48$ ft. One of the unknowns at C has now been determined.

Next, Eq. (26) may be applied between C and either A or E since the value of H at either of the latter points is known. The value of the effective head at C is $H = p + 25 + 1.48$ while $H'_{A-C} = 6V^2/2g = 6 \times 1.48 = 8.88$ ft. Now from Eq. (26)

$$40 - 8.88 = H_C = p + 26.48.$$

Hence

$$p = 4.64 \text{ ft.}$$

If the rate of discharge is also desired it can easily be found. Since $V^2/2g = 1.48$, $V = \sqrt{2g1.48} = 8.025\sqrt{1.48} = 9.78$ ft. per second. Hence $q = 0.196 \times 9.78 = 1.92$ cu. ft. per second.

It will always be found more convenient to consider the velocity head $V^2/2g$ as a single quantity and to carry it through and find its value, rather than to substitute the numerical value of $2g$ earlier in the solution. If desired, the velocity may be found from $V^2/2g$ as shown. Often, however, as in finding the pressure at C, it is $V^2/2g$ and not the value of V that is required.

EXAMPLES

65. Compute the pressures at B and D in Fig. 63.

Ans. 35.6, 23.7 ft.

66. Suppose that all other data for Fig. 63 remains unchanged except the diameter at C. What will this diameter be if there is a vacuum of 20 in. of mercury at C?

Ans. 2.86 in.

67. Suppose the diameter at C in Fig. 63 remains 6 in. and all other data are likewise unchanged except the elevation of C. How far above E can C be placed to produce a vacuum of 20 in. of mercury?

Ans. 52.24 ft.

51. PROBLEMS

68. In the siphon shown in Fig. 64 the loss of head from the intake to B is 4 ft. and that from B to the discharge end of the pipe is 3 ft. Find the

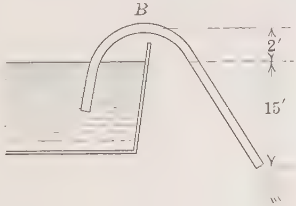


FIG. 64.

rate of discharge and the pressure head at B if the pipe is of a uniform diameter of 6 in.

Ans. $p = -14$ ft.

69. The diameter of the pipe in Fig. 65 is 4 in. and that of the stream discharging into the air at E is 3 in. Neglecting all losses of energy, what are the pressures at B, C, and D? (Velocity assumed negligible at A.)

Ans. -12.6, 27.4, 67.4 ft.

11.6 28.9

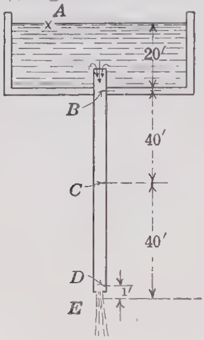


FIG. 65.

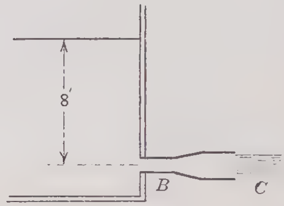


FIG. 66.

70. At B in Fig. 66 the diameter is 1 in. while the diameter of the stream at C is 1.5 in. Assuming no friction loss, what are the values of the velocity and pressure at B?

Ans. 51.1 ft. per second, -32.56 ft.

71. If the head in Fig. 66 were 6 ft. instead of 8, what would be the velocity and the pressure at B? What is the ratio of the rate of discharge to the value obtained if the tube were cut off at B?

Ans. 44.2 ft. per second, -24.42 ft., 2.25.

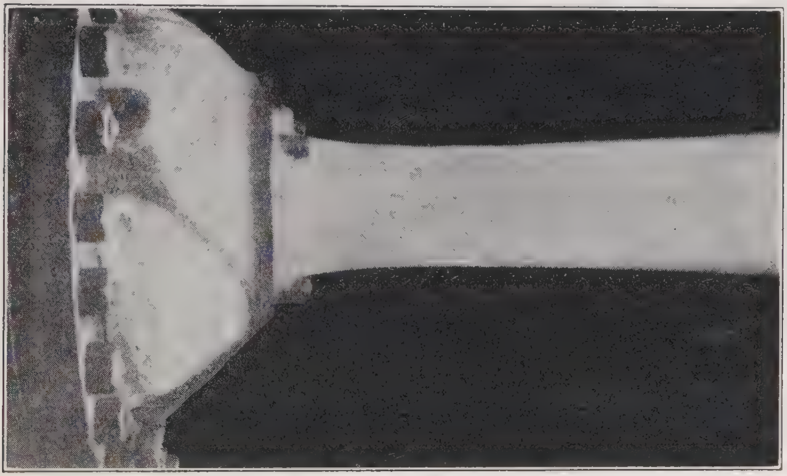
72. If the head in Fig. 66 were 6 ft., and the pressure at B were the minimum value possible of -34 ft., what would be the velocity at B? Is this the maximum possible velocity for this head? With this velocity at B and a 1-in. diameter, what must be the diameter at C?

Ans. 50.7 ft. per second, 1.606 in.

CHAPTER VI

APPLICATIONS OF HYDROKINETICS

52. Definition of a Jet.—A jet is a stream bounded by a fluid of a different kind. The jets with which we are concerned in practical hydraulics are streams of water entirely surrounded by air. It is evident that the pressure to which the water in a jet is subjected is exactly equal to the pressure exerted upon its boundaries by the surrounding air.



From a photograph by W. R. Eckart, Jr.

FIG. 67.—Jet from 7½-inch nozzle. (Head = 822 ft., velocity = 227.4 ft. per sec.)

53. Jet Coefficients.—Due to frictional resistance the actual velocity of a jet is always less than would otherwise be the case. The velocity which would be attained if friction did not exist may be termed the ideal velocity.[1] The ratio of the actual velocity to the ideal velocity is called the *coefficient of velocity.*

[1] This is frequently called "theoretical velocity" by others, but the author feels that this is a misuse of the word "theoretical." Any correct and sensible theory should allow for the fact that friction exists and affects the result. Otherwise it is not theory but merely an incorrect hypothesis.

The area of the opening through which the jet issues is something that is readily determined, but in many cases the area of the jet cannot so readily be measured without special equipment. Hence it is desirable to know the relation between the area of a jet and the area of the opening through which it came. This factor, the ratio of the area of the jet to the area of the opening, is called the *coefficient of contraction.* The word "contraction" is used because the jet usually contracts and is smaller than the opening, as may be seen in Fig. 68. In case the jet does contract, the section of minimum area, the "vena contracta," is the section whose area is considered in the calculations. The velocity of a jet is also understood to be the velocity found at this point.

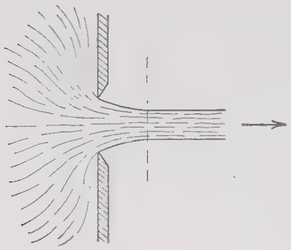


Fig. 68.—The "vena contracta."

The coefficient of contraction may be unity, indicating that the area of the jet is equal to the area of the opening from which

From a photograph by the author.

Fig. 69.—Discharge from end of a straight pipe. (Mixture of water, mud, and rocks building up the Calaveras earth dam.)

it issued. This is the case when the sides of the stream are parallel before it issues from the opening, as when the discharge is from the open end of a pipe as shown in Fig. 69. Of course after passing the point of minimum section the jet diverges again

due to the loss of velocity from frictional resistance. This is seen in Figs. 67, 70, and 78.

The product of the coefficient of velocity and the coefficient of contraction is called the *coefficient of discharge*. It is the ratio of the actual rate of discharge to the ideal rate of discharge that would be obtained if there were no friction and if the jet did not contract. It may be seen that

$$c = c_c \times c_v.$$

54. Jet Contraction.—The jet shown in Fig. 68 contracts after it leaves the orifice because the stream lines are converging as they approach it. Since the tendency of each particle of water is to continue in its original direction of motion, the stream lines continue to converge after they pass the orifice. However, they cannot cross each other so they eventually become parallel and produce a section of minimum area called the "vena contracta."

The water at the jet boundaries in contact with the air is necessarily subjected only to atmospheric pressure; and at the point of minimum cross-section, where the stream lines are parallel, the pressure is atmospheric across the entire section. But in the plane of the orifice the particles of water are flowing in curved paths, which are concave towards the outer boundary. Due to centrifugal action, the pressure increases from the boundary to the center of the stream and hence the *average* pressure of the jet in the plane of the orifice is greater than atmospheric.[1] Thus at the orifice the pressure is higher and the velocity lower than at the point of minimum cross-section, so that both the equation of energy and the equation of continuity are satisfied.

The *pressure* within the jet, with which one is here concerned, is a very different thing from the *force* which the jet is capable of exerting upon any object which it strikes.

Since the contraction of the jet is due to the converging of the approaching stream lines, it may be prevented altogether by causing the particles of water to approach in an axial direction

[1] The fact that in turbulent flow particles of water do not follow ideal stream lines does not prevent one from making use of such a conception as an aid in a practical analysis. However, in the case of a jet the flow is much more nearly stream line than almost any other case commonly met with, except for flows below the critical velocity, and hence the assumptions above are very near the truth. For an explanation of centrifugal action see chap. XI.

solely, as in the case of the pipe in Fig. 69, which is an extreme type of illustration. The construction shown in Fig. 75 (*d*) and also in Fig. 77 (*b*) and (*c*) serves materially to reduce and even totally to eliminate all contraction. In fact the amount of contraction is very sensitive to small changes such as slightly rounding the otherwise sharp edge of an orifice or applying a little oil to an orifice before the water is run through it. Thus values of the coefficient of contraction vary through a much wider range than do values of the coefficient of velocity.

Also if an orifice is placed too near the bottom or side of the tank, the stream lines are prevented from converging properly

From a photograph by the author.

Fig. 70.—Jet from hydraulic giant.

from that direction and hence the coefficient of contraction is increased.

55. Flow through Orifices, Tubes, and Nozzles.—An *orifice* is any opening in the wall of a containing vessel. The only restriction is that the thickness of the wall shall be only a small fraction of the diameter or other linear dimension of the opening.

A *tube* is a pipe, whose length is not more than two or three diameters, and may be treated in the same manner as the orifice. Tubes may be straight, converging, or diverging.

A *nozzle* may be defined as a converging tube placed on the end of a pipe.

Let the general equation of energy, Eq. (26), be written between points (1) and (2) of Fig. 71, assuming the pressure is atmospheric at both points and that the area of the vessel is such that the velocity at (1) may be neglected. Also let a loss of head be assumed between the two points which will be considered as being proportional to the square of the velocity of

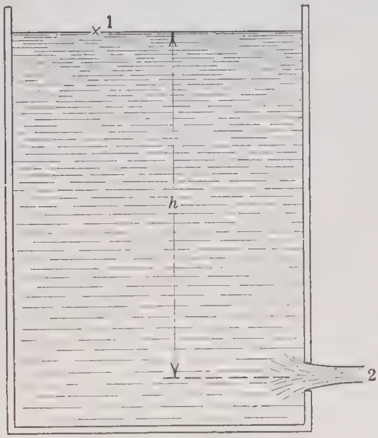


FIG. 71.

the jet V, and introduce such a factor k that $H'_{1-2} = kV^2/2g$. The result is

$$H_1 = 0 + h + 0, \qquad H_2 = 0 + 0 + \frac{V^2}{2g}.$$

Hence

$$H_1 - \frac{kV^2}{2g} = \frac{V^2}{2g}.$$

From this,

$$\frac{V^2}{2g} = \frac{H_1}{(1 + k)}$$

or,

$$V = \frac{1}{\sqrt{1 + k}}\sqrt{2gH_1}. \tag{29}$$

If there were no frictional resistance to flow, the value of k would be zero. Thus the *ideal* velocity is $V = \sqrt{2gH_1}$. It is the ideal velocity that is to be multiplied by the coefficient of velocity to obtain the true velocity. Hence

$$c_v = \frac{1}{\sqrt{1 + k}}. \tag{30}$$

In this equation we have the relation between the coefficient of velocity and the coefficient of loss. By squaring both sides and rearranging this may also be written

$$k = \frac{1}{c_v^2} - 1. \tag{31}$$

Since in this case $H_1 = h$, Eq. (29) may now be written in its more usual and more convenient form[1]

$$V = c_v\sqrt{2gh}. \tag{32}$$

If V and A denote the velocity and area of the jet respectively, while A_o denotes the area of the orifice, it may be seen that

$$q = AV = (c_cA_o)(c_v\sqrt{2gh}) = cA_o\sqrt{2gh}. \tag{33}$$

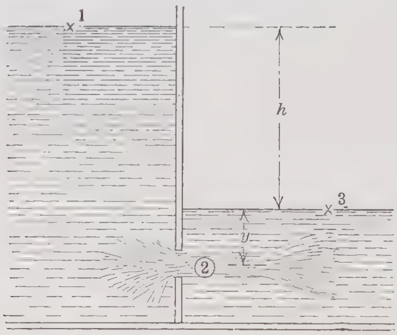


FIG. 72.

56. Submerged Orifice.—For a submerged orifice as shown in Fig. 72, for points (1) and (2) the following could be written:

$$H_1 = 0 + (h + y) + 0, \quad H_2 = y + 0 + \frac{V^2}{2g}.$$

This is based on the assumption that the pressure at (2) is equal to y, which may not be strictly correct but any discrepancy here will be covered by the value of the coefficient. Since y cancels out

$$V = c_v\sqrt{2gh}. \tag{32}$$

[1] In case the jet discharges into a medium under a different pressure from that on the surface of the liquid in the vessel in Fig. 71, the value of h for the pressure difference should be corrected, so that

$$V = c_v\sqrt{2g(h + p_1 - p_2)}.$$

The coefficients for a submerged orifice would be different from those for an orifice discharging into the air. It is possible that the contraction coefficient would be materially larger and the velocity coefficient somewhat smaller.

57. With Velocity of Approach.—In the preceding discussion it was assumed that the cross-section area of the vessel, from which the water issued, was so large that the velocity at (1) was negligible. In case this is not so, it will be necessary to consider the velocity head at (1). The velocity at such a point is called *velocity of approach.*

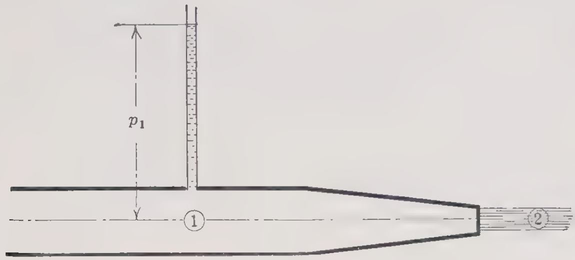


FIG. 73.

Practically the velocity of approach is usually negligible in all cases, except where there is an orifice or a nozzle on the end of a pipe, as shown in Fig. 73. For points (1) and (2),

$$H_1 = p_1 + 0 + \frac{V_1^2}{2g}, \; H_2 = 0 + 0 + \frac{V_2^2}{2g},$$

and

$$H'_{1-2} = k\frac{V_2^2}{2g}.$$

Applying the general equation, Eq. (26),

$$H_1 - k\frac{V_2^2}{2g} = \frac{V_2^2}{2g}.$$

From this,

$$V_2 = \frac{1}{\sqrt{1 + k}} \sqrt{2gH_1} = c_v\sqrt{2gH_1}. \qquad (34)$$

This is identical with Eq. (29), but here H_1 equals the value given above and not h as in Art. 55. It may be seen that p_1 has replaced the h used in that article, but the two are really interchangeable, as h in that case may be said to represent the pressure near the orifice. The real difference is in the addition of $V_1^2/2g$.

Inserting in this equation the value of H_1 the result is

$$V_2 = c_v\sqrt{2g\left(p_1 + \frac{V_1^2}{2g}\right)}. \tag{35}$$

In case the velocity in the pipe is known in some way but the area of the jet is unknown this equation could be used directly to find the velocity of the jet.[1]

If the velocity in the pipe is not known, it is then necessary to know the area of the jet or the orifice, as well as the area of the pipe. By the equation of continuity $A_1V_1 = A_2V_2 = c_cA_oV_2$. Substituting for V_1 in Eq. (35) and rearranging

$$V_2 = c_v\sqrt{\frac{2gp_1}{1 - c_v^2(A_2/A_1)^2}}. \tag{36}$$

Note that if the area of the mouth of the nozzle A_o were given, it would be necessary to know or estimate the value of c_c so as to get A_2, which is the area of the jet at the "vena contracta."

The rate of discharge may be obtained by multiplying V_2 by either A_2 or its equivalent, c_cA_o. Using the latter value and reducing,

$$q = cA_o\sqrt{\frac{2gp_1}{1 - c^2(A_o/A_1)^2}}. \tag{37}$$

If the area of the jet, A_2, were given and not the area of the opening, A_o, the simplest procedure would be to refer to Eq. (36) and insert the value of A_2 in it to obtain the rate of discharge.

Equation (36) might be simplified by omitting the c_v under the radical sign and compensating for it by using a slightly different numerical value of the coefficient outside the radical. But this numerical value would then be different from the one that applies in Eq. (35). Equation (37) could likewise be simplified in a similar way, but again the numerical value of c that would be employed would be inconsistent with values of c used under other circumstances.

[1] If the velocity in the pipe were known, the rate of discharge could as readily be computed from it as from the jet velocity, unless there was some uncertainty about the area at section (1) or some uncertainty about the exact value of the velocity at that point. Since $V_1^2/2g$ is, as a rule, relatively small compared with p_1 a reasonable amount of error in an estimation of the value of V_1 makes but small difference in the value of V_2 computed by this equation. If, also, these values were known precisely and in addition the area of the jet were known, then the velocity of the jet could be computed by the equation of continuity. In Eq. (35), as in Eq. (29), the velocity of the jet is independent of its size.

EXAMPLES

73. Water issues from a vertical orifice (one in a vertical plane) under a head of 16 ft. The diameter of the orifice = 2 in. When measured it was found that q = 33 cu. ft. per minute. What is the coefficient of discharge? If the coefficient of velocity is assumed to be 0.96, what is the value of the coefficient of contraction? What will be the diameter of the jet?

Ans. 0.786, 0.817, 1.8 in.

74. The discharge from an orifice under a head of 230 ft. was found to be 180 cu. ft. per minute. The jet was found to be 2.16 in. in diameter while the diameter of the circular orifice was 2.25 in. What are the coefficients of velocity, contraction, and discharge?

Ans. 0.97, 0.92, 0.89.

75. The velocity of water in a 6-in. pipe is 12 ft. per second. At the end of the pipe is a nozzle whose velocity coefficient is 0.98. If the pressure in the pipe at the base of the nozzle is 10 lb. per square inch, what is the velocity of the jet? What is the diameter of the jet? What is the rate of discharge?

Ans. 39.6, 3.3 in., 2.35.

76. A jet 2 in. in diameter is discharged through a nozzle whose velocity coefficient is 0.98. In the pipe at the base of the nozzle there is a pressure of 10 lb. per square inch, the diameter of the pipe at that point being 6 in. What is the velocity of the jet? What is the rate of discharge?

Ans. 38.1, 0.83.

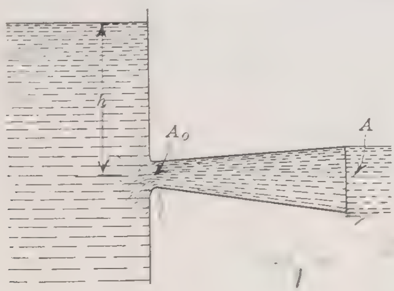


FIG. 74.

58. Flow through Diverging Tube.—The problem of flow through a diverging tube is sufficiently different from the previous cases to warrant a separate treatment. Referring to Fig. 74, let A_o and V_o be the area and velocity respectively at the point of entrance to the tube, where the cross-section is a minimum, while A and V denote the area and velocity respectively of the jet at the end. This area may be considered as the same as the area of the mouth of the tube.

Considering first the ideal case with no friction losses, $V = \sqrt{2gh}$ and $q = A\sqrt{2gh}$. From the equation of continuity, $A_oV_o = AV$. Therefore, $V_o = (A/A_o)V$. Since the velocity

at the throat (point of minimum diameter) is greater than that at the end of the tube, it follows by Bernoulli's theorem that the pressure is correspondingly less than that of the atmosphere.

In the actual case $V = c_v\sqrt{2gh}$ or $= c\sqrt{2gh}$, since the coefficient of contraction is unity. Therefore actually

$$q = A_oV_o = AV = cA\sqrt{2gh}.$$

The coefficient of discharge (which is the same as that of velocity in this case) is here seen to represent the ratio of actual to ideal values, in accordance with our definition. So far the treatment has been exactly the same as the preceding cases.

Suppose, however, that the area of the throat is used, instead, so that

$$q = c_oA_o\sqrt{2gh}.$$

It is then apparent that $c_o = c(A/A_o)$, and while c must always be less than unity in any actual case, the value of c_o may be greater than unity. In fact values of over 1.55 have been attained. Actual values of c, however, are only about 0.46.

It is apparent that this last expression for q is absolutely irrational since the area A_o of one cross-section is multiplied by the ideal velocity $\sqrt{2gh}$ at a *different* cross-section. As has been stated, it may be proved by Bernoulli's theorem that the velocity at the throat is higher than $\sqrt{2gh}$.

The merit of this last expression for q, involving a coefficient c_o greater than unity, is that it enables us to make a direct comparison between the quantity discharged through a standard orifice or tube and that discharged through a diverging tube with the same area A_o. Thus if the coefficient of discharge for a standard orifice is 0.60, while c_o for a diverging tube is 1.55, it follows that the latter will discharge 1.55/0.60 times as much as the former through an opening of the *same* size in the *wall of the vessel.*

EXAMPLE

77. Prove that, in the ideal case, the pressure at the throat of a diverging tube is less than the atmospheric pressure by $[(A/A_o)^2 - 1]h$.

59. Values of Orifice Coefficients.—The value of the coefficient of velocity is always less than unity, though it may often approach that value very closely. In some well-made nozzles and sharp-edged orifices the coefficient of velocity may be as high as 0.98 and occasionally 0.99 may be attained.

A *standard orifice* is one with a sharp edge as in Fig. 75 (*a*). It is called a standard orifice because one will give practically the same results as another of the same size. Any other form of orifice such as (*c*) in Fig. 75 would give different results depending upon the thickness of the plate, the roughness of the material, etc.; hence the coefficients for each individual one would have to be determined if accurate computations were desired.

If the plate is thin and the inner corner is square and sharp, the orifice in Fig. 75 (*b*) may also be considered a standard orifice. But if the plate is too thick the condition in Fig. 75 (*c*)

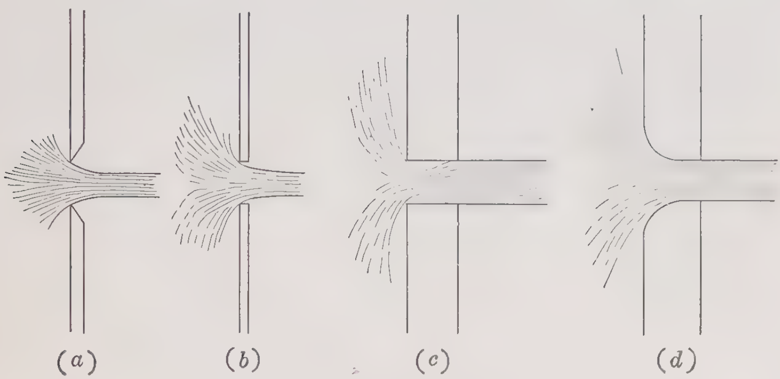


FIG. 75.—Types of orifices.

is met with and the velocity coefficient will be less than in the former case due to the greater frictional resistance the water encounters. Rounding the edge as in Fig. 75 (*d*) reduces the eddy losses and hence increases the coefficient of velocity.

The contraction coefficient is much more sensitive to variations in the nature of the orifice than is the velocity coefficient. But it should be noted that contraction affects only the size of the jet and not its velocity. The coefficient of contraction is the least in (*a*) and (*b*) and may be unity in (*c*) and (*d*). Hence the latter forms of orifices may discharge much more water than the former. Thus the type of orifice to be used depends upon whether a large discharge is desired or the maximum velocity. Of course if an orifice is used as a means of measuring rate of discharge only the standard orifice should be employed unless the special orifice can be calibrated.

Minimum values of the coefficient of contraction are in the neighborhood of 0.62, so that the minimum value for the coefficient of discharge is about 0.60. This applies to sharp-edged orifices such as (*a*) and (*b*) in Fig. 75. For (*c*) and (*d*) if there is no contraction, the coefficient of discharge coincides with the coefficient of velocity and may be as high as 0.98 in (*d*) and somewhat less in (*c*).

Values of coefficients of discharge for standard circular and square orifices are given in Tables I and II respectively. These orifices are to be sharp-edged and to be in vertical planes. The orifice should be so situated that it is flush with a flat wall on the water side free from all obstructions, projections, and sides for a distance in all directions of at least three times the diameter of the orifice. If this is not so the full contraction will not be obtained and the actual coefficient will be larger than given by the tables. The tables show that the coefficients are different for

Table I.—Discharge Coefficients for Standard Circular Vertical Orifices According to Hamilton Smith

Head on center of orifice in feet	Diameter of orifice in feet								
	0.02	0.03	0.04	0.05	0.07	0.10	0.20	0.60	1.00
0.4	......	......	0.637	0.631	0.624	0.618			
0.6	0.655	0.640	0.630	0.624	0.618	0.613	**0.601**	**0.593**	
0.8	0.648	0.634	0.626	0.620	0.615	0.610	0.601	**0.594**	**0.590**
1.0	0.644	0.631	0.623	0.617	0.612	0.608	0.600	**0.595**	**0.591**
1.4	0.638	0.625	0.618	0.613	0.609	0.605	0.600	**0.595**	**0.593**
2.0	0.632	0.621	0.614	0.610	0.607	0.604	0.599	**0.597**	**0.595**
2.5	0.629	0.619	0.612	0.608	0.605	0.603	0.599	0.598	**0.596**
3.0	0.627	0.617	0.611	0.606	0.604	0.603	0.599	0.598	**0.597**
3.5	0.625	0.616	0.610	0.606	0.604	0.602	0.599	0.598	0.596
4.0	0.623	0.614	0.609	0.605	0.603	0.602	0.599	0.597	0.596
6.0	0.618	0.611	0.607	0.604	0.602	0.600	0.598	0.597	0.596
8.0	0.614	0.608	0.605	0.603	0.601	0.600	0.598	0.596	0.596
10.0	0.611	0.606	0.603	0.601	0.599	0.598	0.597	0.596	0.595
20.0	0.601	0.600	0.599	0.598	0.597	0.596	0.596	0.596	0.594
50.0	0.596	0.596	0.595	0.595	0.594	0.594	0.594	0.594	0.593
100.0	0.593	0.593	0.592	0.592	0.592	0.592	0.592	0.592	0.592

TABLE II.—DISCHARGE COEFFICIENTS FOR STANDARD SQUARE VERTICAL ORIFICES ACCORDING TO HAMILTON SMITH

Head on center of orifice in feet	Side of square in feet						
	0.02	0.04	0.07	0.10	0.20	0.60	1.00
0.4	.....	0.643	0.628	0.621			
0.6	0.660	0.636	0.623	0.617	**0.605**	**0.598**	
0.8	0.652	0.631	0.620	0.615	0.605	**0.600**	**0.597**
1.0	0.648	0.628	0.618	0.613	0.605	**0.601**	**0.599**
1.4	0.642	0.623	0.614	0.610	0.605	**0.601**	**0.598**
2.0	0.637	0.619	0.613	0.608	0.605	**0.604**	**0.602**
2.5	0.634	0.617	0.610	0.607	0.605	0.604	**0.602**
3.0	0.632	0.616	0.609	0.607	0.605	0.604	**0.603**
3.5	0.630	0.615	0.609	0.607	0.605	0.604	0.602
4.0	0.628	0.614	0.608	0.606	0.605	0.603	0.602
6.0	0.623	0.612	0.607	0.605	0.604	0.603	0.602
8.0	0.619	0.610	0.606	0.605	0.604	0.603	0.602
10.0	0.616	0.608	0.605	0.604	0.603	0.602	0.601
20.0	0.606	0.604	0.602	0.602	0.602	0.601	0.600
50.0	0.602	0.601	0.601	0.600	0.600	0.600	0.599
100.0	0.599	0.598	0.598	0.598	0.598	0.598	0.598

different sizes, and for a given orifice vary with the head on the orifice, thus illustrating the impossibility of stating general values or laws that hold in all cases. It will be noted that the coefficients decrease as the head increases, and that as higher values are attained the rate of decrease is much smaller. For heads above 100 ft. the values for 100 ft. may doubtless be applied with little error.

60. Coefficients for Short Tubes.—As a measuring device the tube is not quite as good as the standard orifice, since the coefficients of the latter are known with greater accuracy and are less subject to variations. Referring to Fig. 76, the contraction coefficients are all unity on the assumption that the water fills the tube in each case. The discharge coefficients and the velocity coefficients are then identical. Typical values of velocity coefficients are shown in the figure, though these are subject to some variation. The highest value is found in (*a*) since the eddy

losses are reduced to a minimum. The greatest eddy loss is met with in the case of the re-entrant tube shown in (c) and hence its velocity coefficient is the least.[1]

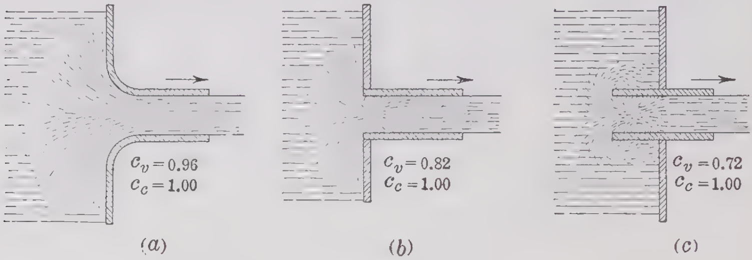


FIG. 76.—Coefficients for tubes.

61. Coefficients for Nozzles.—A nozzle may be a plain conical nozzle as in Fig. 76 (a) or a smooth convex nozzle as shown in Fig. 77 (c). The jet from a nozzle may undergo some contrac-

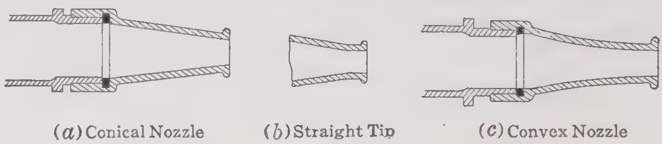


FIG. 77.—Standard nozzles.

tion or, if a small portion near the mouth is of uniform diameter as in Fig. 77 (b), the water may leave in parallel lines and suffer no contraction.

[1] The standard short tube, such as in Fig. 76 (b) will have a length of about 2.5 to 3 diameters. Due to the contraction of the stream as it enters the tube the velocity is increased and the pressure is decreased to a value less than that of the atmosphere by about $0.8h$ at a distance about 0.4 the diameter from the entrance. As the head h is increased this pressure approaches absolute zero and at a head of about 40 ft. the conditions suddenly change, the jet springs clear of the tube, merely touching the sharp edge, and behaves as a jet from a standard sharp-edged orifice. On lowering the head the jet will continue to spring clear of the tube until very low values are reached. When the jet fills the tube, the discharge is very turbulent, due to the eddies. If the re-entrant tube in (c) is short and extends into the vessel a distance about equal to the diameter and does not extend outside of the vessel, it is known as Borda's mouthpiece. This acts as a standard orifice, except that the contraction is greater. The coefficient of discharge is 0.53, which is the minimum value found with any tube or orifice. See article by SCHODER on "Hydraulics" in MARKS, "Mechanical Engineers' Handbook."

The velocity coefficients of well-made nozzles are very high, being practically equal to those of a standard circular orifice. We may reasonably assume an average value of the velocity coefficient of 0.98, though even this is often exceeded.[1]

From a photograph by the author.

Fig. 78.—Jet from hydraulic giant washing out material for earth fill dam.

Nozzles may be used as water-measuring devices the same as standard orifices, and are especially useful for that purpose when

[1] Freeman, John R., *Trans. A. S. C. E.*, vol. 21, p. 303, 1889; *Trans. A. S. C. E.*, vol. 24, p. 492, 1891.

Eckart, W. R., Jr., *Inst. Mech. Eng.*, Jan. 7, 1910.

Fleming, V. R., *Proc. Fifth Meeting of Ill. Water Supply Assoc.*, 1913.

Daugherty, R. L., "Hydraulic Turbines," 3 ed. p. 114.

high heads are employed. They may also be used to furnish jets at high velocities for fire purposes, for power, or for hydraulic mining and similar work such as shown in Fig. 78.

The height to which a good fire stream can be thrown by a nozzle is from about two-thirds to three-fourths of the effective head at the base of the nozzle. The proportion is higher for large jets than for small ones, for smooth nozzles than rough ones, and for low pressures than for high pressures.

62. Loss of Head in Nozzle.—Although a nozzle does not produce an abrupt change of velocity, it nevertheless causes a certain loss of head by virtue of which its velocity coefficient is

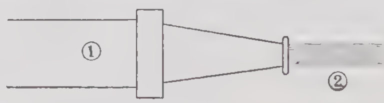


FIG. 79.—Loss in nozzle.

less than unity. For Fig. 79 may be written $H' = H_1 - H_2$. Since $V_2 = c_v\sqrt{2gH_1}$,

$$H_1 = \frac{1}{c_v^2}\frac{V_2^2}{2g},$$

and

$$H_2 = \frac{V_2^2}{2g}.$$

Therefore for the nozzle,

$$H' = \left(\frac{1}{c_v^2} - 1\right)\frac{V_2^2}{2g}, \tag{38}$$

giving for k the value

$$k = \frac{1}{c_v^2} - 1,$$

exactly as in the case of the orifice in Art. 55. Note that in Eq. (38) the loss of head in the nozzle is based upon the jet velocity.

63. Efficiency of Nozzle.—Since a nozzle is frequently employed for power purposes, we may be interested in its efficiency. The efficiency of a nozzle may be defined as the ratio of the power in the jet to the power delivered to the nozzle. But it has been seen that for a given rate of discharge, power is directly proportional to head. Thus referring to Fig. 73,

$$e = \frac{H_2}{H_1}.$$

But from Eq. (34) $H_2 = V_2^2/2g = c_v^2 H_1$. From this it may be seen that

$$e = c_v^2. \tag{39}$$

This would be exactly true if all particles of water in the jet possessed the same velocity and hence the same kinetic energy. Actually V_2 is the average velocity of the jet, and it has been stated in Art. 49 that the true kinetic energy of a stream is greater than that obtained by using the square of the average velocity. Hence the true efficiency of a jet from a good nozzle may be about 1 per cent more than the value given by Eq. (39).

EXAMPLES

78. In Fig. 67 the actual measured diameter of the minimum section of the jet was $6^{13}/_{16}$ in., the area of the nozzle opening being 43.02 sq. in. Compute the coefficients of velocity, contraction, and discharge using the values of H_1 and V_2 given. What is the efficiency of the nozzle? What is the horsepower in the jet?

Ans. 0.989, 0.846, 0.837, 97.8 per cent, 5,250 hp.

79. What is the value of the head lost in hydraulic friction in the nozzle of Fig. 63? What is the value of k?

Ans. 17 ft., 0.022.

80. Find values of the coefficient of loss when the velocity coefficient has values of 1.00, 0.99, 0.95 and 0.80. Find c_v when k has values of 0, 0.5, 1.00.

Ans. $k = 0.562$, $c_v = 0.707$ for last values in each set.

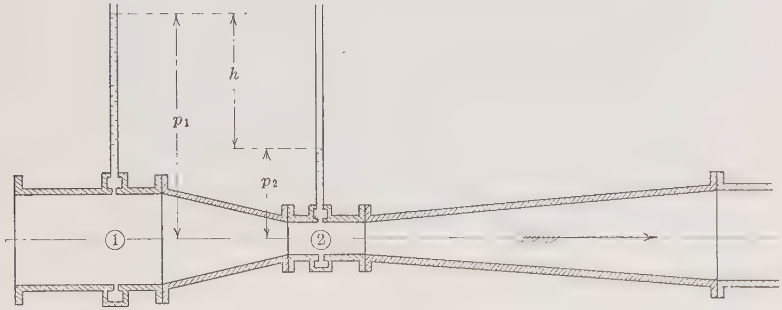


FIG. 80.—Venturi meter.

64. Venturi Meter.—If water is caused to flow through the device shown in Fig. 80, the increased velocity through the "throat" will produce a corresponding pressure drop. This drop in pressure may be made to serve as a measure of the rate of discharge. Such an instrument is called a Venturi meter.

It may be seen that the Venturi meter is very similar in principle to the nozzle. In both there is an increase in velocity of the

water accompanied by a corresponding pressure drop. And in both the rate of discharge may be found to be a function of the pressure drop. The only difference is that the pressure at the throat of the Venturi meter may be either somewhat greater or less than atmospheric, and the stream at that point is not a free jet but is expanded again to fill the pipe below the meter. Hence the equations for the nozzle would seem to apply directly to the Venturi meter, with p_1 equal to the pressure drop in both cases.

Actually the coefficients for the Venturi meter are based upon a formula derived by a slightly different procedure. Thus H_1 is equated to H_2, assuming that H' is zero, and then a coefficient is introduced as the very last step. Assuming the meter to be horizontal so that $z_1 = z_2$, we have[1]

$$p_1 + \frac{V_1^2}{2g} = p_2 + \frac{V_2^2}{2g}.$$

From this,

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = p_1 - p_2 = h.$$

By the equation of continuity $V_1 = (A_2/A_1)V_2$, and hence,

$$V_2 = \sqrt{\frac{2gh}{1 - (A_2/A_1)^2}}.$$

Since there is some slight loss of head between (1) and (2) the true velocity will be less than this and so it is multiplied by a velocity coefficient. The result is

$$V_2 = c_v\sqrt{\frac{2gh}{1 - (A_2/A_1)^2}}. \tag{40}$$

This may be seen to differ from Eq. (36) in that the term $(A_2/A_1)^2$ is not multiplied by c_v^2 but both Eqs. (36) and (40) could be made to yield the same numerical value by using a slightly different value of c_v for the two. Custom has based values of c_v upon Eq. (40) for the Venturi meter and upon Eq. (36) for the nozzle.

With the Venturi meter q is desired, not V_2, and hence, multiplying Eq. (40) by A_2 and replacing c_v by c, the result is

$$q = cA_2\sqrt{\frac{2gh}{1 - (A_2/A_1)^2}}. \tag{41}$$

[1] In case the meter is not horizontal, as shown in Fig. 80, it will be found that $(p_1 + z_1) - (p_2 + z_2) = h$, so the results are the same as here presented.

For a given meter A_1 and A_2 are known quantities and, if $K' = A_2\sqrt{2g}\sqrt{1-(A_2/A_1)^2}$, this may be reduced to

$$q = cK'\sqrt{h}. \tag{42}$$

The coefficient c may be assumed to be 0.995 for very large meters and 0.98 for small ones. If the throat area becomes reduced by a thin deposit of scale the effect is to cause the meter to read too high a value of h and hence require a lower value of c to compensate for the change in areas. Otherwise these coefficients may be relied upon to give a result that is very accurate. The coefficient is practically a constant, though it increases slightly with the velocity. If c is assumed constant for any given meter, it is convenient to replace cK' by K, which gives

Courtesy of Builder's Iron Foundry.

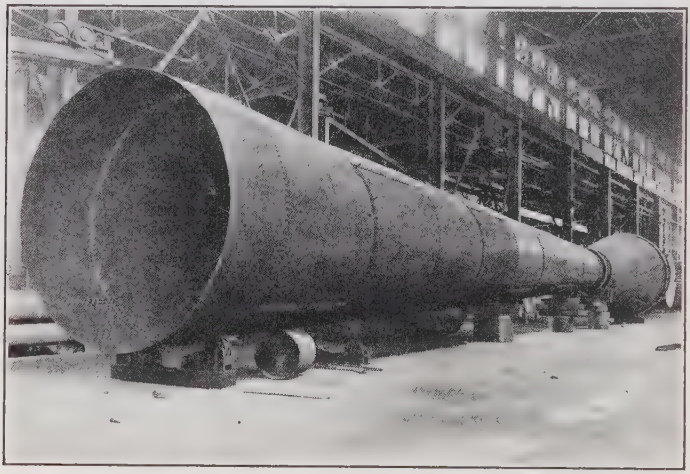
Fig. 81.—Venturi meter in wood pipe line.

$$q = K\sqrt{h}. \tag{43}$$

The Venturi meter, invented by Clemens Herschel in 1886, affords a most valuable and accurate means of measuring water,

especially in large quantities.[1] By a suitable recording device it is possible to make a continuous record of the flow of water through any pipe line in which a Venturi meter is installed. The sole objection to its permanent use in a pipe is that it must necessarily cause some slight friction loss or resistance to flow. If this loss be expressed as $H' = kV_2^2/2g$, then values of k range from about 0.1 to 0.2. The higher values of k naturally go with smaller values of A_2/A_1. Another way of expressing the loss is that it is equal to $\frac{1}{10}$ to $\frac{1}{8}h$ ordinarily.

The usual ratio of the diameter of the throat to the diameter of the pipe is about 1 to 3, making the ratio $A_2/A_1 = \frac{1}{9}$. But in



Courtesy of Builder's Iron Foundry.

FIG. 82.—Venturi meter of riveted steel.

order to reduce the resistance as much as possible and also to avoid producing pressures at the throat below atmospheric, it is quite common to make the diameters in the ratio 1 : 2, making $A_2/A_1 = \frac{1}{4}$. Of course this reduces the magnitude of h for a given rate of discharge and hence makes the readings less accurate, especially for very low discharges. In special cases a ratio as low as $1 : 1\frac{1}{3}$ is used.

A diverging stream is always less stable than a converging stream, that is it is more readily broken up into whirlpools and eddies, and hence more loss of energy takes place in the portion of the meter on the downstream side of the throat. This is

[1] *Trans. A. S. C. E.*, vol. 17, p. 228, 1887.

clearly shown in Fig. 60. In order to minimize this, the downstream portion is made to taper much more gradually than the upstream side.

EXAMPLES

81. A Venturi meter with a 4-in. throat is to be used in a 12-in. pipe line. Assuming a value of $c = 0.985$, determine the value of K for this meter.
Ans. 0.693.

82. If a differential manometer employing mercury (sp. gr. = 13.57) were to be used, determine the value of K for the Venturi meter in problem 81, replacing h by y (Fig. 13) in inches of mercury.
Ans. 0.709.

83. Suppose the throat of the meter in problem 81 were to be 6 in. the pipe remaining 12 in. Compute the value of K.
Ans. 1.600.

84. Suppose that 5 cu. ft. per second is flowing through the Venturi meter. What are the values of h in problems 81 and 83?
Ans. 52.0, 9.79 ft.

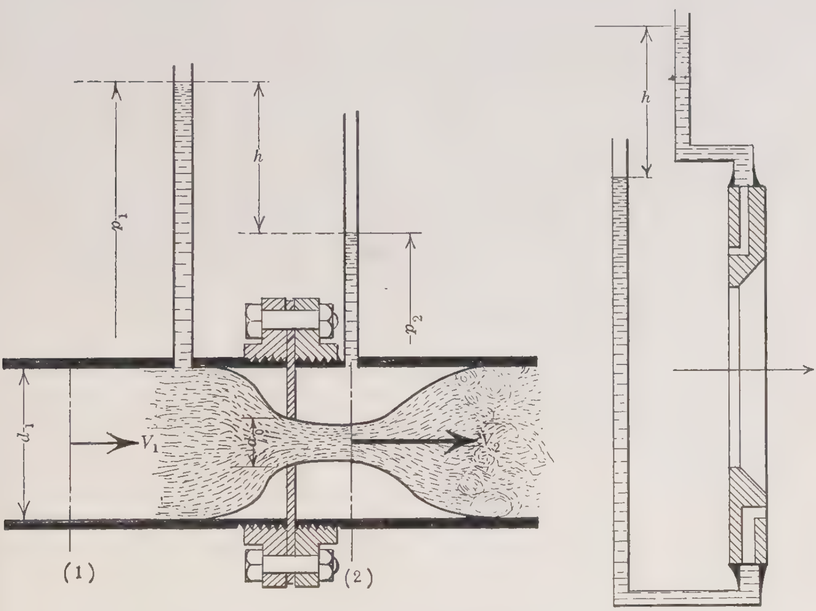


FIG. 83.—The orifice meter.

65. The Orifice Meter.—A satisfactory meter may be made by inserting a thin-plate, sharp-edged orifice in a pipe line, as shown in Fig. 83. Below the orifice there would be a jet of water, which would be surrounded by a body of water with very little velocity.

Beyond the section of minimum contraction there would be considerable eddying and turbulence as the stream expanded and the water slowed down to the normal velocity in the pipe. The section of normal flow would be found a distance of 3 or 4 pipe diameters downstream, the conditions being very similar to those shown in Fig. 102. It has been found that the proper place to take the pressure readings is at a distance 0.8 the pipe diameter upstream and 0.4 the pipe diameter downstream. At the latter place the velocity is a maximum and the pressure a minimum.[1]

An alternate pair of connections for measuring the pressure differential may be made in the orifice plate itself, as is shown in the sketch on the right-hand side of Fig. 83. In some extensive experiments made with a 4.5-in. orifice in a 6-in. pipe, the author found the differential reading to be exactly the same for the two methods of measurement. The latter method has the advantage of making the meter entirely self-contained. That is, it is necessary merely to slip in the plate between the flanges of a pipe line, and no other work on the pipe is required.

The orifice meter may be seen to be very similar to the Venturi meter in its use and in its theory. One important difference, however, is that at the point of minimum cross-section area, where the pressure p_2 is measured, the area A_2, which enters into the Venturi meter formula, is unknown. It must be expressed in terms of A_o, the known area of the orifice, and a coefficient of contraction. It will, therefore, be found more convenient to use the slightly different formula derived for the orifice (or nozzle) with velocity of approach, as this equation involves the area of the orifice and not the area of the jet. Replacing the letter p_1 by its equivalent h, Eq. (37) may be written either in terms of the orifice area A_o or the pipe area A_1. Thus

$$q = cA_o\sqrt{\frac{2gh}{1 - c^2(A_o/A_1)^2}} = A_1\sqrt{\frac{2gh}{(1/c^2)(A_1/A_o)^2 - 1}}. \quad (44)$$

In the case of the jet discharging into air, the coefficient c is practically a constant for any given type of orifice or nozzle. But in the case of discharge into water in a pipe, as in Fig. 83, the coefficient seems to vary considerably with the ratio of the diameter of the pipe to the diameter of the orifice and also with

[1] Davis and Jordan, *Univ. Illinois*, *Bull.* 109, Engineering Experiment Station.

the absolute size of the pipe used. In a general way c varies from 0.595 to 0.64, for values of d_1/d_0 from 2 to 8, but for smaller values of this ratio the coefficient increases rapidly, reaching a value of 0.771 for a ratio of 1.22.

In order to avoid this difficulty Davis and Jordan have devised an empirical formula which holds for values of d_1/d_o from 2 to 6. It is

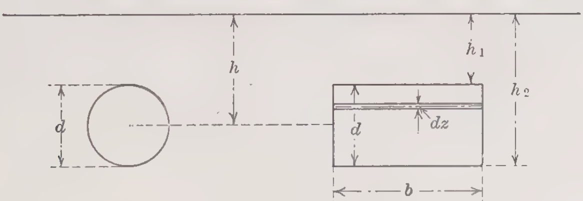
$$q = A\frac{4.85\sqrt{h}}{\left(\frac{d_1}{d_o}\right)^2 - \left(\frac{d_o}{d_1}\right)^3}. \tag{45}$$

For smaller values of this ratio one would need to use the rational equation with the proper value of the coefficient.[1]

The disadvantage of the orifice meter is that it causes more loss of head, all other things being equal, than a Venturi meter. Davis and Jordan also propose an empirical formula for this loss, which is

$$H' = 0.0366F\left[\left(\frac{d_1}{d_o}\right)^2 - \frac{d_o}{d_1}\right]^{2.02} \times V_1^2 \tag{46}$$

where F is a factor depending upon the size of the pipe. It is 0.98 for a 4-in., 1.02 for a 6-in., and 1.04 for a 12-in. pipe. This expression is said to be correct within about 2 per cent, but for a ratio $d_1/d_o = 1.2$ it gives a result about 3 per cent too small.



(a) Circular Orifice (b) Rectangular Orifice

FIG. 84.

66. Large Vertical Orifice.—In the case of an orifice whose vertical dimensions are large as compared with its depth below the free surface it is necessary to proceed as follows: Choose an elementary area dA such that all portions are at the same depth z below the free surface. Now by Art. 55 the rate of discharge through this strip may be expressed as

$$dq = c\sqrt{2gz}\,dA.$$

[1] In the case of the 4.5 in. orifice in the 6-in. pipe tested by the author with volume measurements in a tank of 5,600-cu. ft. capacity, the results were given precisely by the equation $q = 0.642h^{0.485}$.

The rate of discharge through the entire orifice may be obtained by integrating the above. Thus

$$q = c\sqrt{2g}\int z^{1/2}dA. \tag{47}$$

In the case of a rectangular orifice (Fig. 84) take $dA = bdz$, and after integrating obtain

$$q = c\tfrac{2}{3}\sqrt{2g}\ b(h_2^{3/2} - h_1^{3/2}). \tag{48}$$

If h = depth of center of gravity below the free surface,

$$h_2 = h + \frac{d}{2};\ h_1 = h - \frac{d}{2}.$$

Expanding $(h + d/2)^{3/2}$ and $(h - d/2)^{3/2}$ by the binomial theorem and substituting in Eq. (48) the following is obtained

$$q = cbd\sqrt{2gh}\left[1 - \frac{d^2}{96h^2} - \frac{d^4}{2048h^4} - \ .\ .\ .\ \right]. \tag{49}$$

The expression in brackets is a rapidly converging series and its value is always less than unity. When $h = d$, the value of this factor is 0.989, while for $h = 2d$, its value becomes 0.997. Thus for any head greater than $2d$ the rate of discharge may be obtained by the simpler formula $q = cA\sqrt{2gh}$.

In similar manner the rate of discharge through a circular orifice of area A is given by the expression

$$q = cA\sqrt{2gh}\left[1 - \frac{d^2}{128h^2} - \frac{5d^4}{16{,}384h^4} - \ .\ .\ .\ \right]. \tag{50}$$

It may be found that when $h = 2d$ the value of this series is 0.998, thus indicating that the use of the exact formula is unnecessary for heads above that value. In Tables I and II the coefficients in black-face type are to be used in the exact formulas, all other coefficients to be used in the formulas of Art. 55.

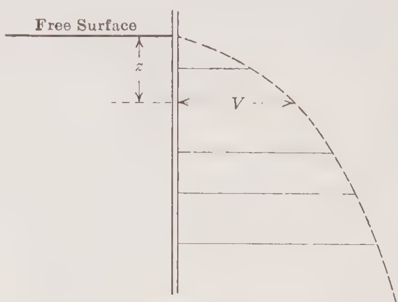


FIG. 85.

67. Weir.—A weir is a special form of orifice, its distinguishing feature being that it is placed at the water surface so that the

head on its upper edge is zero. Thus the usual formulas for orifices given in Art. 55 can no longer be applied and the methods of Art. 66 must be employed. The weir is one of the most widely accepted standard devices for the measurement of water.

If it be assumed that the velocity of water through an orifice varies as the square root of the depth, the curve in Fig. 85 would give a true representation of the flow. However, the particles of water at the surface of the weir opening do not remain at rest but flow with considerable velocity. It may also be observed that the level of the water at this point drops below its normal value, as shown in Fig. 86. Also it must be noted that the stream lines flowing through the weir are not necessarily normal to the plane of the weir; hence it is hardly correct to multiply their velocities

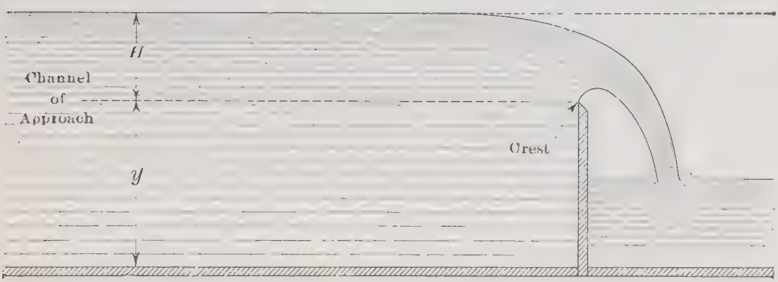
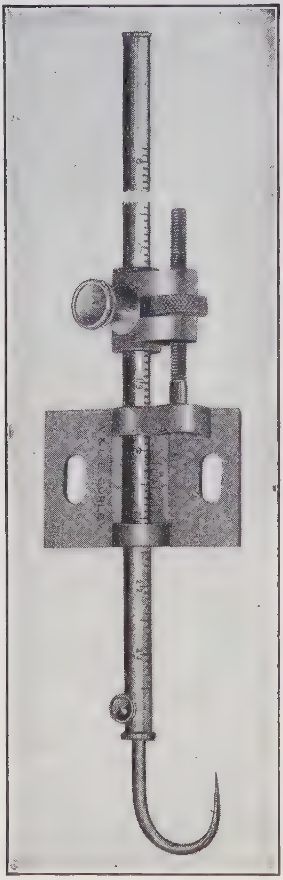


FIG. 86.—Weir.

by areas in the plane of the latter. For these and other reasons it is impossible to derive by theory weir formulas which are exactly correct, but they serve as expressions which may be made to yield correct results by the proper choice and use of experimental coefficients.

It might seem natural to measure the depth of water flowing over the crest of a weir, but in practice it is difficult to do this with any degree of accuracy. It is found more feasible to measure the elevation, above the weir crest, of the water surface at some distance back from the weir, where the water is relatively quiet. Thus all weir formulas express the rate of discharge as a function of H (Fig. 86). This measurement must be taken at a point far enough back to avoid the effects of the surface curve. This distance should be at least $6H$. The usual instrument for measuring H is the hook gage, one form of which is shown in Fig. 87. The gage should be mounted on some rigid support.

In using it the sharp-pointed hook is submerged beneath the surface and then carefully raised until a slight distortion may be seen on the water surface. The hook should then be lowered until this distortion barely disappears. From this reading the value of H is obtained by subtracting the "hook gage zero," which is the reading of the gage when its point is just level with the crest of the weir, as the lower edge is called.



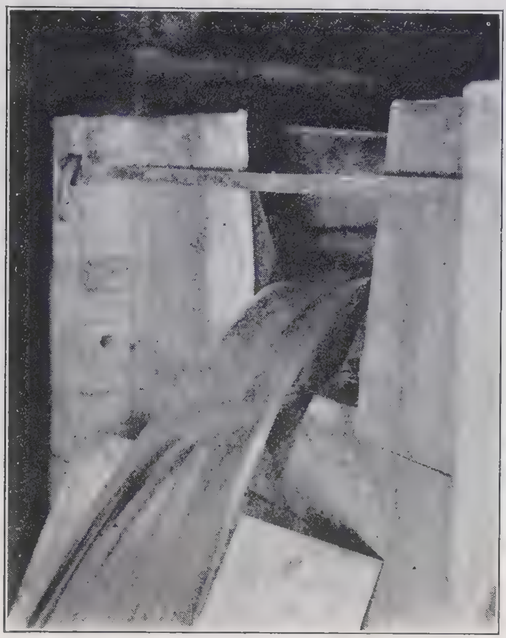
W. & L. E. Gurley.

FIG. 87.—Hook gage.

The Triangular Weir.—The triangular weir such as is shown in Fig. 88 is useful for measuring relatively small rates of discharge, as a reasonable value of H may be obtained by employing a sufficiently small vertex angle. But for discharges much above 2 or 3 cu. ft. per second excessively high values of H are necessary and other types of weir would then be used.

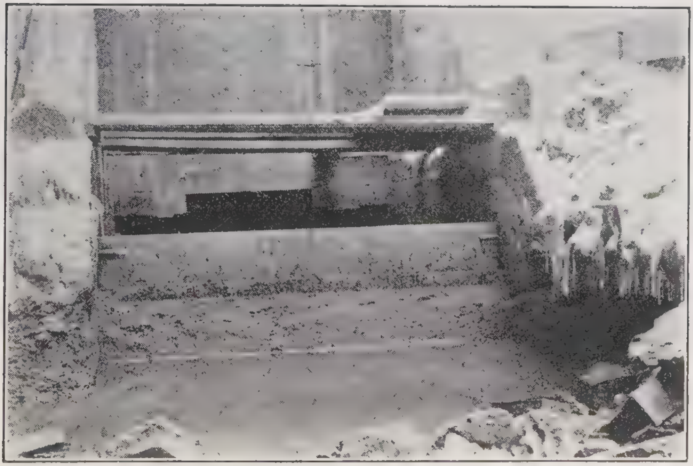
The Suppressed Rectangular Weir.—Probably the most common type of weir is one whose shape is rectangular. If the width of the weir is the same as that of the channel of approach, as in Figs. 89 and 90, the stream of water flowing over the crest will not undergo any lateral contraction, that is the end contractions are suppressed. With this type of weir it is customary to extend the sides of the channel beyond the crest so that the falling stream is bounded by them. If these sides are not so extended the stream will expand somewhat and the discharge for a given value of H will be slightly larger than in the standard type.

It is necessary in this or any type of weir to insure that the weir is "ventilated," that is that air has access to the under side of the falling water. Otherwise the air will be gradually swept out and the water will tend to cling to the face of the weir instead of spring clear of it. For a given value of H the rate of discharge



From a photograph by the author.

FIG. 88.—Discharge from a 60° triangular weir.



From a photograph by the author.

FIG. 89.—Rectangular weir without end contractions.

would then be greatly increased and the usual coefficients would no longer apply.

From a photograph by the author.

Fig. 90.—Rectangular weir without end contractions.

From a photograph by F. H. Fowler.

Fig. 91.—Rectangular weir with end contractions.

In order to insure that the water shall spring clear, that is that perfect crest contraction shall be attained, it is necessary to have a sharp edge on the weir plate. This may be produced by

beveling the edge of a metal plate down to a knife-edge. However, a perfectly sharp square shoulder is just as good as a knife-edge if the plate is not too thick for the water to clear the other shoulder. For very low values of H the knife-edge would still permit crest contraction to take place when the flat edge of a plate would not.

Contracted Rectangular Weir.—When the width of the weir is less than that of the channel of approach, as in Fig. 91, the contraction that occurs at each end causes the real width of the stream of water to be less than that of the weir itself. Such a weir is called a contracted weir.

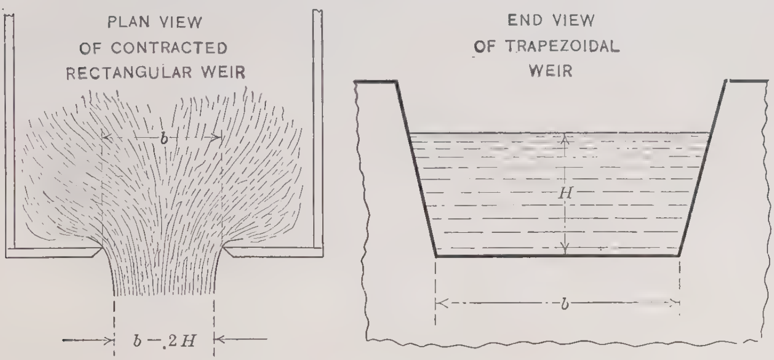


FIG. 92.

Trapezoidal or Cippoletti Weir.—The trapezoidal weir is one in which the sides of the notch are not vertical but diverge so that the width at the water surface increases. If the side slopes have the ratio of 1:4 the weir is called a Cippoletti after an Italian engineer of that name who proposed it. The advantage of this type of weir will be stated later.

68. The Triangular Weir.—Figure 93 is a triangular weir with any vertex angle θ. The rate of discharge through an elementary strip of area dA will be

$$dq = c\sqrt{2gz}\,dA.$$

Now $dA = xdz$ and by similar triangles $x:b = (H - z):H$. Hence $dA = (b/H)\ (H - z)dz$. Substituting in the above the following is obtained for the entire notch

$$q = c\sqrt{2g}\,\frac{b}{H}\int_0^H (H - z)z^{1/2}dz.$$

Integrating between limits

$$q = c\sqrt{2g}\,\frac{b}{H}\,[\tfrac{2}{3}H^{5/2} - \tfrac{2}{5}H^{5/2}].$$

But $b = 2H \tan \theta/2$. Inserting this and reducing

$$q = \tfrac{8}{15}c\sqrt{2g}\,\tan\frac{\theta}{2}\,H^{5/2}. \tag{51}$$

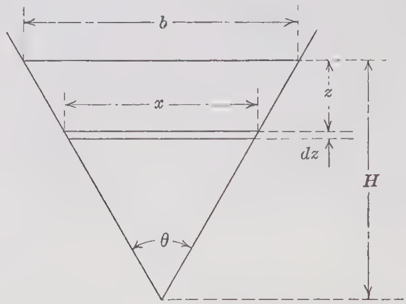


FIG. 93.

This expriesson for any given weir may be reduced to

$$q = KH^{5/2}.\text{[1]} \tag{52}$$

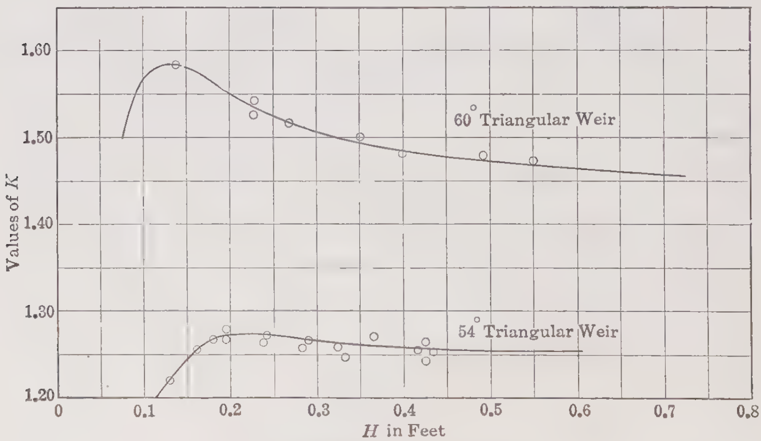


FIG. 94.—Coefficients of triangular weirs.

In Figs. 94 and 95 may be found experimental values of K for several triangular weirs. The 54-, 60-, and one of the 90-deg. weirs are in the laboratory of Sibley College, Cornell University.[2] The

[1] $(H^{5/2} = H^2\sqrt{H}.)$

[2] *Eng. News*, vol. 73, p. 636, 1915.

two lower curves in Fig. 95 were plotted from data in a very valuable paper by James Barr.[1] The weir for which the very lowest curve was constructed had a very fine sharp edge, while the other weir had a square corner and a thickness of about $\frac{1}{16}$ in. Both of these weirs have values of K below that of the Sibley College weir but the difference, of about 1 per cent, may be due to the difference in smoothness of the surface of the plate in which the notch is cut. A rougher surface has been found to decrease the contraction of the stream and thus increase the weir coefficient.

These curves show that the discharge does not vary as the five-halves power of H since c is not a constant. Thompson, who first employed the triangular weir, chose for K a value

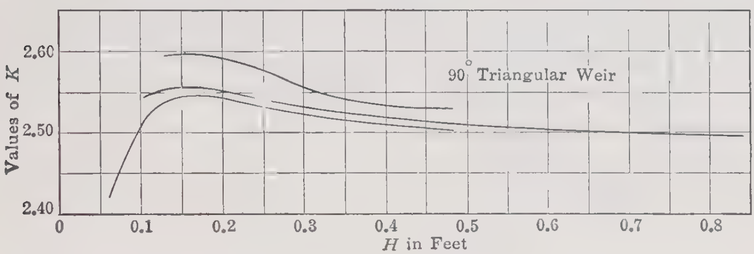


FIG. 95.—Coefficients of 90° triangular weirs.

of 2.54. This may be seen to be a fair average for ordinary weirs, but for greater precision values of K may be read from the curves for the particular value of H. As a fair average for heads above 0.2 ft., it may be found that the curves here shown will give for the 60-deg. weir

$$q = 1.42H^{2.44}, \tag{53}$$

and for the 90-deg. weir

$$q = 2.48H^{2.475}. \tag{54}$$

EXAMPLE

85. What is the rate of discharge of a 54-deg. triangular weir when $H = 0.400$ ft.? With the same value of H what would be the rate of discharge of a 90-deg. triangular weir? (Use K from curves.) What would be the value of H for a rate of discharge of 2.0 cu. ft. per second, if a 60-deg. triangular weir is used? (Use equation.)

Ans. 0.1275, 0.2550, 1.15.

[1] "Experiments upon the Flow of Water over Triangular Notches," *Engineering*, Apr. 8 and 15, 1910.

69. The Rectangular Weir.—For the rectangular weir in Fig. 96 the discharge through the elementary strip of area dA may be given by

$$dq = c\sqrt{2gz}\,bdz.$$

Integrating between limits

$$q = c\sqrt{2g}\,b\int_0^H z^{1/2}dz$$

$$q = \tfrac{2}{3}c\sqrt{2g}\,bH^{3/2}. \qquad (55)$$

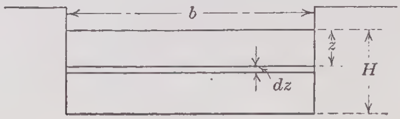


FIG. 96.

This is the fundamental formula. Many variations of it have been suggested in an attempt to express the value of c, which is not necessarily a constant for all values of H. (It may be noted that $H^{3/2} = H\sqrt{H}$.)

70. The Francis Weir Formula.—About 1850 James B. Francis made some very accurate investigations of the flow of water over weirs.[1] As a result of his experiments he selected a value of 0.622 for c in Eq. (55), so that for a suppressed weir

$$q = 3.33bH^{3/2}. \qquad (56)$$

With a contracted weir he concluded that the effect of each contraction was to reduce the effective width of the weir by $0.1H$. Thus for a contracted weir

$$q = 3.33(b - 0.1nH)H^{3/2}. \qquad (57)$$

The usual type of contracted weir will have two end contractions, giving n a value of 2 in Eq. (57), but there might be a weir with one end contracted and one end without contraction. Equation (57) is strictly empirical and applicable only within limits. If H is greater than one-third b it is impossible for perfect end contraction to occur and hence the conditions upon which Eq. (57) is based no longer exist.[2]

When the cross-section area of the channel of approach is relatively small, there may be a velocity of flow in it that is high

[1] "Lowell Hydraulic Experiments."

[2] For flow through narrow rectangular notches and other valuable practical information on weirs, see SCHODER in MARKS, "Mechanical Engineers' Handbook," 2nd ed., p. 268.

enough to affect the result. This velocity is called the velocity of approach. Francis corrected for this by replacing $H^{3/2}$ in both Eqs. (56) and (57) by $[(H + h_v)^{3/2} - h_v^{3/2}]$, where h_v is the velocity head in the channel. In practical work the last term is often dropped. There is no real good theoretical foundation for any expression involving velocity of approach. A modified Francis formula for the suppressed weir is

$$q = 3.33b(H + \alpha h_v)^{3/2} \tag{58}$$

in which α is given values ranging from 1 to 2. If the velocity of water in the channel at the section where H is measured is uniform over the cross-section, a value of 1 should be used for α. But if the surface velocity is much higher than the bottom velocity, the value of α should be greater than unity. This is because the true velocity head which affects the discharge over the weir is greater than the h_v which is based upon the average velocity in the section. The value of α is sometimes taken as the ratio of the surface velocity to the average velocity. Note that the area of the section is the product of the total width of the channel by $(H + y)$ in Fig. 86. In using Eq. (58) first solve Eq. (56) to obtain an approximate value of q. Divide this by the area of the section, where the hook gage is located, and from this velocity an approximate value of V, and hence h_v, can be obtained. This αh_v is to be added to H and a new and somewhat larger value of q calculated. From this a new value of h_v could be computed, and so on. However, after about two such solutions it would be found that further solution would alter the result very slightly.

Referring to Fig. 97 it may be seen that for the limiting case the area through the weir is $3H^2$, while the minimum cross-section of the channel of approach is $21H^2$ and preferably should be greater. But with this minimum area, the error involved in neglecting velocity of approach is about 1 per cent. Hence, in most practical cases, it is foolish to consider velocity of approach in the use of a weir with end contractions. If desired, however, the value of b in Eq. (58) may be corrected as in Eq. (57) and the resulting equation used for a contracted weir with velocity of approach.

In order to save the necessity of solving Eq. (58) by trial it is possible by a little algebraic manipulation to express the value of h_v in terms of channel dimensions and ultimately reduce the equation to the form given by Bazin.

71. The Bazin Weir Formula.—Bazin, in France, made a valuable series of experiments upon weirs without end contractions and with high velocities of approach. From these he devised a weir formula which expresses the effect of velocity of approach in a much less awkward manner than the Francis formula. His most accurate formula is rather complicated, but for practical work the following approximate formula is sufficient:

$$q = \left[3.25 + \frac{0.0789}{H}\right]\left[1 + 0.55\left(\frac{H}{H + y}\right)^2\right]bH^{3/2}. \qquad (59)$$

The quantity y indicates the height of the weir crest above the bottom of the channel and thus introduces the effect of velocity of approach into the formula in an indirect manner.

EXAMPLES

86. The width of the weir in Fig. 89 is 7.573 ft. Neglecting velocity of approach, what is the rate of discharge when $H = 1.200$ ft.?

If the value of y is 2.85 ft., solve for the rate of discharge by using the Francis formula with $\alpha = 1.0$. Solve with $\alpha = 2.0$.

Ans. 33.15, 34.0, 34.9.

87. Solve problem 86 by the use of Bazin's formula.

Ans. 34.5.

88. Assume that the weir in Fig. 91 is also 7.573 ft. in width. What would be the rate of discharge when $H = 1.200$ ft.? What would be the maximum allowable value of H?

Ans. 32.1.

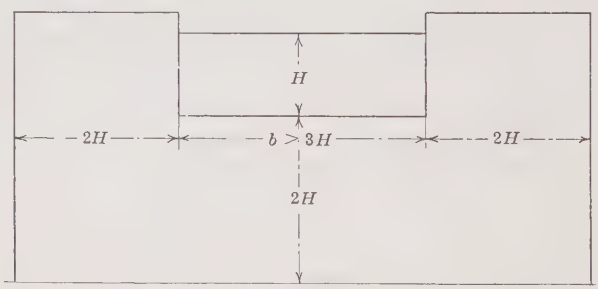


FIG. 97.—Limiting proportions of standard contracted weirs

72. Comments on Weirs.—Eq. (57) for the contracted weir is applicable only to the standard Francis weir, whose limiting proportions are shown in Fig. 97. Unless there is sufficient space left at the two ends and at the bottom of the weir, perfect contraction will not be obtained. These dimensions may be made greater than the values given, but never less. The height

of the crest above the bottom should preferably be at least $3H$. Francis estimated that a height of $2H$ would increase the discharge about 1 per cent. If sufficient space cannot be had to secure perfect end contraction, the end contractions should be entirely suppressed, or one of them suppressed and all the space given to the other end. There is no known coefficient or method for dealing with imperfect contraction. Also the value of H should not exceed one-third of the width b.

For the suppressed weir there are no standard dimensions to be observed. Experiments at Cornell University have indicated that the Francis coefficient in Eq. (56) is applicable for heads up to at least 5 ft. and for heads down to 0.3 ft. For $H = 0.2$ ft. it should be increased 3 per cent, and for $H = 0.1$ ft. it should be increased 7 per cent.

If the area of the channel of approach exceeds $6bH$ it can be shown that the velocity of approach is negligible. Hence it is seen that velocity of approach need not be considered in a contracted rectangular weir. But with a suppressed weir the depth of the channel would have to be $6H$, and that is not often the case. Hence most suppressed weirs have a velocity of approach that needs to be considered. The Francis coefficient was based upon work with weirs having a velocity of approach less than 1 ft. per second. When the velocity of approach is high, the formula of Bazin should be applied.

The most accurate type of weir is a suppressed weir with such a deep channel of approach that the velocity of approach is negligible. A contracted weir for which the velocity of approach is negligible is about in the same class with a suppressed weir with a moderate velocity of approach. End contractions have been held to be a source of error and there appears to be no truly rational way to correct for them. The least desirable type of weir is the one with a high velocity of approach because of the difficulty not only of reading H accurately but also of allowing for the effect of this velocity in a scientific manner.

It almost goes without saying that a weir should be set with its crest level and its plane vertical. An inclination upstream decreases and an inclination downstream increases the discharge for a given H. The crest should be sharp and in good condition.

In using weirs for accurate work it is desirable to study the original experiments upon which the formulas are based and use

the formula that has been derived under circumstances most nearly like those in hand. And it is likewise desirable to duplicate the original investigator's methods. The hook gage, for instance, should be located in the same way and at the same distance from the weir. Unless these precautions are followed one has no assurance that the coefficients given fit his own case.[1]

73. The Cippoletti Weir.—In order to avoid the trouble of correcting for end contractions, the sides of the Cippoletti weir are given such a batter (1:4) that they add enough to the effective width of the stream to offset the contraction $0.2H$ of the contracted Francis weir. Thus computations may be made upon the basis of the width b at the crest by the following formula

$$q = 3.367bH^{3/2}. \tag{60}$$

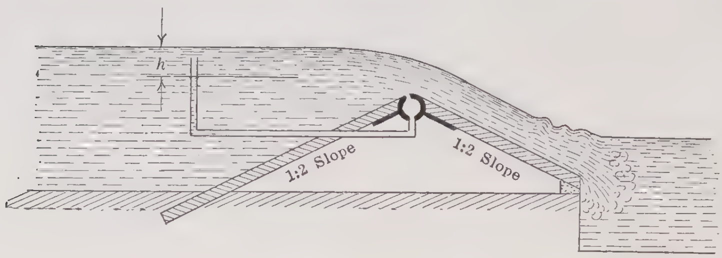


Fig. 98.—The Herschel weir.

74. The Herschel Weir.—A type of weir, shown in Fig. 98, on an entirely new principle has recently been proposed by Clemens Herschel. In this there is no crest contraction, no sharp edge, nor any "ventilation" on the downstream side to prevent the water clinging to the face of the weir. The crest itself is made of a hollow circular pipe into which holes are drilled at the point where the upstream face becomes tangent to the circular pipe. The head is measured as the differential pressure between a section of the stream, where a hook gage would ordinarily be employed, and the interior of this crest. A series of

[1] For information on weirs see:

Horton, R. E., "Weir Experiments, Coefficients, and Formulas." *U. S. Geol. Survey, Water Supply and Irrigation Paper*, 150, Revised as 200.

Hughes and Safford, "Hydraulics."

Parker, "The Control of Water."

tests with flows per foot of length from 0 to 9.55 cu. ft. per second gave the weir formula for this as

$$q = 5.50bh. \tag{61}$$

75. The Pitot Tube.—Among other water-measuring devices is the Pitot tube. This is an instrument which indicates the velocity of water at a point. From the velocity the rate of discharge may be obtained.

The principle of the Pitot tube is illustrated in Fig. 99 and its theory will be discussed in a subsequent chapter. For an open stream only a single tube is necessary, but in a stream of water

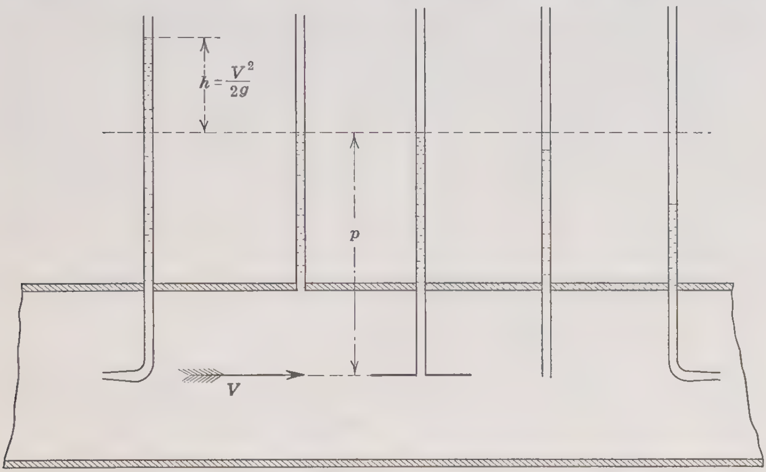


FIG. 99.

under pressure a second tube is necessary to record the pressure alone. The quantity desired is the difference between the two readings, which will be called h. It can be shown by correct theory that if h is the value in feet of water of the dynamic pressure exerted by the impact of the stream against the opening of the tube, the velocity of the water is given by

$$V = \sqrt{2gh}. \tag{62}$$

This has been found by experiment to be true when there is smooth, stream line flow, but in case of turbulent flow a coefficient whose value is about 0.977 should be introduced, so that the following may be written

$$V = 0.977\sqrt{2gh}. \tag{63}$$

The fact that this coefficient is anything less than unity is not

because our theory is at fault nor because of any defect in the instrument itself, but is due to the fact that the instrument records the true velocity at the point even though it is not exactly perpendicular to the tube orifice, while we desire, for practical purposes, the axial component of velocity. Hence the factor is designed to give us the axial component of velocity (OB in Fig. 49 rather than OD).[1]

If the Pitot tube is used in a pipe, the pressure reading should be taken by a piezometer tube which does not project within the walls of the pipe and which is at right angles to it, as in the second tube of Fig. 99. If it is necessary, for some reason, to have the tube project into the stream, a correct reading may be obtained if the piezometer orifice is made in a flat plate, the plane of which may be parallel to the stream lines, as is shown by the third tube in Fig. 99. Or the orifice may be made in the side of a smooth tube, whose axis is parallel to the stream lines and whose closed upstream end is pointed so as to diminish eddy disturbances. The chief source of error in the use of the Pitot tube lies in the measurement of the pressure.

In the case of a jet there is no pressure reading to be subtracted from the Pitot tube reading. Neither is there in the case of an open stream, but in the latter case it may be inconvenient for the observer to secure a reading which is only a few inches above the water surface. Therefore a piezometer tube may be employed here and the two connected by a differential manometer of the type shown in Fig. 13 (b) but using air. By partially exhausting the air through valve V the two columns may be drawn up to any convenient height.

In using the Pitot tube it is often convenient to divide a cross-section of the stream up into parts of equal area and to determine the velocity in the center of each area. The average velocity of the stream will be the average of the observed velocities. But if the areas are not equal the average of the velocities will have no significance. It will then be necessary either to plot a curve from which velocities at other points may be taken or to multiply each observed velocity by the area which it may be assumed to represent. The total rate of discharge of the entire stream is the sum of all such partial discharges. Thus

$$q = \Sigma A'V' \tag{64}$$

[1] Moody, L. F., "Measurement of the Velocity of Flowing Water," *Proc. Eng. Soc. Western Penn.*, vol. 30, p. 319, 1914.

where A' is a portion of the total area and V' is the velocity through that area. If the average velocity is desired, it can be obtained by dividing the rate of discharge by the total area.[1]

EXAMPLE

89. Assume that a Pitot tube and a piezometer tube are connected to two sides of a differential manometer containing mercury. Suppose the Pitot tube is placed in such positions in the stream, which is 10 in. in diameter, that it measures the velocities in five areas of equal magnitude. Suppose these five readings on the differential manometer are 1.50, 2.15, 2.84, 3.62, and 4.05 in. of mercury. Find the rate of discharge of the stream.

Ans. $q = 7.43$ sec. ft.

76. The Current Meter.—For moderate velocities such as are found in canals and natural streams the current meter is well adapted. It consists of a wheel, as in Fig. 100, or in other types a screw, which is rotated by the action of the water. By calibration the relation is determined between the velocity of the water and the rate of rotation of the meter.

In many current meters each revolution is recorded by a click in a telephone receiver at the ear of the observer, the click being produced by the wheel making an electric contact every revolution. In most meters the contact is not made so frequently, every ten revolutions being the number commonly recorded. Other types of meters have some form of mechanical recording device. It is generally better to determine the time necessary for a given number of revolutions rather than to attempt to find the number of revolutions made in some specified time, owing to the difficulty of estimating fractions of a revolution or fractions of the number of revolutions that may be recorded as a unit.

Current meters may be roughly divided into two classes, those with the axis vertical, as in Fig. 100, and those with the axis horizontal.

In comparatively shallow water the meter may be rigidly fastened to a rod, and in this case the weight and tail, as shown

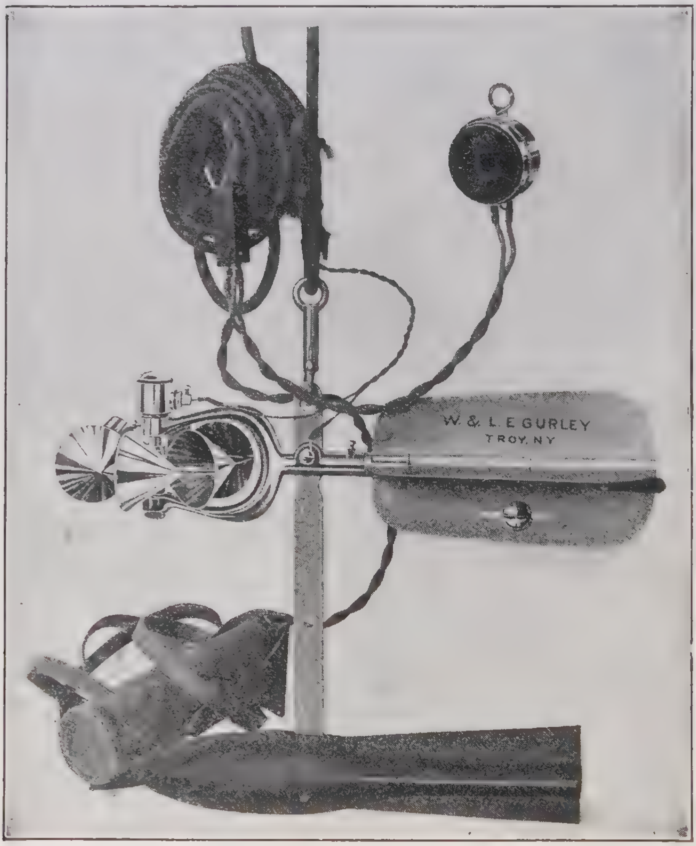
[1] In the case of a circular cross-section, the discharge through an annular ring of area dA is

$$dq = V'dA = Vd(\pi r^2).$$

Hence q is the product of π and the area under a curve of values of V plotted against corresponding values of r^2. In similar manner the true kinetic energy is the product of $\pi w/2g$ and the area under the curve of V'^3 vs. r^2.

in Fig. 100, are unnecessary. But for deeper water where the meter is suspended by a cable the latter are required to hold the meter in the proper position.

Generally it is desired to find the velocity of the water flowing across some sectional area. If the stream lines are not perpen-



Courtesy of W. & L. E. Gurley.

Fig. 100.—Current meter.

dicular to the area in question, it is the normal component of the velocity that is desired rather than the value of the actual velocity. It may be seen that the type of meter shown in Fig. 100 will rotate with equal velocity no matter from which horizontal direction the water may come. It will also be rotated by a cur-

rent that is vertical, or parallel to its axis. And in any case the rotation is always in the same direction. Thus this meter tends to record the value of the velocity regardless of its direction.

In other types, generally with the axis horizontal and the wheel made in some form similar to a screw propeller, the meter records only the component of the velocity parallel to its axis. And in case the meter is located in a portion of the stream where an eddy causes a reverse current the meter will then give a negative reading, since it will be rotated in the opposite direction. Such a type of meter is more accurate in all cases where the flow is irregular or turbulent. However, the type shown in the figure is of excellent mechanical construction and is widely used. For many cases where the stream flow is fairly regular and extreme accuracy is not required, it is quite satisfactory.

In using the current meter the velocities are determined at a number of different points and the total discharge of the entire stream computed in the same manner as in Art. 75.

77. Comments on Measurement of Water.—The accurate measurement of rate of discharge is one of the most difficult problems in practical hydraulics. The only positive way of measuring rate of discharge is to weigh the amount of water discharged in a given time or to determine its volume in suitably calibrated tanks or reservoirs. The former method is applicable only for relatively small rates of discharge, and facilities for the latter are seldom to be had. Also in the latter method the effect of leakage, evaporation, and other factors may sometimes prove troublesome.

The methods that are usually employed are the ones that have been given in this chapter. They are all indirect in that the velocity or the rate of discharge is assumed to be a function of some other quantity which can be measured.

The discharge of water from any tank can be measured by an orifice, tube, or nozzle. When a stream of water flows in an open channel it may be caused to flow over a weir or its velocity throughout any cross-section may be found by a current meter, by floats, or other means. For a stream of water confined within a closed pipe a Pitot tube may be used to determine the velocity across a cross-section, or the water may be run through a Venturi meter. At the end of a pipe line a nozzle might also be placed which would also permit the rate of discharge to be obtained. The discharge from a nozzle may be computed or it

may be measured directly by determining the velocity of the jet with a Pitot tube. The means of measurement that is to be used depends upon the circumstances.

In addition to the methods of measurement that have been described in this chapter, there are other methods, especially chemical methods. One of these is simply a matter of discharging a small quantity of highly colored liquid into the intake of a pipe line and noting the time that it takes for the discoloration to be noted at the other end. Knowing the length of pipe it is easy to compute the velocity of the water. Another valuable method consists of adding a strong salt solution at a known definite rate. Samples of water are taken at a downstream section and analyzed. Knowing the strength of the solution used, its rate of discharge, and the amount of dilution in the main stream, the rate of discharge in the latter may be determined. This method has been used in some cases with a high degree of accuracy and it may offer an easy, cheap, and convenient way of measuring rate of discharge of large quantities of water.[1]

Where water flows over a spillway dam the latter may be used as a special type of weir. The same weir formula as given in Eq. (55) may be applied, if the proper value of the coefficient is known. Since the spillway crest may be of various shapes and dimensions, it is not a standard piece of apparatus like the sharp-crested weir. Hence the value of the coefficient has to be determined for each case either by calibrating the spillway in question or using the results of observations upon another spillway of similar form.

78. Discharge under Varying Head.—If the head varies, the rate of discharge will likewise vary and the total discharge in a given time, or the time required for a given total discharge, must be determined as follows:

Let Q = the total volume in cubic feet of any given body of water, while q = cubic feet per second as usual. Then

$$q = \frac{dQ}{dt} \text{ or } dQ = q dt.$$

Suppose that into this body of water in question there is an inflow at the rate of q_1 cu. ft. per second, while water flows out at the

[1] GROAT, B. F., "Chemihydrometry and Precise Turbine Testing," *Trans. A. S. C. E.*, vol. 80, p. 951, 1916.

rate of q_2 cu. ft. per second. It then follows that the change in the total volume in any time dt is

$$dQ = q_1dt - q_2dt.$$

Also let M = the area of the water surface of the body in question while dz = the change in the level of the surface. Then

$$dQ = Mdz.$$

Equating these two expressions for dQ,

$$Mdz = q_1dt - q_2dt. \tag{65}$$

Now either q_1 or q_2 or both may be variable and functions of z, the variable height of the water surface, or one of the two may have a constant value or be equal to zero. For instance, if the water is discharged through an orifice or pipe line of area A under the head z, the following may be written

$$q_2 = cA\sqrt{2gz},$$

while if it overflows a weir or spillway dam of width b,

$$q_2 = Kbz^{3/2}.$$

In the former case z might correspond to the h in Fig. 111 while the value of c would be determined from the principles of Art. 80 and subsequent articles in case the discharge takes place through a pipe line. In the case of flow over a spillway the z would be the height of the surface of the water above the crest or in other words would correspond to the H in Fig. 86 or of Eq. (55). And in like manner q_1 may also be some function of z.

Equation (65) is perfectly general and if it is possible to express M, q_1, and q_2 as mathematical functions of z, it may then be possible to solve the problem by integration. In other cases the integral may be evaluated by graphical methods. For example, from Eq. (65) may be obtained

$$t = \int_{z_1}^{z_2} \frac{Mdz}{q_1 - q_2}. \tag{66}$$

By integration this will give us the time required for the water level to change from z_1 to z_2. If it cannot be integrated by calculus it may be done graphically by computing values of q_1 and q_2 and plotting values of $M/(q_1 - q_2)$ against corresponding values of z. The area between this curve and the z axis is the value of the integral. Of course without actually plotting it, the value of the area may be computed by various rules of approximation.

EXAMPLES

90. Suppose a ship lock in a canal has vertical sides and that the water discharges through an outlet of area A such that $q = cA\sqrt{2gz}$. Prove that the time required for the water level to fall from z_1 to z_2 is

$$t = \frac{2M}{cA\sqrt{2g}}(z_1^{1/2} - z_2^{1/2}).$$

91. Suppose water in a storage reservoir whose surface area is constant discharges over a spillway for which $q = Kbz^{3/2}$ and that there is no inflow into the reservoir. Prove that the time required for the height of water on the spillway crest to fall from z_1 to z_2 is $t = \frac{M}{2Kb}\left(\frac{1}{\sqrt{z_1}} - \frac{1}{\sqrt{z_2}}\right)$. How long will it be before the flow ceases?

79. PROBLEMS

92. Suppose that a ship lock in a canal is of uniform rectangular cross-section and that it is 300 by 90 by 40 ft. deep. Suppose that the water from this lock is discharged through a tunnel which is 3 ft. in diameter, the coefficient of discharge being 0.50. If the initial head under which water discharges is 35 ft., how long will it take for the level to drop 25 ft. (*i.e.*, from 35 to 10 ft. elevation)?

Ans. 87 min. 30 sec.

93. In problem 92 how large would the tunnel have to be to permit the water level to drop from 35 to 10 ft. in 15 min.?

Ans. 7.27 ft.

94. Water enters a reservoir at a uniform rate of 150 cu. ft. per second and flows out over a spillway whose length of crest is 100 ft. The value of K for this spillway is 3.45. Areas of water surface at various elevations above the crest of the spillway are given in the adjoining table. (*a*) Find the time required for the level to drop from 3 to 1 ft. above the crest. (*b*) Find the final elevation after equilibrium is established. (*c*) How long a time will it take for equilibrium to be established?

Ans. (*a*) 2,052 sec. (*b*) 0.573 ft. (*c*) Infinite time theoretically.

z (Feet)	M (Square Feet)
3.00	860,000
2.50	830,000
2.00	720,000
1.50	590,000
1.25	535,000
1.00	480,000

CHAPTER VII

FRICTION LOSSES IN PIPES

80. Loss of Head in Pipe Friction.—In dealing with such devices as the orifice, nozzle, Venturi meter, etc., the effect of frictional resistance to flow has been compensated for by the introduction of velocity coefficients. This is feasible because all of these devices can be standardized so that the coefficients which have been determined for one may be applied to another of the same type. Velocity coefficients might be had for pipes, also, if the latter were more nearly alike. But actually, pipe lines differ from each other in length, size, degrees of roughness, and other respects to such an extent that the application of velocity coefficients is impractical. Therefore it is necessary to proceed on a different basis.

It is obvious that the loss of head in friction in a pipe is directly proportional to the length. Experience shows that it is also proportional to some power of the velocity, that it varies inversely with the size, and that it increases with increasing roughness of the surface. It is found to be independent of the pressure, in the case of liquids, but is a function of density and viscosity, which in turn are functions of temperature. For the present, however, the variation due to temperature changes will be disregarded.[1]

Two terms may be defined here which are frequently used. By *hydraulic slope* is meant the value of s which is given by

$$s = \frac{H'}{l}. \tag{67}$$

[1] The loss of energy in a pipe line is called a *friction* loss though the phenomena have little in common with the frictional resistance between solids. It is rather a degradation of the available energy of the stream into the unavailable form of eddies and turbulence and heat. The roughness of the pipe doubtless promotes turbulence and hence increases the internal eddy losses. In this chapter the chief concern is with velocities above the critical.

Thus s is an indication of the rate of loss as a function of distance along the pipe.[1]

By *hydraulic mean depth* or *hydraulic radius* is meant the value of m given by the expression

$$m = \frac{A}{\text{wetted perimeter}} \tag{68}$$

where A denotes the cross-section area of the stream, and the wetted perimeter is the length of the perimeter of the channel actually in contact with the water. (In an open stream it would not include the distance across the free surface.) It may be seen that for channels of the same area, the wetted perimeter will vary with the shape of the cross-section. Thus m is an index not only of size but also of shape. It is reasonable to suppose that the frictional resistance in channels of the same area would be proportional to the surface or perimeter in contact with the water, and thus in general inversely proportional to m. Since the value of m may be determined for any shape of cross-section whatever, it affords us a much more general dimension than the diameter d which is naturally confined to conduits of circular cross-section.[2] For a circular pipe flowing *full*, the area is πr^2 and the wetted perimeter is $2\pi r$. Thus

$$m = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4}. \tag{69}$$

81. Exponential Formulas for Pipe Friction.—Experiment shows that the loss of head in pipe friction is represented by an expression of the form

$$H' = f' \frac{l}{d^x} V^n. \tag{70}$$

[1] As a special case, if the pipe is of a uniform diameter, the velocity head is constant and the total energy gradient and the hydraulic gradient are parallel. Then s would be proportional to the slope of the hydraulic gradient. If the pipe is both of a uniform size and horizontal, then s is the tangent of the angle made by the hydraulic gradient with the horizontal. If the slope of the pipe and the hydraulic gradient are the same, as might be the case if the change in elevation were just equal to the friction loss so that the pressure in the pipe was the same along its length, then s is the sine of the angle. In other cases it does not represent directly any natural function of any angle that may appear in a drawing.

[2] If a rectangular area equal to the actual cross-section is constructed with a base equal to the length of the wetted perimeter, then the depth of this rectangle would be m. However, this physical interpretation is of no significance.

In this equation, with correct values of x and n, the factor f' would be independent of d and V and vary only with the roughness of the pipe. The difficulty of applying this equation in practice is that one has to estimate values of f', x, and n. In other words one is dealing with three variables, which depend upon the character of surface.

Experiments show that n varies from 1.72 for *very smooth* pipes up to 1.95 or even 2.0 with very rough ones. For ordinary commercial pipes the range is more nearly 1.8 to 1.95. The value of x is usually given as 1.25, though it should be a variable also and approach 1.0 as n approaches 2.0. The reasons for this conclusion will be given later. If f', x, and n are known for a given kind of pipe, Eq. (70) may be very useful.[1]

Solving Eq. (70) for velocity,

$$V = f'^{\frac{1}{n}} d^{\frac{x}{n}} \left(\frac{H'}{l}\right)^{\frac{1}{n}} = C' m^{\frac{x}{n}} s^{\frac{1}{n}} \tag{71}$$

where C' is an experimental coefficient. As has been explained, the use of m instead of d renders the equation applicable to all shapes of cross-section.[2]

82. The Common Equation for Friction Loss.—The formula that is usually employed for pipe friction is

$$H' = f\frac{l}{d}\frac{V^2}{2g}. \tag{72}$$

Since H' must be a quantity in linear units, as explained in Art. 46, and the velocity head $V^2/2g$ is a linear quantity, it is apparent that fl/d must be an abstract number. Since l/d is a mere ratio of two dimensions, which may be in any units, so long as they are alike, it follows that the term f must also be an abstract number. It is also apparent that, since the exponents in Eq. (72)

[1] SCHODER in MARKS, "Mechanical Engineers' Handbook," gives for average cast-iron and wrought-iron (not riveted) pipes in fair condition, $H' = 0.00038lV^{1.86}/d^{1.25}$, and for rough or riveted pipes, $H' = 0.0005lV^{1.95}/d^{1.25}$.

[2] By inserting values of these exponents for average conditions the Hazen-Williams formula is obtained, which is

$$V = C'm^{0.63}s^{0.54}.$$

Values of C' may be found in Table III. The value of C' is based upon the Chezy coefficient C for $m = 1$ and $s = 0.001$. Thus $C' = C \times 0.001^{-0.04} = C \times 1.318$. The use of this formula is greatly facilitated by employing the Williams and Hazen "Hydraulic Tables" or their special slide rule for its solution.

are not exactly correct, the error introduced will have to be compensated for by a proper value of f. Thus in this equation f is a function not only of the character of the surface but also of d and V. The quantity f is called the *coefficient of pipe friction* or simply *friction factor*.

Solving Eq. (72) for velocity it is found that $V = \sqrt{2gdH'/fl}$. Now replace H'/l by s, d by $4m$, and the remaining factors by C such that

$$C^2 = \frac{8g}{f}. \tag{73}$$

This expression now reduces to

$$V = C\sqrt{ms}. \tag{74}$$

It may be seen that Eqs. (72) and (74) are equivalent, since each may be derived from the other. The equation in the latter form is known as Chezy's formula and C as Chezy's coefficient.

As will appear later, in the solution of pipe-line problems where other factors besides pipe friction alone are to be considered, it will be found that Eq. (72) is more convenient. But Eq. (74) may be used, if desired, especially where pipe friction is the sole item considered. It is the equation that is universally used with open channels, and may be applied to any form of cross-section. Equation (72) may be used for sections other than circular by substituting $4m$ for d.

83. Common Values of Friction Factor.—Because Eq. (72) for pipe friction is irrational, it is necessary that the coefficient of friction f, should vary with diameter and velocity, as well as the character of the surface, as has been stated previously. Concerning the character of the surface and its influence, it may be noted that a given condition of surface is relatively of more importance with a small diameter than a large one, because projections and depressions of the same size are a greater percentage of the diameter. In other words different sizes of pipe of exactly the same material will have different degrees of roughness. This alone would cause the friction factor f to decrease with increasing size of pipe.

One of the formulas most commonly used is that of Darcy, who ignored the effect of velocity and made the coefficient depend only upon the diameter. According to Darcy, the value of f for new, clean, cast-iron pipes may be given as

$$f = 0.02 + \frac{0.02}{d''}. \tag{75}$$

Since f must be an abstract number, the value of the ratio in the last member of the equation must be independent of any units. Since the diameter there appears as *inches*, it follows that this dimension is involved in the numerator also.[1]

This equation gives values of f that are too high, especially for large pipes, but it is on the side of safety. Relative values for other kinds of pipe, even in small sizes, may be estimated by comparison with the values in Table III.

For old, corroded, cast-iron pipes, values given by Eq. (75) should be increased up to a maximum of twice as great, depending upon the judgment of the engineer as to the condition of the surface. It is impossible to formulate any rule for the effect of age, as the chemical composition of the water, the velocity of flow, and other factors make a material difference in the amount of rusting and tuberculation that takes place with time.

Since the largest pipes used by Darcy in his experiments were only 20 in. in diameter, his equation should not be applied to sizes above 2 ft. in diameter at the most. The form of his equation, however, shows that the value of f approaches a constant, as the size of the pipe increases. Hence for large pipes, above 2 or 3 ft. in diameter, and for the average velocities commonly encountered, values may be used which vary only with the kind of pipe, such as are shown in Table III.

TABLE III

For large pipes	f	C	C'	N
New, smooth, cast-iron pipe	0.015	130	170	0.011
Wood-stave pipe, new or old	0.018	120	160	0.012
New riveted pipe	0.022	110	145	0.013
Old, tuberculated, cast-iron pipe or riveted pipe	0.026	100	130	0.014
Any old and rough pipe	0.040	80	105	0.019

The second column in this table gives typical values of C. Since convenient "round" numbers are used, values of f and C are not here related to each other so as precisely to satisfy Eq.

[1] For the sake of uniformity, unless otherwise specified, all answers in this book will be based upon values of f from Eq. (75) and upon the diameters stated, rather than the values corresponding to the nomins. al size

(73). The third column gives values of C' for the Hazen-Williams formula in the footnote of Art. 81. In the fourth column are values of N, which is the scale of roughness in Kutter's formula in Chap. IX.

EXAMPLES

95. What will be the loss of head in a 10-in. pipe line 2,000 ft. long, when the velocity of the water in the pipe is 6 ft. per second? What is the value of s?

Ans. 29.5 ft.

96. What will be the loss of head in a 1-in. pipe line 2,000 ft. long, when the velocity of the water in the pipe is 6 ft. per second? What is the value of s?

Ans. 537 ft.

97. Using Table III, what is the velocity of flow in a 5 ft. new cast-iron pipe, when the loss of head is 1.6 ft. per 1,000 ft. of length? (Solve by both Eqs. (72) and (74).) What will it be when old?

Ans. 5.85, 5.82, 3.6 ft. per second.

84. General Expression for Friction Factor.—The formula $H' = f(l/d)V^2/2g$ is simpler to use than the exponential formulas, and will yield equally correct results, if it is only possible to determine the proper value of f in any case. It has been seen that in this form of equation f is a function not only of the character of the surface but also of the diameter and the velocity. It is known that the friction loss in a pipe line is also a function of the density and the viscosity.

It has been stated that the friction factor f is an abstract number. If it is a function of diameter, velocity, density, and viscosity, then these four quantities must be combined in such a way that they yield a dimensionless value.[1] The only possible combination of these four factors such that all the dimensions cancel is

$$\frac{dVw}{\mu}. \tag{76}$$

In addition to the above, the friction factor f is a function of a dimensionless factor expressing roughness. This is the ratio of the depth of irregularities in the surface to the diameter. So far it has not been possible to evaluate this ratio and to establish

[1] See Appendix I for an explanation of viscosity. In English units viscosity μ is lb./(ft. $\times$ sec.). The above relation is true for any system of units, however, as long as they are homogeneous.

a numerical scale. Hence roughness is allowed for merely according to the experience and judgment of the engineer.

Since all the quantities in Eq. (76) appear with the same exponent, it shows that the factor f must be a function of the same power of each one of them, whatever that function may be.

Attention may now be given to water at the same temperature so that both w and μ are constant, and then the chief concern is only with d and V as variables. If values of H' as given by Eq. (70) and (72) are equated the following is obtained:

$$f = \frac{2gf'}{d^{x-1}\ V^{2-n}}, \tag{77}$$

and, since f' is supposed to vary only with roughness, it follows that f decreases with an increase in either the diameter or the velocity. Furthermore, since f must vary as the same function of either d or V, as stated above, it follows that

$$x - 1 = 2 - n. \tag{78}$$

Thus when $n = 1.75$, it has been found that $x = 1.25$, and when $n = 1.9$, then it should be expected that $x = 1.1$, and so on. Thus, x should vary with n.

Now actual tests on pipes of different diameters do not seem to show this exact agreement between n and x, though they do show that the tendency is in this direction. One reason why actual agreement is not found is that pipes of the same kind of surface are relatively smoother as the diameter increases, as has been explained. Thus the effect of diameter may be more important than that of velocity, but this is because the roughness factor is not constant.

Our present experimental information seems to indicate that the exponents x and n vary not only with the roughness of the surface, but also as functions of d and V. For stream-line flow the value of x is 2 and that of n is 1. Above the critical velocity, the greater the turbulence, the more nearly does x approach 1 and n approach 2. Now turbulence is increased with increasing roughness of surface or with higher velocities or larger diameters because the higher the velocity the farther away is it from the critical velocity, and the larger the diameter the farther away it is from the small sizes that also favor stream-line flow. Hence x diminishes and n increases with increasing roughness, velocity, or diameter. Thus it is apparent that an equation of the form of Eq. (77) may be used with given values of the exponents only

for a limited range. The knowledge of the function that f may be of the quantities mentioned is wholly experimental and appears to be given by an empirical equation of the form

$$f = a + \frac{b}{(dVw/\mu)^y}. \tag{79}$$

As a matter of convenience, expression (76) may be written

$$\frac{dVw}{\mu} = \frac{d''Vs \times 62.3}{12U \times 0.000672} = \frac{d''Vs}{U} \times 7{,}740 \tag{80}$$

where s is the specific gravity relative to that of water at 68°F. or 20°C., at which temperature the density is 62.3 lb. per cubic foot and U is the viscosity relative to that of water at 68°F., which is then 0.000672 lb. per foot-second.

With these more convenient units, Eq. (79) for smooth brass pipe may be expressed as

$$f = 0.0072 + \frac{0.0265}{(d''Vs/U)^{0.355}}, \tag{81}$$

while for ordinary commercial smooth steel pipe

$$f = 0.014 + \frac{0.0238}{(d''Vs/U)^{0.424}}. \tag{82}$$

Curves representing values of f given by these equations are shown in Fig. 101 together with other curves for various types of pipe surface.[1]

In order to use Fig. 101, it is necessary to compute the value of $d''Vs/U$ and then read off the value of f from the proper curve or location on the diagram corresponding to the estimated roughness of the pipe. As a rough guide for this a scale of values of Kutter's N, which is an index of roughness commonly used for open channels and large pipes, is shown at the extreme right-hand side. It is this portion of the diagram that pertains mostly to the flow in large pipes. It may well be assumed, however, that if all

[1] WILSON, MCADAMS, and SELTZER, *Jour. Ind. Eng. Chem.*, vol. 14, No. 2, p. 105.

BUCKINGHAM, *Trans. Amer. Soc. Mech. Eng.*, vol. 37, p. 263, 1915.

SWINDIN, "The Modern Theory and Practice of Pumping," D. Van Nostrand Co.

STANTON and PANNELL, National Physical Laboratory, 1914.

LANDER, *Proc. Roy. Soc. A*, vol. 92, p. 737, 1916.

In addition to these typical curves given by those referred to in the footnote, the author has indicated a few experimental points on this diagram to show the application to specific cases. Thus there is a diameter as small as ¼ in. and one as large as 108 in. There are also two sets of values for a new pipe and again after 10 years of service.

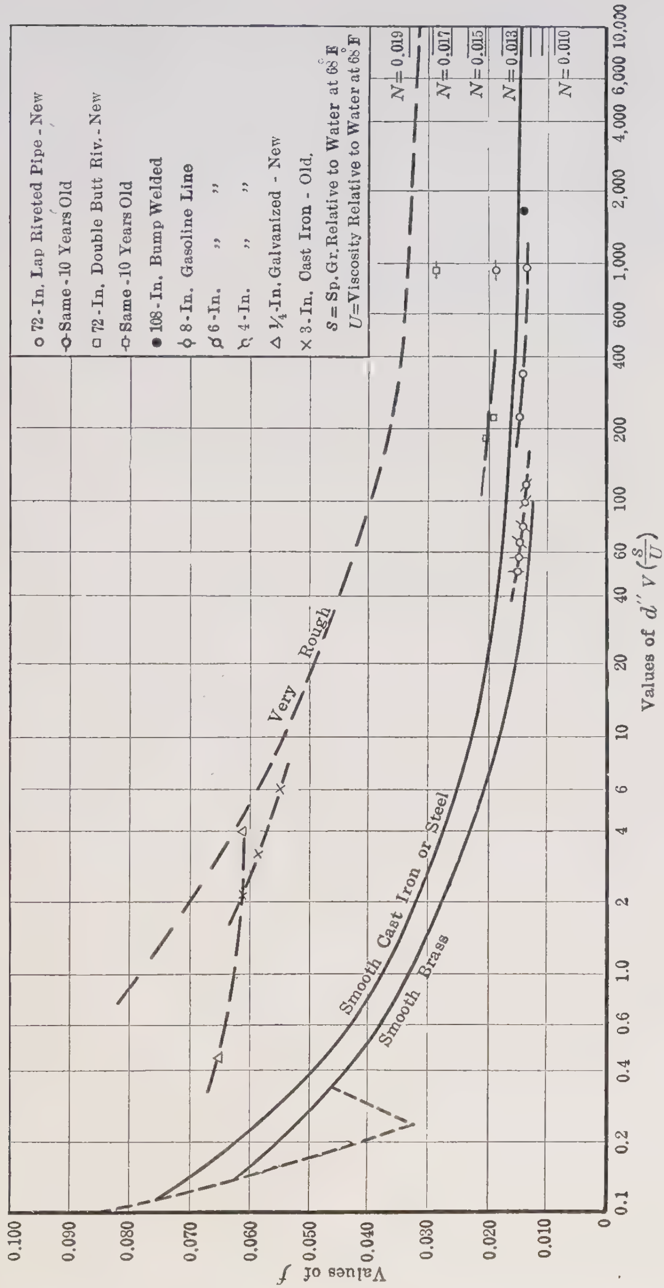


FIG. 101.—Values of friction factor.

experimental results could be properly classified according to some numerical scale of roughness, such as given by this N, values of which are given in Tables III and VI, it would be possible to draw on this diagram a series of similar curves for different values of N and terminating at the right at the positions shown. In dealing with smaller pipes, however, it is well to remember that what may be called the relative roughness is greater for a similar surface.

When the value of $d''Vs/U$ is as low as 0.2, the point is in the region of the critical velocity. The intersection of one of the solid curves with the dotted line at the left indicates the transition from turbulent to stream-line flow. Below the critical velocity values of f may then be taken from this dotted curve for use in Eq. (72), but preferably the equation given in the footnote on page 50 should be used. On this diagram is shown a triangular area indicating the unstable region, similar to that shown in Fig. 51.

The interesting part about the curves in Fig. 101 is that they are not only true for all the range of conditions that are to be found in practical hydraulics, but the values are equally good for all fluids, whether gas, steam, water, or viscous oil. It is only necessary to know the density and the viscosity in order to make use of this diagram. For ordinary purposes the water may be assumed to be at 68 deg. so that both s and U become unity and the main concern is with d'' and V. But for an accurate analysis of experimental data, it is necessary to consider the effect of temperature change upon water as well as other fluids. A few representative values of viscosity will be found in Appendix I.

EXAMPLES

98. Water at 68°F. flows with a velocity of 4 ft. per second through: a ¼-in. smooth brass pipe; a 10-in. smooth steel pipe; a 100-in. lap-riveted steel pipe. Find the loss of head per 1,000 ft. by the aid of Fig. 101.

Ans. 394, 5.52, 0.417 ft.

99. Suppose warm fuel oil with a specific gravity of 0.9 and a relative viscosity of 90 to be pumped through the 10-in. line in problem 98 with a velocity of 4 ft. per second. Find the value of f and the ratio of the friction loss to that of water when both are in pounds per square inch.

Ans. 0.043, 2.093.

85. Minor Losses.—Whenever the velocity of a flowing stream is altered either in direction or in magnitude such alteration sets up excess eddy currents or turbulence and thus creates a loss of

energy in excess of the usual pipe friction. The magnitude of this loss is a function of the abruptness of the velocity change. Such a loss of head is often called a minor loss because in a pipe line of considerable length the pipe friction itself is of such a value that the losses of head due to these other factors may be relatively insignificant.

Thus assuming a pipe line whose length is 10,000 diameters and with a friction factor of 0.02 the value of $f\,l/d$ is 200. Suppose that the loss of head due to other causes amounts to $1.2V^2/2g$. It may be seen that this is only 0.6 per cent of the pipe friction, which is a negligible amount, especially in view of the fact that the value of f may not be known within 10 per cent.

On the other hand, if the length is 500 diameters, the value of $f\,l/d$ is 10, and it does make an appreciable difference in the computed result whether the $1.2V^2/2g$ is omitted or not. Even in this case, however, the value of the minor losses is equivalent to a possible variation in the friction factor.

But if the length is only 10 diameters, the pipe friction is only $0.2V^2/2g$, and it is seen that the so-called minor losses now become the most significant factors. Thus the importance of such items is seen to be a matter of relative proportions.

The most common source of minor losses are described in the remainder of this chapter.

86. Loss of Head at Entrance.—Referring to Fig. 102, it may be seen that, as water from the reservoir enters the pipe, the stream lines tend to converge, much as though this were a jet issuing from a sharp-edged orifice, so that at B a maximum velocity and aminimum pressure is found. It may be supposed that at B the central stream is surrounded by water which is in a state of turbulence but has very little forward motion. Between B and C the water is in a very disturbed condition as the stream expands and the velocity decreases, while the pressure rises. From C to D the flow is normal.

It is seen that the loss of energy at entrance is not confined to the section at A but is distributed along the length AC, a distance of several diameters. The increased turbulence and vortex motion in this portion of the pipe causes the friction loss to be much greater than in a corresponding length where the flow is normal, as is shown by the drop of the total energy line. Of this total loss a portion, H'_f, would be due to the normal pipe

friction. Hence the difference between this and the total, or H'_e, is the true value of the extra loss caused at entrance.

It is apparent that at section C the pressure is less than the static pressure, due to the head in the reservoir, by an amount equal to the velocity head acquired plus the total friction loss in AC.

If the pipe were to project into the reservoir, instead of being flush as shown, the stream lines would have a still greater tendency to converge and the entrance loss would be greater. On the other hand, a bell-mouthed entrance properly rounded eliminates this contraction and greatly diminishes the entrance loss. These three types are similar to the three tubes shown in

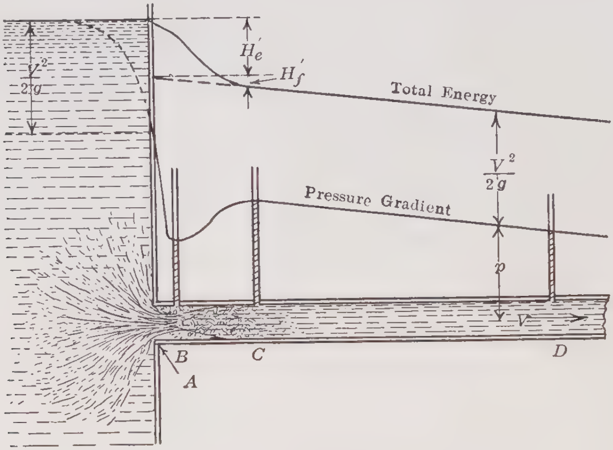


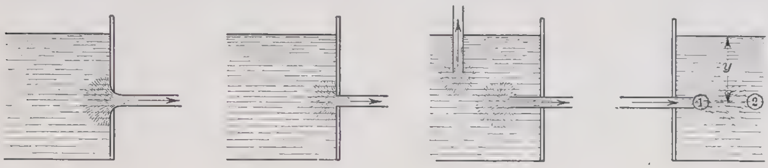
Fig. 102.—Conditions at entrance.

Fig. 76, page 84, and coefficients of loss for the pipes are computed by Eq. (31) from velocity coefficients determined by experiments on such short tubes. It is customary to consider the pipe friction loss in this short length as negligible in comparison with the entrance loss.

Entrance losses may also be materially reduced by a very short conical mouthpiece on the end of the pipe. It has been found that the most efficient proportions of such a device is with a central angle (*i.e.*, angle of vertex of cone) between 30 and 60 deg. and an area ratio of only 1:2 is necessary. This gives an entrance loss of $0.18V^2/2g$.[1] Coefficients of

[1] Seely, F. B., *Univ. Illinois Bull.* 96, Engineering Experiment Station.

velocity for short tubes with square entry have been found from 0.82 to 0.80, which gives a coefficient of loss of from 0.47 to 0.56. For projecting entry tubes discharging into air, coefficients of velocity vary from 0.72 to 0.75 with corresponding loss factors of 0.93 to 0.78, respectively. On the basis of experiments on short tubes discharging under water, Seely recommends a coefficient of loss of 0.62 for a projecting pipe.



(*a*) $k = 0$ (*b*) $k = 0.5$ (*c*) $k = 1.0$ (*d*) $k = 1.0$

FIG. 103.—Entrance and discharge losses.

If the loss of head at entrance may be expressed as

$$H' = k_e\frac{V^2}{2g} \tag{83}$$

values for k_e may be obtained as follows:

Bell-mouthed entrance	$k_e = 0.04$	(0)
Conical entrance	$= 0.18$	
Non-projecting pipe	$= 0.47$ to 0.56	(0.5)
Projecting pipe	$= 0.62$ to 0.93	(1.0)

For practical purposes where the entrance losses are quite secondary in importance, the values given in parentheses and also indicated in Fig. 103 may be used.

87. Loss of Head at Discharge.—In the case of a pipe discharging freely into the air, as in Fig. 104 (*a*), one may write $H_1 = H_2 + H'$, where $H_2 = V^2/2g$ and $H' = (k_e + f\,l/d)V^2/2g$. On the other hand, point (2) may be considered to be taken at such a place that the velocity with which the water issues from the pipe has been reduced to zero. In that case $H_2 = 0$. But now, not only does the loss in the pipe up to B have to be considered, but also the loss between the end of the pipe and the place where point (2) is taken to be. This loss is evidently $V^2/2g$, since that is the difference in value between the heads at these two points. Thus $H' = (1 + k_e + f\,l/d)\,V^2/2g$. But when these values are substituted for H_2 and H' in the equation, $H_1 = H_2 + H'$, the result is seen to be identical for the two cases,

since the $V^2/2g$ has merely been transferred from H_2 to H'. This difference in grouping is merely a matter of the point of view, but it is necessary to be consistent and not use the velocity head in both terms at the same time.

In Fig. 104 (*b*) is shown a nozzle on the end of a pipe. The velocity head only in the issuing stream is had here, the same as

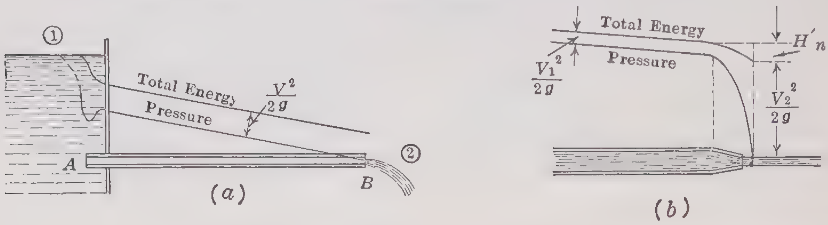


FIG. 104.—Conditions at discharge into air.

in the case above. The only difference is that the velocity of the jet is much higher than the velocity within the pipe. The drop of the energy gradient marked H'_n is the friction loss within the nozzle itself and is similar to the friction within the pipe. It has nothing to do with discharge loss.

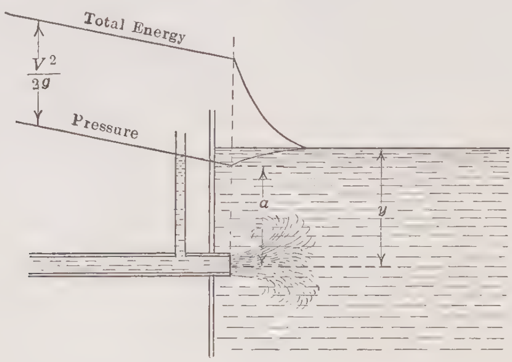


FIG. 105.—Conditions at discharge under water.

In Fig. 105 is shown a pipe discharging under water. If point (2) were taken at the end of the pipe, as in the preceding illustration, $H_2 = a + V^2/2g$ would be written, where a is the pressure head at that point and the datum plane is taken through the center of the pipe so that $z = 0$. Experiments indicate that the value of a is less than the depth of the water.[1] If this is so,

[1] WISLER, C. O., *Eng. News-Record*, 22, vol. 87, 1921.

it is clear that not all of the velocity head is lost at discharge but that some of it is converted into pressure head. If, on the other hand, point (2) were taken at the more convenient location at the surface of the water, using the same datum plane as before, $H_2 = 0 + y + 0$ would be written. The difference between the values for H_2 at these two different points is $a + V^2/2g - y$ and is obviously the energy lost between the mouth of the pipe and any point where the water is at rest. Thus the discharge loss is $H'_d = V^2/2g - (y - a)$.

The discharge of a pipe line under water is a limiting case of sudden enlargement, the final cross-section area being assumed infinite and the velocity zero.[1] The equation for sudden enlargement, when applied to this case, reduces to

$$H'_d = 1.098 \frac{V^{1.919}}{2g} = 0.017V^{1.92}. \tag{84}$$

This may also be written in the form

$$H'_d = k_d \frac{V^2}{2g}, \tag{85}$$

where $k_d = 1.098/V^{.081}$. If it is assumed, however, that k_d cannot be greater than 1.0, it may be seen that this should not be used for velocities less than 3.16 ft. per second.[2] Below that velocity the value of k_d may be taken as unity. For higher velocities the following may be used:

TABLE IV

V	3.16	5	10	15	20	40	60
k_d	1.00	0.96	0.91	0.88	0.86	0.81	0.79

In view of the fact that in most practical problems the value of this loss is of minor importance, unless the pipe is quite short, it is perfectly safe to assume

$$k_d = 1.0.$$

[1] The empirical equation for sudden enlargement based upon Archer's experiments at the University of California is said by Wisler to have been found by experiments at the University of Michigan to hold very accurately in this special case.

[2] It may be noted that this velocity is lower than the minimum values reported in either of the two sets of experiments cited.

This loss could be very materially reduced by placing a diverging mouthpiece on the discharge end of the pipe.

88. Loss Due to Sudden Contraction.—The phenomena attending the sudden contraction of a stream are shown in Fig. 106, which shows a marked drop in pressure due both to the increase in velocity and to the loss of energy in turbulence.[1] It is noted that in the corner upstream at section C there is a rise in pressure due to the fact that the stream lines are here curving so that the

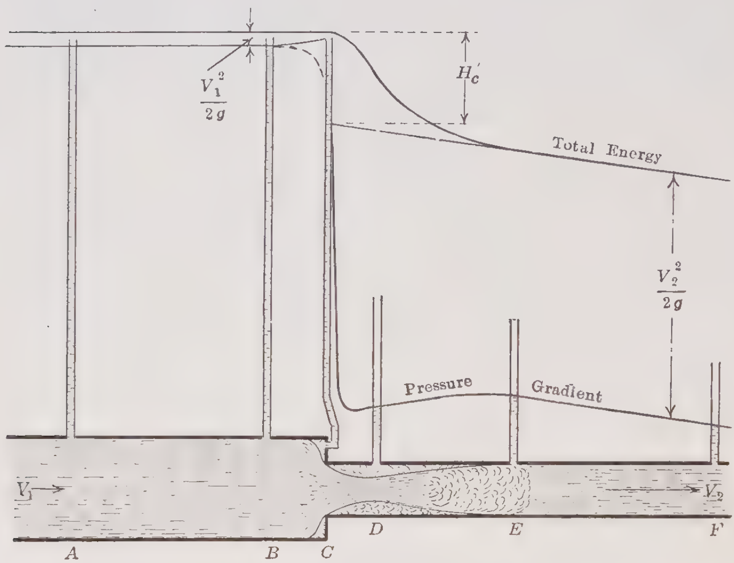


Fig. 106.—Loss of head due to sudden contraction.

centrifugal action causes the pressure at the pipe wall to be greater than in the center of the stream, for which the pressure variation from B to C is indicated by the dotted line.

From C to E the conditions are similar to those described for entrance. The loss of head may be represented by

$$H'_c = k_c \frac{V_2^2}{2g}$$

where k_c has the following values:

[1] Figures 106 and 107 are plotted to scale from observations made by the author with the same velocities in each case.

TABLE V

d_2/d_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k_c	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06

89. Loss Due to Sudden Enlargement.—The conditions at a section of sudden enlargement are shown in Fig. 107. There is a rise in pressure due to the decrease in velocity, but this rise is not as great as it would be if it were not for the loss in energy. There is a state of extreme turbulence from C to F beyond which the flow is normal. The drop in pressure just beyond section C, which was measured by a piezometer not shown in the drawing,

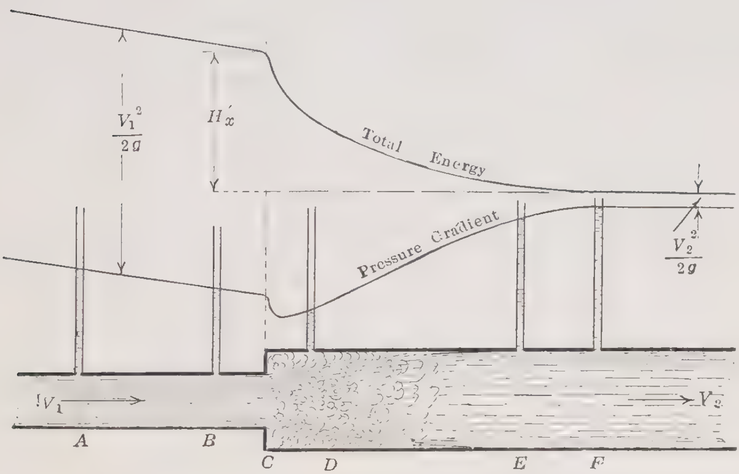


FIG. 107.—Loss of head due to sudden enlargement.

is due to the fact that the pressures at the wall of the pipe are in this case less than those in the center of the pipe.

It may be observed from the scale of the drawings that the loss of head due to sudden enlargement is greater than that due to sudden contraction. This is an illustration of the well-known fact that the losses of energy accompanying a decrease in velocity are always greater than those accompanying an increase. This is due to the less stable flow and the greater turbulence in the former case.

It has been found that the loss of head due to a sudden enlargement may be represented by[1]

$$H'_x = 1.098\,\frac{(V_1 - V_2)^{1.919}}{2g} = 0.017(V_1 - V_2)^{1.919}, \quad (87)$$

which may also be written in the form

$$H'_x = k_d \frac{(V_1 - V_2)^2}{2g}, \quad (88)$$

where k_d has the same values as in Table IV of Art. 86, for values of V identical with values of $(V_1 - V_2)$. If desired, the above may also be written in the form

$$H'_x = k_d\left(\frac{A_2}{A_1} - 1\right)^2 \frac{V_2^2}{2g} = k_d\left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2g}. \quad (89)$$

In most practical cases no appreciable error will result from assuming $k_d = 1.0$.

90. Loss for Gradual Contraction.—In order to reduce the foregoing losses, abrupt changes of cross-section should be avoided. Inspection of Fig. 106 shows that the principal cause

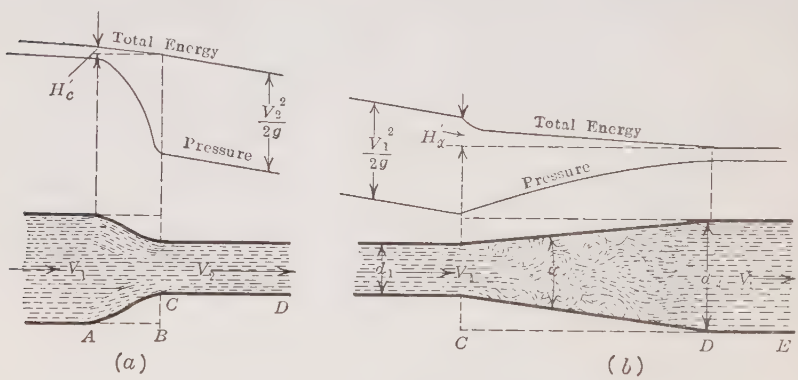


FIG. 108.—Gradual changes of area.

of loss is due to the contraction of the stream as it enters the smaller pipe. This may be prevented by changing from one diameter to the other by means of the smooth curve AC of Fig. 108 (a). With this construction there will be no additional loss in the length CD. Due, however, to the higher average velocity in the portion AC, there will be a slightly greater loss than in the same length of pipe of the original diameter. This loss is identical with that in a nozzle of the same form as the

[1] ARCHER, W. H., *Trans. A. S. C. E.*, vol. 76, p. 999, 1913.

portion AC, and is practically negligible. Ordinarily it will be about $0.04V_2^2/2g$.

For ease in manufacture the curved portion AC of Fig. 180 (a) may be replaced with a frustum of a cone without any appreciable increase in the loss. If the angle of the cone is too great the conditions of sudden contraction will be approached, due to the tendency of the stream lines to converge downstream from C. On the other hand, if the angle of the cone is too small, the distance AB will be unduly large and, since the average velocity in this conical section will be higher than that in the larger straight pipe, the friction loss in the reducer will be somewhat greater than in the length of straight pipe which it replaces. A total angle of from 20 to 40 deg. is probably about the proper amount of taper.

81. Loss in Diffuser.—A diffuser to join the smaller to the larger diameter may also be given a curved outline, or it may be a frustum of a cone, as in Fig. 108 (b). The loss of head will be some function of the angle of divergence and also of the ratio of the two areas, the length of the diffuser being determined by these two variables. Because of the great importance of this problem in such cases as draft tubes for turbines, diffuser passages for centrifugal pumps, and other practical applications, it seems desirable to devote some space to the consideration of the factors involved.

First for a given angle α the loss in the diffuser must increase as the ratio d_2/d_1 increases, due to the greater length of CD. Next with a given ratio of d_2/d_1, the length of the section CD, and the loss within it, will vary with the angle α.

In flow through a diffuser, the total loss may be considered as made up of two factors. One is the ordinary pipe friction loss which may be represented by

$$H' = \int \frac{f}{d}\frac{V^2}{2g}(dl).$$

(Note that (dl) is a differential length of the cone, while d is the diameter at any section.) In order to integrate the above it is necessary to express the variables f, d, and V as functions of l. For our present purpose it is sufficient, however, merely to note that the friction loss increases with the length of the cone. Hence for a given ratio of areas, the larger the angle of the cone, the less its length and the less the pipe friction, which is indicated

by the curve marked F in Fig. 109. However, in flow through a diffuser there is an additional turbulence loss set up by induced currents which produce a vortex motion over and above that which normally exists.[1] This additional turbulence loss will naturally increase with the degree of divergence, as is indicated by the curve marked T in Fig. 109. The total loss in the diverging cone is then represented by the sum of these two losses, marked k'. This is seen to be a minimum at 6 deg. for the particular case chosen, which is for a very smooth surface. If the surface were rougher, the value of the friction F would be increased. This not only increases the value of k', which is now

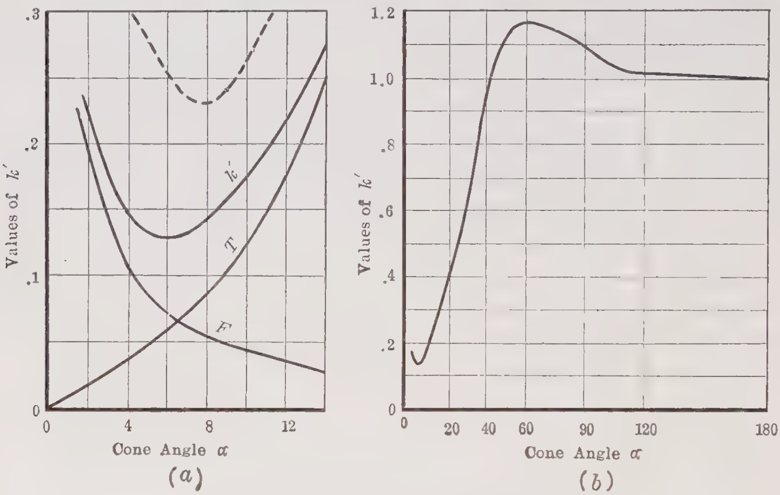


Fig. 109.—Losses in conical diffuser.

indicated by the dotted curve, but also shifts the angle for minimum loss to 8 deg. Thus the best angle of divergence increases with the roughness of the surface.

It has been found that this excess turbulence loss, produced by secondary or induced currents set up, is independent of the roughness of the surface of the cone. But it is a function of the viscosity of the fluid. The higher the viscosity the less the amount of eddying in the fluid and the less the loss. Decreasing the amount of this turbulence loss by using a liquid of higher viscosity than water tends to shift the angle of minimum loss to a higher value,

[1] Nedden, F. zur, "Induced Currents of Fluids," *Trans. A. S. C. E.*, vol. 80, p. 844, 1916.

as may be readily seen. But with a higher viscosity the normal pipe friction would also be greater and this likewise calls for a greater angle. Thus the best angle of divergence increases with an increase of viscosity.

It has been seen that the loss due to a sudden enlargement is very nearly represented by $(V_1 - V_2)^2/2g$. The loss due to a gradual enlargement is expressed as

$$H' = k'\frac{(V_1 - V_2)^2}{2g} = k'\left(\frac{A_2}{A_1} - 1\right)^2\frac{V_2^2}{2g} = k'\left(1 - \frac{A_1}{A_2}\right)^2\frac{V_1^2}{2g} \cdot \quad (90)$$

Values of k' as a function of the cone angle α are shown in Fig. 109 (*b*), for a wider range than appears in (*a*).[1] It is of interest to note that at an angle slightly above 40 deg. the loss is the same as that for a sudden enlargement, which is 180 deg., and that between these two the loss is greater than for a sudden enlargement, being a maximum at about 60 deg. This is because the induced currents set up are worse within this range.

EXAMPLE

100. For a diameter ratio of 1:2 and a velocity of 20 ft. per second in the smaller pipe, find the loss of head due to: (*a*) sudden contraction, (*b*) sudden enlargement, (*c*) expansion in conical diffuser with total angle of 20 and 6 deg.

Ans. (*a*) 2.05 ft., (*b*) 3.07 ft., (*c*) 1.40 ft., 0.453 ft.

92. Other Minor Losses.—In addition to the various losses which have just been described in detail, there are other sources of extra friction loss to which but little space will be devoted, since they involve nothing different in principle. Thus when water flows around a bend there is a distortion of the normal velocity distribution which sets up eddy currents and disturbances which persist downstream for something like 80 diameters. As a rough rule, it may be said that values of k may be taken as 0.2 for 90-deg. bends with ratios of the radius of curvature to the diameter of the pipe from 2 to 10, that $k = 0.8$ for a ratio of 1, and may be assumed as 0.75 for an ordinary ell. These values of k are to be multiplied by $V^2/2g$, where V is the average velocity in the pipe, and give us the friction loss in the bend and also the

[1] GIBSON, A. H., *Engineering* (*London*), Feb. 16, 1912. These values were based on area ratios of 1:9, 1:4, 1:2.25 and gave one curve up to an angle of about 30 deg. Beyond that the three ratios gave three curves which differed as much as about 18 per cent at 60 deg., where the turbulence was a predominating factor, and then drew together again as 180 deg. was approached. The curve here shown is a composite of those three.

downstream portion of the pipe in excess of the loss in the same length of straight pipe. For a tee the value of k may be taken as 1.50.

The loss of head in gate valves wide open may be found by using values of k which range from 0.80 to 0.06 as the size increases from ½ to 12 in., and globe valves wide open produce from fifteen to forty times as much loss as gate valves of the same size.[1]

In some cases it may be convenient to express all of these various minor losses as equivalent to so much additional length of straight pipe. Thus, if $kV^2/2g$ is equated to $f(l/d)V^2/2g$, it is seen that $l/d = k/f$. Hence the length of pipe, or preferably the value of l/d, can be found that would correspond to any given value of k and added to the actual length.

93. PROBLEMS

101. A pipe 12 in. in diameter and 300 ft. long has a flush entry and discharges freely into the air. If the velocity is 10 ft. per second, what is the loss of head?

Ans. 12.42 ft.

102. Prove that for a given rate of discharge, the friction loss in a pipe varies inversely as the fifth power of the diameter, assuming f to be constant.

103. Prove that $d = \sqrt[5]{lfq^2/39.7H'}$.

[1] CORP and RUBLE, *Univ. Wis. Bull.* 1, vol. 9.

CHAPTER VIII

FLOW THROUGH PIPES

94. Pipe Line Discharging into Air.—To illustrate the method of solution for flow through a pipe line of uniform diameter discharging freely into the air, a numerical example is given. Referring to Fig. 110, let h = 260 ft., the diameter of the pipe = 10 in., and the length = 5,000 ft. Consider the entrance to be non-projecting and assume the value of k_e to be 0.5 for this case. Let (1) refer to a point at the surface of the water in the reservoir, while (2) refers to the stream issuing from the end of the pipe. Then $H_1 = 0 + 260 + 0$, $H_2 = 0 + 0 + V^2/2g$, and $H' = (0.5 + 0.022 \times 5000 \times 12/10)V^2/2g = 132.5V^2/2g$. Inserting

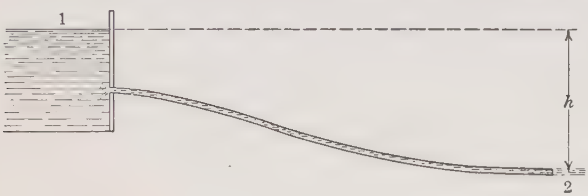


FIG. 110.

these values in the equation $H_1 = H_2 + H'$, $260 = V^2/2g + 132.5V^2/2g$ is obtained. Hence $V^2/2g = 260/133.5 = 1.95$. $V = 8.02\sqrt{1.95} = 11.2$ ft. per second. Therefore, $q = 0.545 \times 11.2 = 6.11$ cu. ft. per second. It may be seen that with this length of pipe it would have made very little difference if the entrance loss had been neglected altogether, and even the velocity head at (2). This would have been equivalent to the assumption that

$$h = f\left(\frac{l}{d}\right)\frac{V^2}{2g}, \tag{91}$$

which is often used for long pipe lines, because in such cases the loss in pipe friction alone is approximately equal to h. But this approximate equation can be used only for long pipes of uniform diameter and when the velocity of the stream at (2) is no greater than in the pipe itself.

In case the pipe is made up of sections of different diameters, the procedure would be to write an expression for the loss of head in each length in terms of the dimensions applying to it, that is

$$H' = f_1\left(\frac{l_1}{d_1}\right)\frac{V_1^2}{2g} + f_2\left(\frac{l_2}{d_2}\right)\frac{V_2^2}{2g} + \text{etc.}$$

By the equation of continuity these might then all be placed in terms of one velocity head. The solution of such a problem is very similar to the example which is worked out in the next article.

By way of presenting some general laws it may be observed that for any pipe line, no matter how complex, the loss of head may be expressed as

$$H' = y\frac{V^2}{2g} \tag{92}$$

where y is a factor evaluated according to the various details of the pipe and V is the velocity at any one section. That is $y = \Sigma k + \Sigma f\, l/d$. From this,

$$V = (1/\sqrt{y})\sqrt{2gH'}.$$

If the general equation of energy is written between points (1) and (2) of Fig. 110 and V is now considered the velocity at (2), then

$$h - V^2/2g = y\, V^2/2g,$$

from which

$$V = \left(\frac{1}{\sqrt{1 + y}}\right)\sqrt{2gh}, \tag{93}$$

which is seen to be similar in form to the formula for the velocity of flow through an orifice. By introducing the area A,

$$q = AV = \left(\frac{A}{\sqrt{y}}\right)\sqrt{2gH'} = K'\sqrt{H'} \tag{94}$$

$$\doteq \left(\frac{A}{\sqrt{1 + y}}\right)\sqrt{2gh} = K\sqrt{h}, \tag{95}$$

where K' and K are factors representing the quantities indicated in the equations.

In case the pipe does not discharge into the air at (2), but the water there is under some pressure at that section, then h must be interpreted as the drop of the hydraulic gradient. Thus the relations above are perfectly general.

Inspection of Eq. (91) shows that for a given head and length of pipe, the velocity will vary somewhat with the diameter of

the pipe. For by Eq. (75) f decreases as d increases and in Eq. (91) the ratio l/d also becomes smaller with larger diameters; therefore the entire coefficient of $V^2/2g$ becomes smaller as the size of the pipe increases. Hence for the same value of h, V will increase as the diameter of the pipe increases, and it may be shown that V varies as $d^{0.5 \text{ to } 0.6}$.

EXAMPLES

104. Suppose that in Fig. 110 the pipe projects into the reservoir at entrance and discharges freely into the air at (2), the size of the jet being equal to the diameter of the pipe. If $h = 40$ ft., $d'' = 12$ in., and $l = 50$ ft., compute the rate of discharge considering all losses.

Ans. 22.7 cu. ft. per second.

105. In problem 104 if $l = 1{,}000$ ft., all other data remaining the same, compute the rate of discharge considering all losses. Compute the rate of discharge by the approximate method, neglecting minor losses.

Ans. 8.2 cu. ft. per second; 8.56 cu. ft. per second.

95. Pipe Line with Nozzle.—Suppose in the example of the preceding article, it is assumed that there is placed on the end of the pipe a nozzle which discharges a jet 2.5 in. in diameter and that the velocity coefficient of the nozzle is 0.95. There are two velocities to deal with instead of one, so let V_1 = velocity in the pipe and V_2 = velocity of the jet. By the equation of continuity, $A_1V_1 = A_2V_2$ and thus $V_2 = (10/2.5)^2V_1 = 16V_1$ and $V_2^2 = 256V_1^2$. The loss of head in the nozzle may be found by Eq. (31) to be $(1/0.95^2 - 1)V_2^2/2g = 0.11V_2^2/2g$. If (2) now refers to the jet, while n refers to a point in the pipe near the nozzle then,

$$H_1 = 0 + 260 + 0,$$

$$H_n = p_n + 0 + \frac{V_1^2}{2g},$$

$$H_2 = 0 + 0 + \frac{V_2^2}{2g} = 256\frac{V_1^2}{2g},$$

$$H'_{1-2} = 132.5\frac{V_1^2}{2g} + 0.11\frac{V_2^2}{2g} = 160.6\frac{V_1^2}{2g}.$$

Writing the equation now between (1) and (2),

$$260 = 256\frac{V_1^2}{2g} + 160.6\frac{V_1^2}{2g} = 416.6\frac{V_1^2}{2g}.$$

Thus

$$\frac{V_1^2}{2g} = \frac{260}{416.6} = 0.625$$

and

$$V_1 = 8.02\sqrt{0.625} = 6.33 \text{ ft. per second.}$$

It would have been equally easy to have substituted for V_1 in terms of V_2 and to have found $V_2^2/2g = 160$ and $V_2 = 101.6$ directly. With the procedure used, they can now be found by use of the equation of continuity. The rate of discharge is $q = 0.545 \times 6.33 = 0.034 \times 101.6 = 3.45$ cu. ft. per second. This shows that the addition of the nozzle has reduced the discharge, but has given a much higher jet velocity.

Writing the energy equation between (1) and n,

$$p_n + \frac{V_1^2}{2g} = 260 - 132.5\frac{V_1^2}{2g}.$$

Since the numerical value of $V_1^2/2g$ has been obtained, this may be reduced to

$$p_n = 260 - 133.5 \times 0.625 = 176.6 \text{ ft.}$$

which is the pressure head at the base of the nozzle. The solution may now be considered as completed, but, as a check, it may be noted that

$$H_n = 176.6 + 0.625 = 177.2$$

and

$$V_2 = 0.95 \times 8.02\sqrt{177.2} = 101.6 \text{ ft. per second.}$$

It may be observed that if the value of f were to be taken from the curves in Fig. 101, the solution would not be quite so direct, since f is there a function of the unknown velocity. The procedure in such a case is to select a value of f for what is believed to be the probable value of the velocity and solve, as above. The velocity obtained may then be used to find a new value of f, and so on until the result checks with the assumption. A sufficient degree of accuracy for all practical purposes, however, considering the uncertainty as to the precise value of the roughness element, may be attained in one or two trials.

EXAMPLE

106. A pipe line 400 ft. long and 6 in. in diameter discharges a 2-in. jet into the air at a point which is 200 ft. lower than the water surface at intake. The entrance to the pipe is a projecting one and the velocity coefficient of the nozzle is 0.97. Find the pressure head in the pipe at the base of the nozzle. What is the rate of the discharge?

Ans. 160.8 ft., 2.16 cu. ft. per second.

96. Discharge under Water.—In case the pipe line discharges under water, as shown in Fig. 111, the point (2) is most con-

veniently taken at the surface of the water, where everything may be supposed to be known. Thus if $h = 100$ ft., the diameter $= 10$ in., and the length $= 400$ ft., then $H_1 = 100$ and $H_2 = 0$, taking the datum plane at the surface of the lower reservoir. Assuming at entrance, $k = 1$, and at discharge $= 0.9$, the total losses are given as, $H' = (1 + 0.022 \times 400 \times 12/10 + 0.9)V^2/2g = 107.5V^2/2g$. Since $100 - 0 = 107.5V^2/2g$, $V^2/2g = 100/107.5 = 0.93$ is found, from which $V = 8.02\sqrt{0.93} = 7.72$ ft. per second. Referring to Table IV it may be seen that the value of k at discharge should have been taken as somewhat larger, but the effect upon the answer here would be negligible.

It may be observed that the solution is independent of the location of either end of the pipe, as the flow is a function only of the difference of the two water levels.

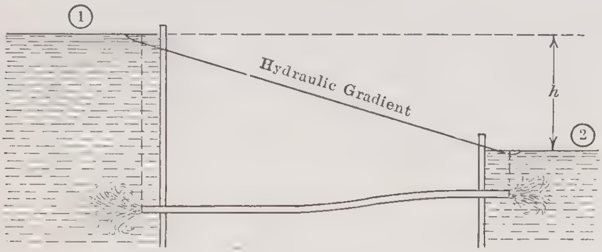


FIG. 111.

If the coefficient of loss at discharge were assumed to be 1 the solution would be identical with that of a pipe discharging into the air.

97. Size of Pipe for Given Discharge.—It is possible to find the diameter of pipe necessary for a given rate of discharge by direct solution, but a fifth degree equation is involved.[1] It will therefore be found slightly easier and simpler to solve for the diameter by a method of "cut and try" rather than to attempt the solution of the higher degree equation, though the latter is not difficult.

The procedure of "cut and try" is first to assume any diameter that seems reasonable, and by the usual methods compute the actual rate of discharge that this diameter would give. Then compare this with the value of q desired, and if it is too large or too small assume a new size of pipe and repeat until the computed

[1] If f is expressed by Eq. (75) this equation will be of the sixth degree.

rate of discharge is approximately equal to that specified. Of course one should use only commercial sizes of pipe and not attempt to get a diameter to a fraction of an inch. Naturally if only commercial sizes of pipe are used the computed rate of discharge may not agree precisely with the value desired, and so the size pipe that gives the nearest to that should be chosen. If the conditions of the problem are such that at least a certain rate of discharge must be obtained, then a size pipe should be used which will give a slightly greater value than this rather than one just under it. In the problems of the text, sizes of pipe are used in whole inches. Actual standard pipe dimensions are given on page 321.

If good judgment is used, it should be possible to get the correct answer within two or three trials at most. In order to do this it is necessary to compare carefully the rate of discharge computed with the value specified and then to estimate how much the area of the pipe might need to be increased or diminished to yield the proper result. In doing this it must be borne in mind that the velocity is not the same in pipes of different sizes. The larger the pipe the higher the velocity of flow, and hence it will discharge more in proportion to its area than a smaller pipe. This is not a matter that is worth making any computations on for this purpose, but it might be noted that, all other things being equal, the rate of discharge varies about as $d^{2.6}$. Therefore, in assuming a new diameter it is not advisable to go quite so far from the former value as would be necessary if the velocity remained the same in value, in which case q would vary as d^2.

As an example, suppose it is desired to find the size of pipe to deliver 50 cu. ft. per second with a total fall available of 60 ft., the length of pipe being 300 ft. Consider an entrance coefficient of 0.5 and that the pipe discharges freely into the air. Our equation may then be put into the form $(1.5 + f300/d)V^2/2g = 60$. Assume any reasonable value, such as 20 in., for the diameter. Then $(1.5 + 0.02 \times 300 \times 12/20)V^2/2g = 60$. From this $V^2/2g = 60/5.1 = 11.75$, $V = 8.7$, and $q = 19$. (Observe that the quantity which this size pipe will actually deliver is found without making any use of the desired discharge at all.) Now it is seen that a rate of discharge is required about 2.5 times as great. Since the velocity will increase with a larger diameter a pipe will be tried whose area is only about twice as great, or a

diameter of 30 in. With this $(1.5 + 0.02 \times 300 \times 12/30)V^2/2g = 60$, from which $V = 9.95$, and $q = 48.8$. This is close to the required amount. A pipe 31 in. in diameter may be seen to discharge something in excess of $(31/30)^2 \times 48.8$ and will, therefore, be the correct size to use.

If the length of the pipe were so long that the approximate formula, Eq. (91) could be used, then the procedure might be somewhat more direct (see example 103). Thus assume the length above to be 20,000 ft. and the required rate of discharge to be only 5 cu. ft. per second. The result obtained would be $(f \times 20{,}000/d)V^2/2g = 60$, and since $V = q/0.785d^2$, this may be reduced to $d^5 = 210f$. Assuming $f = 0.02$, then $d = 4.2^{0.2} = 1.33$ ft. or 16 in. If necessary, the value of f used might then be corrected and a new solution made. That is not warranted in this instance, however.

EXAMPLES

107. What must be the diameter of pipe to discharge 6.5 sec. ft. under a head of 120 ft., if the length of pipe = 50 ft. and the entrance coefficient is 1?

Ans. 6 in.

108. What size is required if the length is 5,000 ft.?

Ans. 12 in.

98. Economic Size of Pipe.—Where the physical conditions fix the value of the head to be lost in pipe friction, the size of pipe for a given rate of discharge is to be determined as in Art. 97. But in case the pipe line is to deliver the water from a pump the friction head may have any value whatever, while if it supplies water to a turbine the head lost may also be of any magnitude up to the value of the total head available minus the velocity head in the pipe. Practically, however, it should be restricted to less than one-third the total head (see Fig. 117).

If the rate of discharge is assumed to be constant, it is clear that the larger the pipe the less the velocity of flow and hence the less the value of the head lost. Since lost head means a waste of power in pumping water, or a loss of power which might otherwise be developed by a power plant, the problem becomes one of determining the proper value to be assigned to this item.

The larger the pipe the more it costs as is shown in Fig. 112. The values plotted in this curve, however, are a certain percentage of the total cost, being the annual fixed charges and including

interest on the investment, depreciation, etc. This curve in general is a discontinuous function since the costs of different commercial sizes do not follow a mathematical equation. Also the curve is subject to abrupt breaks where the increasing size may compel the change from one type of pipe to another of different construction. For each size of pipe the loss of head may be determined and hence the amount of power lost. If this horsepower lost is then multiplied by the annual value of a horsepower, a second curve showing the annual loss due to pipe friction may be plotted. The sum of these two items is the total annual cost

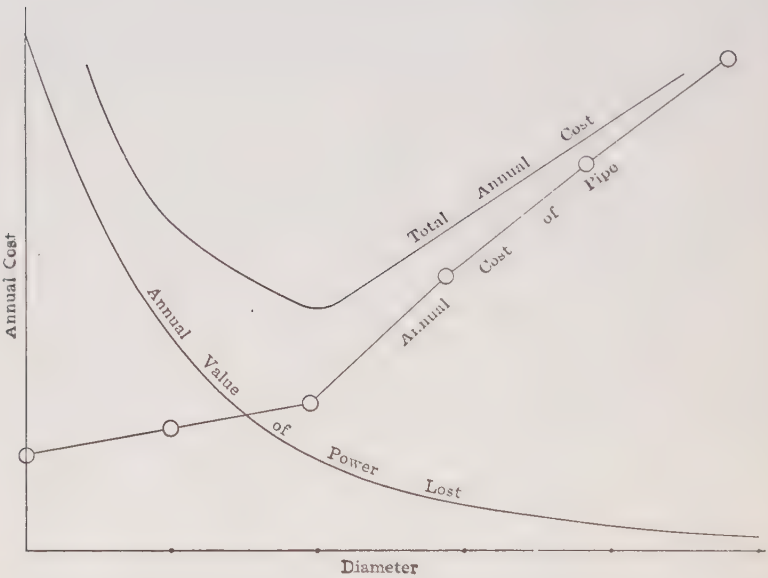


Fig. 112.

of the pipe line. The size for which this total is a minimum is the most economical.

It must be pointed out that an accurate solution of this problem may be difficult in practice, especially in the case of a water power plant. The chances are that the rate of flow through the pipe line will not be constant, since the load on the plant will vary, and hence the load factor must be known before one can compute the total amount of power lost per year. And even then it is hard to fix the exact money value of such power lost.

Since in a given pipe line the power lost varies as q^3, it is readily seen that the constant flow to be used is the cube root of the mean of the cubes of the actual rates of flow.

It must also be noted that in some cases the economic solution may not be the best. In case the value of a unit of power is small and the fixed charges are high the resulting pipe size would be relatively small and the loss of head large. This means that the velocity of the water would be high and, as will be seen later, this may cause trouble due to surges and water hammer when any change is made in the flow in governing the turbine. Also when the loss of head is large the variation in head from full load to no load is large, as may be seen in Fig. 117. This may also be undesirable in the operation of the turbine. Hence for these reasons a larger-size pipe may be used.

The higher the pressure in a pipe line the greater the cost of a pipe of given diameter. Thus in Fig. 112 the curve for the annual cost of pipe is higher with increasing heads and the most economic diameter is, therefore, smaller. Hence the penstock for a high-head plant should not be uniform but should diminish in size as the head increases, while at the same time the thickness of the metal should increase. For such a case, the values in Fig. 112 should not be for the entire length of the pipe, but only for a short length, say 1 ft., and the solution repeated for various intervals. There is a more convenient method of solution of the problem for this case, but the explanation requires too much space to be given here.[1]

EXAMPLE

109. A water supply of 300 cu. ft. per second is available for a power plant under a static head of 1,200 ft. The penstock is of riveted steel ($f = 0.022$) and 7,000 ft. long. Assume the fixed charges on investment to be 10 per cent per year and the annual value of a horsepower to be $20 under the conditions of operation, treating the case as though the flow were constant. Fill in the table and determine the most economical size.

Ans. 90 in.

[1] It will be found in the mid-November, 1924, number of *Mechanical Engineering* of the A. S. M. E., in papers by H. L. Doolittle and the author.

Diameter, inches	Cost	V	H′	Annual value of power lost	Annual fixed charges	Total annual cost
70	$192,000					
80	250,000					
90	317,000					
100	390,000					

99. Compound Pipes.—In the case of flow through compound or parallel pipes, such as in Fig. 113 the following fundamental relations exist. The sum of the flow through all the compound pipes equals the total flow in the main. And since the pressures

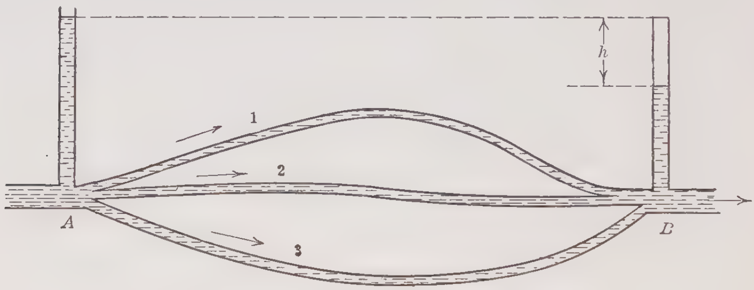


FIG. 113.

at A and B are common to all the pipes it follows that the loss of head in each pipe is the same. Or

$$1.\ q_0 = q_1 + q_2 + q_3.$$

$$2.\ h = \text{same for all.}$$

The loss of head in any section of any pipe is $H' = \left(f\frac{l}{d} + n\right)\frac{V^2}{2g}$, where n is a factor to account for any minor losses and in long pipes may be neglected as shown in Art. 85. Solving for V the following is obtained,

$$V = \sqrt{\frac{2gH'}{f\frac{l}{d} + n}}$$

$$q = AV = A\sqrt{\frac{2gd}{fl + nd}}\sqrt{H'} = K'\sqrt{H'}$$

as in Eq. (94). In long pipes also the difference between H' and h may be considered to be negligible so that Eqs. (94) and (95) are practically equivalent. Since K is a constant for any given pipe its value may be computed in each case and the following written

$$
\begin{aligned}
q_1 &= K_1\sqrt{h} \\
q_2 &= K_2\sqrt{h} \\
q_3 &= K_3\sqrt{h} \\
\hline
q_0 &= K_0\sqrt{h}
\end{aligned}
$$

where $K_0 = K_1 + K_2 + K_3$.

If the value of h be given and all dimensions of the pipes are known it is then easy to find the rate of discharge in each separate pipe. If the total rate of discharge, q_0, be given the value of h may be computed and then the flow in each pipe can be found. If the dimensions of one or more of the pipes are unknown, however, a solution by trial may be necessary. If any water is supposed to be withdrawn between A and B, it will then be necessary to combine this problem with that in Art. 100.

EXAMPLE

110. In Fig. 113 suppose that water enters at A from a large standpipe and that B is located 50 ft. above a given datum plane. The three pipes are of the following dimensions: 1,200 ft. of 6-in. pipe, 1,000 ft. of 8-in. pipe, and 1,200 ft. of 10-in. pipe, while the diameter at B is 16 in. If 14 cu. ft. of water per second are delivered at B under a pressure head of 100 ft., what must be the elevation of the water surface in the standpipe above the datum plane?

Ans. 242 ft.

100. Branching Pipes.—Suppose the water flowing in pipe AB in Fig. 114 divides at B, a portion flowing through BC into the reservoir shown, while the rest flows on through pipe BD to some destination not shown. Suppose the pressure at D to be indicated by a piezometer column. (In reality both branches are similar since the condition would be practically the same if the second pipe discharged at D into a reservoir whose water surface were the same as the top of the water in this tube.) There are two fundamental relations also. The flow in the main AB

is equal to the sum of the flow in the branches. And the pressure at B is a value common to all three pipes. That is

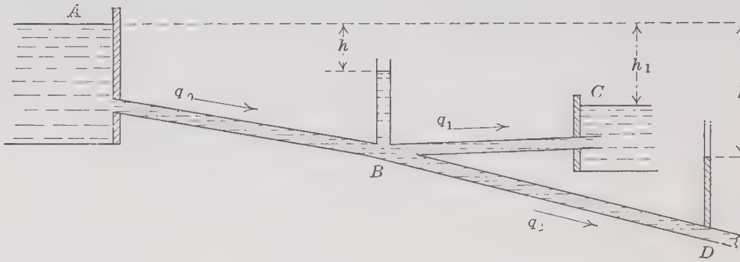


FIG. 114.

1. $q_0 = q_1 + q_2$.
2. p_B (or h) = common to all.

From Eq. (95), we have

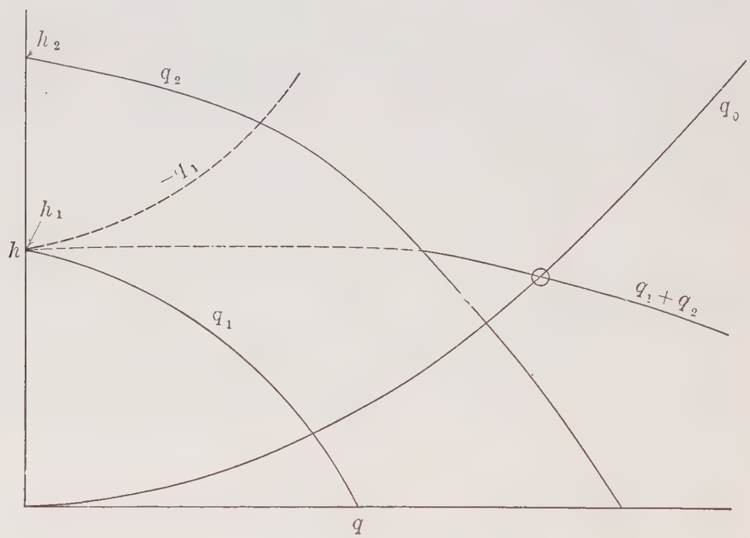


FIG. 115.

$$q_0 = K_0\sqrt{h}$$
$$q_1 = K_1\sqrt{h_1 - h}$$
$$q_2 = K_2\sqrt{h_2 - h}.$$

These equations may be verified by writing the general equation of energy between A and B, B and C, and C and D. In

the case of reasonably long pipes we may disregard the minor losses and also velocity heads at such points as B and D. This problem is not so readily solved as the one in the preceding article because the factor under the radical sign is different for each pipe. Also in the former case there may be different rates of flow and hence different values of h. But in this case there is only one value of p_B or h for equilibrium and hence there can be only one rate of flow, if other dimensions are fixed.

The solution of this problem is illustrated by Fig. 115. The value of h at which equilibrium is attained is given by the intersection of curves for q_0 and $q_1 + q_2$. It should be noted that if the conditions are such that h is greater than h_1, for instance, the flow in BC would be opposite to that assumed and the curve for q_1 would then be as indicated by the dotted line. In this case values of q_1 should then be added to q_0.

EXAMPLE

111. Suppose that in Fig. 114, AB consists of 1,500 ft. of 12-in. pipe, BC of 800 ft. of 6-in. pipe, and BD of 1,200 ft. of 8-in. pipe. The value of h_1 is 20 ft., B is 35 ft. below the level of the water surface at A, and D is 60 ft. below. When the pressure head at B is 25 ft., find the values of the flow in each pipe and the pressure head at D.

Ans. $q_0 = 3.49$, $q_1 = 0.817$, $q_2 = 2.67$, $p_D = 12.6$ ft.

101. Pipe with Laterals.—Assume a pipe main from which water is withdrawn by laterals along its course. Then either V or d or both must vary. In such a case the loss of head between any two points may be determined as follows. Differentiating the expression for loss of head an expression is obtained for the loss of head in any infinitesimal distance dl. Thus

$$dH' = \frac{f(dl)}{d}\frac{V^2}{2g}.$$

The integration of this between the proper limits of l will give us the value of the head lost in that distance. Thus

$$H' = \frac{1}{2g}\int f\frac{V^2}{d}(dl). \tag{96}$$

If it is possible to express f, V, and d as functions of l the integration of the above equation will give the value of H'. If an integration by calculus is not possible values of fV^2/d may be plotted as a function of l. The area between this curve and the axis for l is the value of the integral.

Special Case.—If the pipe is of uniform diameter and the laterals are uniformly spaced and may be assumed to take off water uniformly along its length, the above may readily be integrated. If the velocity of the water entering the length considered is V_1 and that leaving it is V_2 while the total length is l, the above conditions give

$$\frac{(dl)}{dV} = \frac{l}{V_2 - V_1},$$

since the velocity decreases uniformly along the length of pipe. Substituting this value of $d(l)$ in Eq. (96),

$$H' = \frac{f}{2gd} \times \frac{l}{V_2 - V_1}\int_{V_1}^{V_2} V^2 dV$$
$$= \frac{1}{3}\frac{fl}{2gd}\frac{V_1^3 - V_2^3}{V_1 - V_2}.$$

If the terminal of the main is a dead end so that the value of V_2 is zero, this expression is further simplified and indicates that the loss of head is one-third the loss that would exist if the entire amount of water entering at (1) flowed clear through the pipe and discharged at (2).

EXAMPLE

112. In Fig. 114, suppose that the branch BD were closed at D and discharged uniformly through laterals along its length. What would then be the pressure head at D, using same data as in problem 111?

Ans. 37.7 ft.

Fig. 116.—Varying hydraulic gradient with different rates of discharge.

102. Power Delivered by a Pipe.—In Fig. 116 consider a point C which is located near the end of a pipe line. When no flow occurs, due to the closure of a valve or other device beyond C, the pressure at C is a maximum, being equal to CX. But when flow occurs the pressure at C drops to the value CY, and the greater the rate of discharge the steeper will be the hydraulic gradient and the less will be the pressure at C. If the nozzle, or other device beyond C, be removed entirely, making

C a point at the very end of the pipe, the pressure will then be reduced to zero. In Fig. 117 are shown the decrease in pressure head at C and the increase in velocity head at C as the rate of discharge is caused to increase by opening wider whatever device is below C. Now the total head at C is the sum of the pressure head and the velocity head, but it is seen to decrease continually with increasing discharge until it reaches a minimum value which is the velocity head when the pipe is wide open.

It has been seen that power is a function of both q and H and may be expressed as wqH_C. In the case under consideration

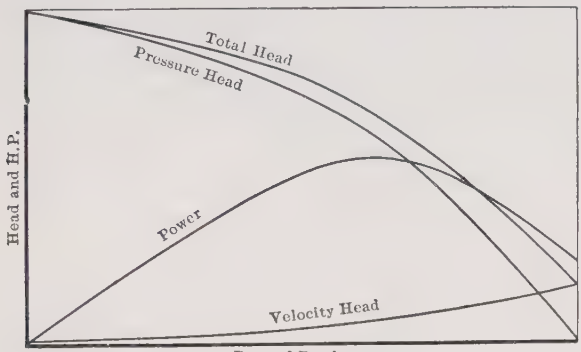


FIG. 117.—Head and power delivered by a pipe.

H_C decreases as q increases. When q is zero H_C is a maximum but the power is zero. And when q is a maximum the power is small due to the small value of H_C. Somewhere between these two extremes the product of these two variables reaches a maximum as shown by Fig. 117. It can be shown that the power is a maximum when the flow is such that one-third of the static head is consumed in friction, provided H' varies as V^2 or $H' = bq^2$. Let z be the static head or total fall.

From Eq. (27),

$$P = wqH_C = wq(z - H') = wqz - wbq^3$$

$$\frac{dP}{dq} = z - 3bq^2 = z - 3H'.$$

For a maximum value of P,

$$z - 3H' = 0$$

or

$$H' = \frac{z}{3}.$$

The efficiency of a pipe line may be defined as the ratio of the power delivered to the power supplied. But power is proportional to head, and hence the efficiency is H_C/z, where $z = CX$ in Fig. 116. In the case of maximum power delivered one-third of CX has been consumed in friction, hence the efficiency is only $66\frac{2}{3}$ per cent. If economy of water is no object it would be desirable to transmit power under these conditions as the cost of the pipe line would be small in proportion to the power delivered. But under usual conditions it is undesirable that one-third of the energy of the water be wasted, and hence such a size of pipe line would be employed that it could deliver the water available with a loss of only a few per cent. With ordinary power-plant practice the efficiency of the pipe lines leading to the turbines is about 95 per cent.

EXAMPLES

113. A pipe line 2,000 ft. long is 5 ft. in diameter. If the fall from the reservoir to the end of the pipe is 120 ft., what is the maximum amount of power the pipe could deliver?

Ans. 3,200 hp.

114. What amount of power would the pipe in problem 113 deliver if its efficiency were 95 per cent?

Ans. 1,765 hp.

115. What size pipe would be required to deliver the water discharged in problem 113 if the efficiency of the pipe were to be 90 per cent?

Ans. 6.4 ft.

103. Pipe Line with Pump.—In case a pump lifts water from one reservoir to another, as in Fig. 118, it not only does work in lifting the water the height z but it also has to overcome the friction loss in the suction and discharge piping. This friction head is equivalent to some added lift so that the effect is the same as if the pump lifted the water a height $z + H$, without loss. Hence the power delivered to the water by the pump is

$$W(z + H'). \tag{97}$$

The power required to run the pump is greater than this, depending upon the efficiency of the pump. Although the pump

actually lifts the water a height z, it is said to work against a head h whose value is

$$h = z + H'. \tag{98}$$

In case the pump discharges a stream of water through a nozzle, such as in Fig. 119, the water has not only been lifted a height z but it has also received kinetic energy proportional

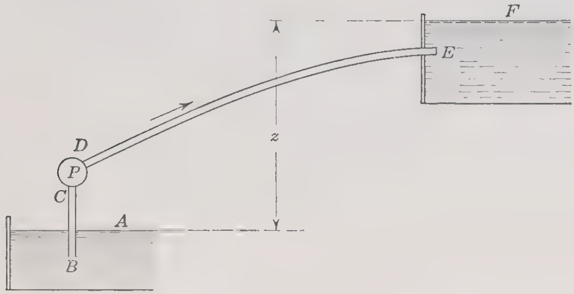


FIG. 118.—Pipe line with pump.

to $V_2^2/2g$, where V_2 is the velocity of the jet. Thus the power delivered to the water by the pump is

$$W\left(z + \frac{V_2^2}{2g} + H'\right). \tag{99}$$

And the head against which the pump works is now

$$h = z + \frac{V_2^2}{2g} + H'. \tag{100}$$

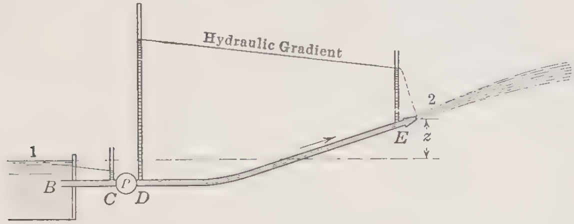


FIG. 119.—Pipe line with pump.

The difference between the two cases in Figs. 118 and 119 is really slight. In Eq. (97) the velocity head at E has been considered to have been lost, while in Eq. (99) the velocity head in the jet has not yet been lost. Thus H' in Eq. (97) includes the velocity head of discharge while the H' in Eq. (99) does not.

EXAMPLES

116. A 10-in. pipe line is 3 miles long. If 4 cu. ft. of water per second are to be pumped through it, the total actual lift being 20 ft., what will be the horsepower required if the pump efficiency is 70 per cent?

Ans. 240 hp.

117. In Fig. 118 assume $d'' = 10$ in., $BC = 20$ ft., $DE = 3{,}000$ ft., and $z = 135$ ft. If $q = 7$ sec. ft. and the pump efficiency is 80 per cent, what is the power required?

Ans. 340 hp.

118. In problem 117, if the elevation of C above the water surface is 13 ft. and that of D is 15 ft., compute the pressures at C and D.

Ans. $p_C = -19.48$ ft., $p_D = +323$ ft.

104. Pipe Line with Turbine.—The type of machine that is usually employed for converting the energy of water into mechanical work is called a turbine. In flowing from the upper body of water in Fig. 120 to the lower, the water loses its

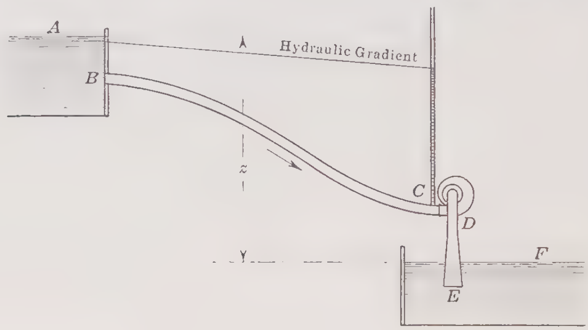


FIG. 120.—Pipe line with turbine.

potential energy due to the elevation z. This energy, which the water loses, is expended in two ways. A part of it is consumed in hydraulic friction in the pipe and the rest of it is delivered to the turbine. Of that which is delivered to the turbine, a portion is lost in hydraulic friction within the machine and the rest is converted into mechanical work.

The power delivered to the turbine is decreased by the friction loss in the pipe line, and its value is given by

$$W(z - H'). \tag{101}$$

The power delivered by the machine is less than this depending upon both the hydraulic and mechanical losses of the turbine. The head under which the turbine operates is

$$h = z - H'. \tag{102}$$

In the case of a turbine the only loss of head H', which is deducted is that in the supply pipe. The draft tube, as the conduit which leads the water away from the turbine is called, is considered an integral part of the machine and hence h should cover losses in it as well as in the turbine case itself.

In applying these equations it should be noted that the particular location of the turbine is immaterial so long as it is not set so high above the lower water level that the pressure at the top of the draft tube approaches absolute zero in value. But as long as this is avoided the turbine can make use of the entire fall to the lower water level by the use of an air-tight draft tube. The higher the turbine is situated, within the limit specified, the less the pressure will be at intake, but this is offset by an increased suction on the discharge side.

EXAMPLES

119. In Fig. 120 assume $d'' = 12$ in., $BC = 200$ ft., and $z = 120$ ft. The entrance to the pipe at the intake is flush with the wall. (*a*) If $q = 8$ sec. ft., what is the head supplied to the turbine? (*b*) What is the power delivered by the turbine if its efficiency is 75 per cent?

Ans. (*a*) $h = 112.2$ ft. (*b*) 76.5 hp.

120. A turbine operating under a total fall of 120 ft. ($z = 120$ ft.), is supplied with water through 300 ft. of 8-in. pipe. If the rate of discharge be such that 30 ft. of head is lost in friction in the pipe, what will be the power delivered to the turbine?

Ans. 49.2 hp.

105. Equation of Energy with Turbine or Pump.—The general equation of energy, derived in Art. 45, may be applied equally well to a pipe line in which there is any form of turbine or pump between the two sections considered. But the equation should always be applied with the water flowing from point (1) to point (2) regardless of the relative positions of these two points. In the preceding article H' represents the energy lost by the water in pipe friction, while h represents the energy lost by the water within the turbine. (Of the latter a part is lost within the turbine in hydraulic friction and a part is converted into mechanical work, but it is all lost so far as the water is concerned.) Hence the following may be written for the turbine

$$H_1 - H_2 = H' + h.$$

This is really equivalent to Eq. (102) where A and F correspond to points (1) and (2) in the above equation, for $H_A - H_F = z$.

In the case of the pump the h represents energy put into the water by the pump between points C and D in Fig. 118 and hence is a negative loss. For the pump, therefore, may be written

$$H_1 - H_2 = H' - h.$$

This is equivalent to Eq. (98) where $H_A - H_F = -z$, or to Eq. (100) where $H_1 - H_2 = -(z + V_2^2/2g)$.

As an illustration consider a special case where a turbine of known capacity is placed in a pipe line of known dimensions and it is then desired to determine the rate of discharge. Since in the pipe line $H' = yV^2/2g$ as in Eq. (92), $H' = Mq^2$ may be written where M is a constant whose value may be determined from the dimensions of the pipe. It may be shown (Art. 169) that the rate of discharge through any turbine may be expressed as $q = k\sqrt{h}$ where h is the net head utilized by the turbine, and k would be known for a given turbine. Hence may be written $h = (q/k)^2 = Bq^2$, where B is another constant whose value can be determined. Now referring to Fig. 120

$$H_1 - H_2 = H_A - H_F = H' + h$$

$$z = Mq^2 + Bq^2$$

$$q = \sqrt{\frac{z}{M + B}}.$$

After q is determined the net head on the turbine may readily be found and everything is then known. The method can readily be extended to other combinations.

EXAMPLE

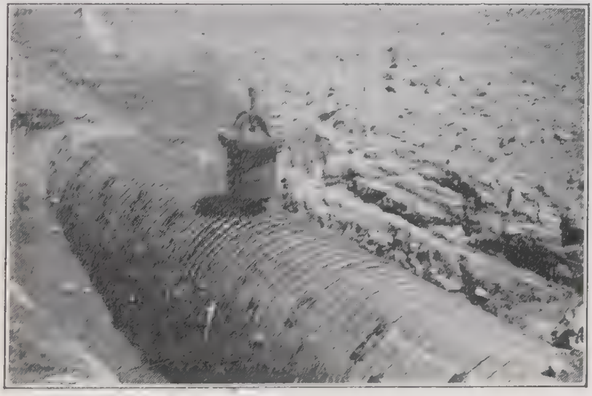
121. Assume the total fall from one body of water to another to be 120 ft. The water is conducted through 200 ft. of 12 in. pipe with the entrance flush with the wall. At the end of the pipe is a turbine and draft tube which discharged 5 cu. ft. of water per second when tested under a head of 43.8 ft. in another location. What would be the rate of discharge through the turbine and the net head on it under the present conditions.

Ans. $q = 8$ cu. ft. per second. $h = 112.2$.

106. Effect of Air at Summit.—In Fig. 58 is shown a pipe line having a "summit" at D, which is above the hydraulic gradient, indicating that the pressure at this point is less than atmospheric. In practice this would be avoided, for not only might the excess external pressure cause this portion of the pipe to collapse, but the accumulation of air at this point might interfere with or even stop the flow entirely. All ordinary water

carries air in solution and readily gives it up at a point of low pressure so that air would collect in time, though it were all expelled by some means in the beginning. Therefore in designing a pipe line, whenever any portion of it is found to be above the hydraulic gradient, an attempt would be made to change the profile so that this may be avoided. In case this is impossible then provision must be made for exhausting the air occasionally, if full flow is to be maintained.

If the summit is below the hydraulic gradient, air could still collect, though not so readily since water under pressure tends



Courtesy Redwood Manufacturers Co.

Fig. 121.—Air valve on wooden pipe line.

to absorb air. But under such conditions it is very easy to release the air, since it will escape if an opportunity is offered it. A valve for such a purpose is shown in Fig. 121. Such valves usually have a float, the dropping of which, as air collects and lowers the water surface, causes a valve to open. When the air escapes, the water level rises and the float closes the valve again. The valve in Fig. 121 is also constructed so as to admit air into the pipe in case a vacuum should accidentally occur in any way. This will prevent the pipe from collapsing in such an event. In many cases it is highly desirable that pipe lines be furnished with suitable air valves for both these purposes.

In Fig. 122 is shown how a vacuum might accidentally occur, when normally the pipe is under a positive pressure. It has been seen that the greater the velocity of flow through a pipe line the less the pressure will be at any point. Hence if some event,

such as the bursting of the pipe at C, permits a larger flow of water, the hydraulic gradient will be much steeper than normal. This means that it will be lowered, and it may be lowered sufficiently to be below portions of the pipe as in Fig. 122.

Also if the admission of water to a pipe line is shut off by the closure of a gate valve at intake, the water which is already

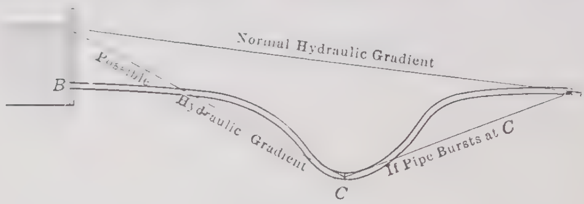


Fig. 122.

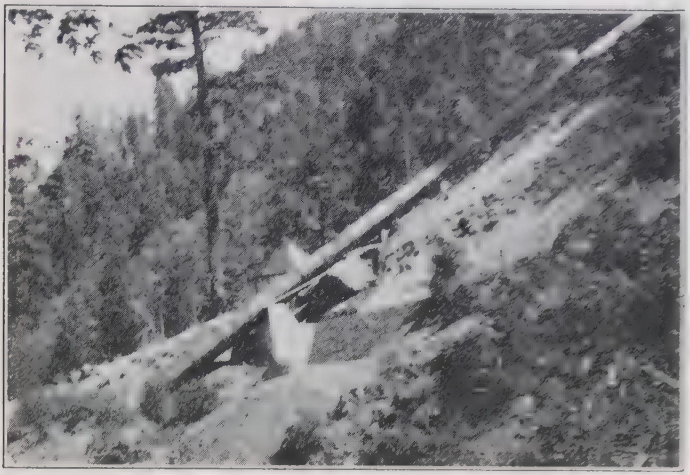
in the pipe will tend to run out. As no more water can get in to take its place, a vacuum might be created unless air were admitted. Hence, some device is usually provided just below the valve at the intake, and in some cases at other points, to admit air upon such an occasion.

From a photograph by the author.

Fig. 123.—Cast-iron pipe line.

107. Construction of Pipe Lines.—Cast-iron pipes have been employed for the last 200 years and are very satisfactory for ordinary waterworks purposes where moderate heads are employed. They are very durable and require but little attention. While it is sometimes used under higher pressures, cast

iron is not considered desirable for heads above 400 ft. nor is it suitable for very large diameters on account of low tensile strength and possible defects in casting. For temporary purposes or for cheaper installations pipes are sometimes made

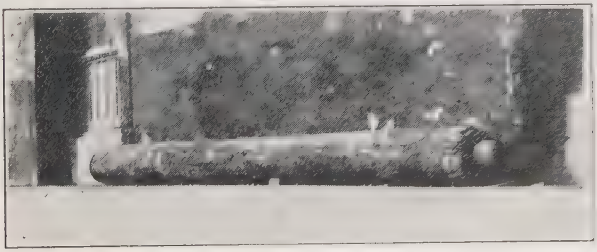


From a photograph by the author.

Fig. 124.—Riveted steel pipe under head of 1,300 ft. leading to Drum power house of Pacific Gas & Elec. Co. in California.

of very light weight riveted steel, usually coated with some material in order to enable them to resist corrosion.

For high pressures cast iron is unsuitable and steel pipe is used. These may be riveted as in Fig. 124, or they may be



From a photograph by the author.

Fig. 125.—Old wooden water pipe at New Orleans made from cypress log

welded in special cases. Riveted-steel pipe offers more resistance to flow than a new cast-iron pipe on account of the projecting rivet heads and the overlapping of the plates, but an

old riveted-steel pipe and an old cast-iron pipe are about the same since both become coated alike with tubercles. A steel pipe is not considered as durable as a cast-iron pipe, but for high heads it is necessary to use it.

For heads under 200 or 300 ft. wood-stave pipe offers many advantages. It is cheaper than a metal pipe for the same service. The resistance to flow is less than a riveted-steel pipe and about the same as a new, smooth, cast-iron pipe, but it has the advantage that its capacity does not decrease with age. The early

Courtesy of Redwood Manufacturers Co.

FIG. 126.—Construction of wood-stave pipe.

types of wooden pipe used were simply hollow logs as shown in Fig. 125. Some of these were used for many years. Modern wood pipe is generally built up of staves as shown in Fig. 126. The staves are so arranged that the joints are "broken." In order to make a watertight joint, a thin steel tongue is inserted in a saw cut across the end of each stave. This piece of steel is slightly wider than the stave so that when the bands are tightened up it will sink into the staves on either side a distance of about $\frac{1}{8}$ in. or more. In the life of a wood-stave pipe the encir-

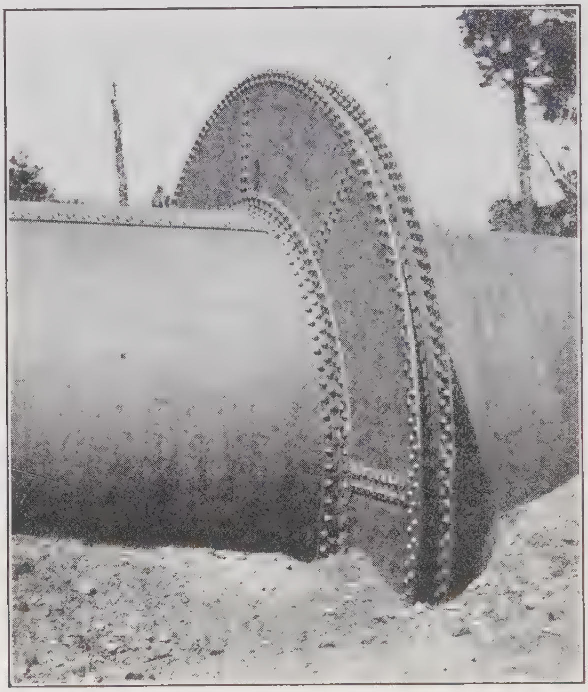
cling metal bands often have to be renewed. It is essential that a wooden pipe be kept filled with water, if it is to have a long life, as wood does not rot rapidly if it is kept continually wet or continually dry. It rots more quickly when it is exposed to alternation in these conditions. The life of a wood pipe is not as long as that of a heavy cast-iron pipe but it may be as long as that of a steel pipe. However, statistics of these matters are lacking

From a photograph by the author.

Fig. 127.—Curves in wood-stave pipe. In the Sierra Nevada Mts. of California.

and subject to much dispute. The wood pipe is free from corrosion and from electrolysis and is not attacked by acids in the water. Hence, it is often used for carrying liquids that could not be handled by a metal pipe. It is possible to introduce broad sweeping curves into a wood-stave pipe without any special devices or fittings, as shown in Fig. 127.

Metal pipes are subject to expansion and contraction due to temperature changes and provision must often be made for this. In the case of a cast-iron pipe line the amount of play afforded at each joint is usually sufficient. But a riveted-steel pipe line has no such flexibility and expansion devices may be employed. One type of expansion joint is shown in Fig. 128, which is suitable only for low pressures. It may be seen that the circular plates can spring enough to permit the necessary endwise motion



From a photograph by the author.

FIG. 128.—Expansion joint in 8.5 ft. riveted steel pipe under low head.

of the pipe. For higher pressures a joint such as in Figs. 129 and 130 may be used.

When water is lifted by a pipe line to a greater height than the initial water level, as in Fig. 131, the pipe is called a siphon. Of course it is necessary to exhaust the air by some means in order to start the flow, and if the flow is to continue the air which collects at the summit must be removed from time to time. There are times when such a device cannot be avoided.

From a photograph by the author.

Fig. 129.—Expansion joint in high pressure riveted steel pipe line.

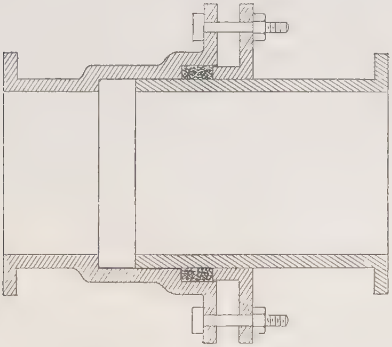


Fig. 130.—Expansion joint.

By analogy a pipe line such as shown in Fig. 132 is called an "inverted siphon," and it is usually found where it is necessary

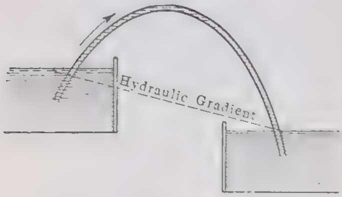


Fig. 131.—Siphon.

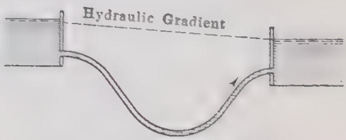


Fig. 132.—Inverted siphon.

From a photograph by the author.

Fig. 133.—Riveted steel siphon. Lake Spaulding development of Pacific Gas & Elec. Co.

to carry water across a valley or depression as in Fig. 133. It is, however, quite common to call a pipe so situated simply a "siphon."

108. PROBLEMS

122. A pipe line 850 ft. long discharges freely into the air under a fall of 40 ft. (Assume a projecting pipe at entrance.) (*a*) If $d'' = 6$ in., find the rate of discharge. (*b*) If $d'' = 12$ in., find the rate of discharge.

Ans. (*a*) $q = 1.55$ sec. ft. (*b*) $q = 8.85$ sec. ft.

123. Suppose a pipe line runs from one reservoir to another, both ends of the pipe being under water. Assume the intake end is non-projecting. If the difference in water levels is 110 ft., the length of pipe is 500 ft., and the diameter 10 in., what will be the rate of discharge? What will the capacity be when the pipe is old?

Ans. 11.95 cu. ft. per second, 8.68 cu. ft. per second.

124. A pump delivers water through 300 ft. of 4-in. fire hose to a nozzle which throws a 1-in. jet. The velocity coefficient of the nozzle is 0.98 and the value of f for the hose may be assumed to be 0.025. The nozzle is 20 ft. higher than the pump. It is required that the velocity of the jet be 70 ft. per second. What will be the necessary pressure at the pump?

Ans. 45.7 lb. per square inch.

125. The steel siphon shown in Fig. 133 is 8.5 ft. in diameter. It is 1,900 ft. long and carries 300 cu. ft. of water per second. What must be the difference in water level at the two ends? (It is arranged as in Fig. 132.)

Ans. 2.8 ft.

126. The pipe line shown in Figs. 129 and 206 has an average diameter of 62 in.; is 6,272 ft. long; and the difference in level between the power house and the intake is 1,375 ft. When the pipe delivers 300 cu. ft. of water per second, what is its efficiency? What is the horsepower delivered to the plant?

Ans. 93.8 per cent, 44,000 hp.

CHAPTER IX

UNIFORM FLOW IN OPEN CHANNELS

109. Open Channels.—An open channel is one in which the stream is not completely enclosed by solid boundaries and therefore has a free surface subjected only to atmospheric pressure. The flow in such a channel is not dependent upon some external head but rather upon the slope of the channel and of the water surface.

From a photograph by the author.

FIG. 134.—Canal of the Pac. Gas & Elec. Co. with one bank rock lined.

The principal types of open channels are: natural streams or rivers, artificial canals, and sewers, tunnels, or pipe lines not completely filled.

The accurate solution of problems of flow in open channels is much more difficult than in the case of pressure pipes. Not only is reliable experimental data more difficult to secure, but there is a wider range of conditions than is met with in the case of pipes. Practically all pipes are round, but the cross-sections of open channels may be of any shapes from circular to the irregular

forms of natural streams. It is probable that the shape of the cross-section affects the flow in a way that is not covered by the factor, m, the hydraulic mean depth (see Art. 80). In

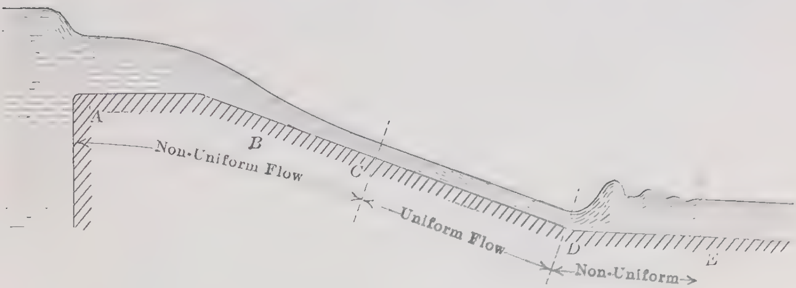
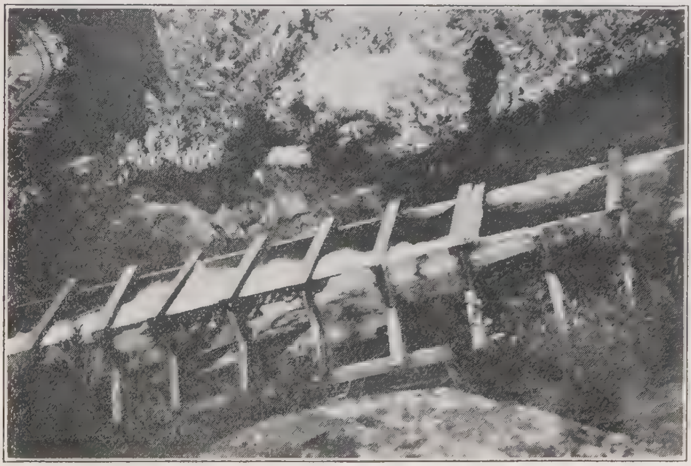


FIG. 135.

pipes the degree of roughness ordinarily ranges from that of new, smooth, cast-iron or wood-stave pipes, on the one hand, to that



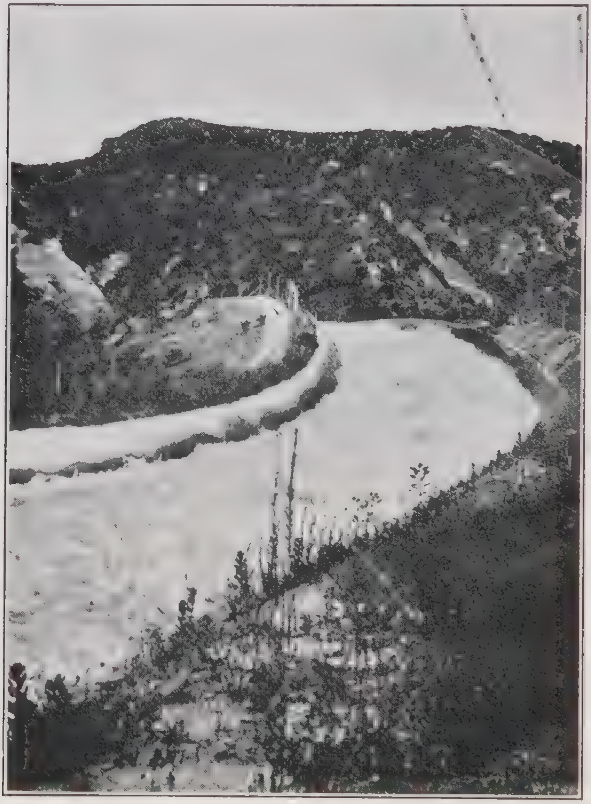
From a photograph by the author.

FIG. 136.—Non-uniform flow in wooden flume.

of old, corroded pipes, on the other. But with open channels the surfaces vary from smooth timber (Fig. 136) to the rough and irregular beds of some rivers. Hence the choice of friction fac-

tors is attended with greater uncertainty in the case of open channels than in the case of pipes.

110. Uniform Flow.—If the shape and size of any water cross-section is identical with that of every other section in the length of channel under consideration, the flow is said to be *uniform*.



From a photograph by the author.

Fig. 137.—Cascade on Los Angeles aqueduct.

Such cases are shown in Figs. 52 and 134. Uniform flow must not be confused with *steady* flow. The former requires that the conditions at any time be the same from place to place; the latter requires that the conditions at every section be constant with respect to time. We might have steady flow for both uni-

form and non-uniform flow as shown in Fig. 135. Uniform flow is obtained only when a channel is uniform for a considerable distance so that the water has a chance to adjust itself. The channel in Fig. 136 is uniform but the flow is non-uniform in the portion shown because the water has just entered it and has not yet attained a condition of equilibrium. The conditions are similar to the lower portion of the channel shown in Fig. 135. On the other hand, the flow is non-uniform in Fig. 137 because the slope of the channel varies.

111. Hydraulic Gradient.—It is quite evident that in the case of an open channel the hydraulic gradient coincides with the water surface. For if a piezometer tube be attached to the side of the channel the water will rise in it until its surface is level with that of the water in the channel.

112. Equation for Uniform Flow.—The equation that is most generally used for steady uniform flow in open channels is one that is also often used for flow in long pipes. It is

$$V = C\sqrt{ms}$$

the derivation of which was given in Art. 82. In this formula, C is a coefficient dependent upon the roughness of the surface in contact with the water, and it is also often given as a function of other variables as well. In the case of an open channel with uniform flow, s is the slope of the water surface.

Recognizing that the velocity does not vary exactly as the square root of m or of s, exponential formulas such as Eq. (71) are sometimes used. But if Eq. (74) is employed it is seen that C must then be a function of m and s, since Eq. (74) does not involve the correct exponents of m and s.

113. Kutter's Formula for C.—The formula for C that has probably been more widely used than any other is that of Kutter and Ganguillet, two Swiss engineers. This formula is based upon a wealth of data from small artificial canals up to natural streams as large as the Mississippi, and for this reason it is believed to be applicable to a wide range of conditions. But any formula that attempts to cover too large a field must necessarily be a mere average of a number of scattered values and, though giving approximate values at least for any combination of factors, it cannot be expected to give exact values in individual cases. Hence too great reliance must not be placed upon values given by the use of this or any other such empirical formula.

In Art. 112, it was pointed out that, since Eq. (74) is not a true expression of the law of flow, the value of C must be a function of m and s as well as the roughness of the surface. The formula of Kutter takes all three factors into consideration. It is

$$C = \frac{41.65 + 0.00281/s + 1.811/N}{1 + (41.65 + 0.00281/s)N/\sqrt{m}}. \tag{103}$$

In this expression the factor N is a coefficient of roughness, values of which are given in Table VI.

TABLE VI.—VALUES OF N IN KUTTER'S AND MANNING'S FORMULAS

Nature of surface	N
Planed and smoothly laid timber	0.009
Planed timber, not perfectly true	0.010
Wood-stave pipes	0.011
Smooth cement	0.011
Smooth iron pipes	0.011
Rough timber, good brickwork	0.013
Slightly rough iron pipes	0.015
Rough brickwork, cut stones	0.015
Good rubble masonry	0.017
Tuberculated iron pipes	0.017
Rough brick and stonework	0.017
Smooth earth channels	0.017
Coarse gravel, well packed	0.020
Large earth channels, good condition	0.022
Small earth channels, good condition	0.025
Channels in fair condition	0.030
Channels in bad order, with weeds, etc	0.035
Channels encumbered with drift	0.045

In order to save tedious computation when Eq. (103) is used, various sets of tables have been published and also a number of graphical solutions have been devised.[1] In Table VII will be found values of C determined by Eq. (103). Intermediate values may be found by interpolation with as much accuracy as the conditions warrant.

[1] One of the simplest of these is the diagram published by KARL R. KENNISON.

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TABLE VII.—VALUES OF C COMPUTED FROM KUTTER'S FORMULA

Slope s	N	Hydraulic mean depth, m, in feet												
		0.2	0.4	0.6	0.8	1	1.5	2	3	4	6	8	10	15
0.00005	0.009	100	124	139	150	158	173	184	198	207	220	228	234	244
	0.010	87	109	122	133	140	154	164	178	187	199	206	212	220
	0.011	77	97	109	119	126	139	148	161	170	182	189	195	205
	0.012	68	88	98	107	114	126	135	148	156	168	175	181	189
	0.013	62	79	90	98	104	116	124	136	145	156	163	169	179
	0.015	51	66	76	83	89	99	107	118	126	137	144	149	158
	0.017	44	57	65	71	77	87	94	104	111	122	129	134	142
	0.020	35	46	53	59	64	72	79	88	95	105	111	116	125
	0.025	26	35	41	46	49	57	62	71	77	85	91	96	104
	0.030	21	28	33	37	40	47	51	59	64	72	78	82	90
	0.035	18	24	28	31	34	40	44	50	56	63	68	72	79
0.0001	0.009	112	136	149	158	166	178	187	198	206	215	221	226	233
	0.010	98	119	131	140	147	159	168	178	186	195	201	205	212
	0.011	86	106	118	126	132	144	151	162	169	178	184	188	195
	0.012	76	95	105	114	120	130	138	149	155	164	170	174	181
	0.013	69	86	96	103	109	120	127	137	143	152	158	162	169
	0.015	57	72	81	88	93	103	109	119	125	134	139	143	150
	0.017	48	62	70	76	81	89	96	104	111	119	124	128	135
	0.020	39	50	57	63	67	75	81	89	94	102	107	111	118
	0.025	29	38	44	48	52	59	64	71	76	84	88	92	98
	0.030	23	31	35	39	42	48	53	59	64	71	75	78	85
	0.035	19	25	30	33	35	41	45	51	55	61	66	69	75
0.0002	0.009	121	143	155	164	170	181	188	200	205	213	218	222	228
	0.010	105	125	138	145	151	162	170	179	185	193	198	201	207
	0.011	93	112	122	131	136	146	154	163	168	176	182	185	190
	0.012	83	100	111	118	123	133	140	149	155	162	167	170	176
	0.013	74	91	100	107	113	122	129	137	143	150	155	158	164
	0.015	61	76	85	91	96	105	111	119	125	132	137	140	145
	0.017	52	65	73	79	83	91	97	105	111	117	122	125	131
	0.020	42	53	60	65	69	77	82	89	94	100	105	108	113
	0.025	31	40	46	50	54	60	64	72	76	82	87	89	95
	0.030	25	32	37	41	44	49	54	59	63	69	73	76	82
	0.035	21	27	31	34	37	42	45	51	55	60	64	67	72
0.0004	0.009	126	147	157	166	172	183	190	199	204	211	215	219	224
	0.010	110	129	140	148	154	164	170	179	184	191	196	199	203
	0.011	97	115	126	133	138	148	154	162	168	175	179	183	187
	0.012	87	104	113	121	125	135	141	149	154	161	165	168	172
	0.013	78	94	103	110	115	124	130	138	142	149	153	157	162
	0.015	65	79	87	93	98	106	112	119	124	130	135	138	143
	0.017	54	68	75	81	85	93	98	105	110	116	120	123	128
	0.020	44	55	62	67	70	78	83	89	94	99	104	107	110
	0.025	32	42	47	51	55	61	65	71	76	81	85	88	92
	0.030	25	33	38	42	45	50	54	59	63	69	73	75	80
	0.035	21	27	31	35	37	42	45	51	55	60	64	66	70
0.0010	0.009	129	150	161	169	175	184	191	199	204	211	214	218	222
	0.010	113	131	142	150	155	165	171	179	184	190	194	197	202
	0.011	99	117	127	134	139	149	155	163	168	174	178	181	186
	0.012	89	105	115	122	127	136	142	149	154	160	164	167	171
	0.013	81	96	104	111	116	124	134	138	142	149	152	155	160
	0.015	66	80	88	94	99	108	112	119	124	130	134	136	141
	0.017	57	69	76	82	86	93	98	105	110	116	120	122	127
	0.020	45	56	63	68	71	78	83	89	93	99	103	105	110
	0.025	34	43	48	52	56	62	66	71	75	81	85	87	91
	0.030	27	34	39	42	45	50	54	59	63	68	72	74	78
	0.035	22	28	32	35	38	43	46	51	54	59	63	65	68
0.0100	0.009	130	151	162	170	175	185	191	199	204	210	213	217	222
	0.010	114	133	143	151	156	165	171	179	184	190	193	196	200
	0.011	100	119	129	135	141	149	155	162	167	173	176	180	184
	0.012	90	107	116	123	128	136	142	149	154	160	163	166	170
	0.013	81	98	106	112	117	125	130	138	142	148	151	154	159
	0.015	66	80	89	95	100	108	112	119	124	129	133	134	138
	0.017	57	70	77	82	87	94	99	105	109	115	118	121	126
	0.020	46	57	64	68	72	79	83	89	93	99	102	105	108
	0.025	34	44	49	53	56	62	66	71	76	81	84	86	90
	0.030	27	35	39	43	45	51	55	59	63	68	71	74	77
	0.035	22	29	33	35	38	43	46	51	55	59	62	65	68

Although C is a function of the slope, it will be found that its variation with values of s is not great. The difference between values of C for $s = 0.0001$ and $s = 0.0010$ is a matter of from 10 to 15 per cent at the very most. The equation also shows that as s increases in value its influence upon the value of C decreases. For all values from $s = 0.0010$ up to $s = 0.100$, or even greater, the change in the value of C is negligible (see Fig. 138, which is constructed for several values of m when $N = 0.017$). Similar results would be obtained with any other value of N.

From the range of experiments upon which it is based, Kutter's formula would appear to be applicable for values of m up to 10 ft., for velocities up to 10 ft. per second, and for slopes greater than $s = 0.0001$. Outside of these limits reliable data are lacking and Kutter's formula should be used with caution.

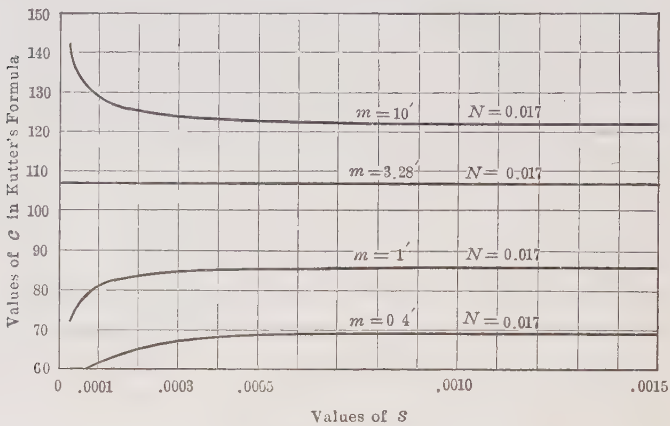


Fig. 138.—Relation between C and s.

114. Manning's Formula for C.—Inspection of Fig. 138 shows that the value of C as given by Kutter's formula is affected by the slope to a very small extent. Thus but very little error would be involved in disregarding it altogether. As a matter of fact there are reasons for believing that the term involving the slope was based upon incorrect data and that it is therefore a source of error in the equation. This is borne out by the fact that in Fig. 138 the value of C is shown as increasing with s in some cases and decreasing in others, while certain theoretical considerations would indicate that it must increase with values of s in the same way that true values of f decrease with increasing

values of V. Thus equating values of V given by Eqs. (71) and (74),

$$C = C'm^{\frac{x}{n} - \frac{1}{2}} s^{\frac{1}{n} - \frac{1}{2}}. \tag{104}$$

With the same values of x and n as used in the Hazen-Williams formula, and considering that C' varies only with the surface, this gives $C = C'm^{0.13}s^{0.04}$.

But for exactly the same surface, the relative roughness of large and small open channels will probably vary through a wider range even than in the case of circular pipes, and thus the effect of size will predominate to a greater extent than the above exponents would indicate. Hence Manning disregards the effect of s altogether and makes C a function of m only.

The formula of Manning gives practically the same values as does that of Kutter and is probably as accurate as our present information warrants. Manning's formula is

$$C = \frac{1.49}{N} m^{1/6} \tag{105}$$

in which N is the same as in Kutter's formula. Values of N are given in Table VI.

Eq. (105) gives a better idea of the way C varies with N and m than can be obtained from an inspection of Eq. (103). Since these two equations give values of C which are approximately equal to each other, it follows that in Kutter's formula C varies approximately inversely as N and directly as $m^{0.17}$.

For practical use it is better to compute V directly, rather than to determine C separately by Manning's formula. Substituting the value of C given by Eq. (105) in Eq. (74),

$$V = \frac{1.49}{N} \sqrt[3]{m^2} \sqrt{s}. \tag{106}$$

Values of $m^{2/3}$ may be found on page 321.

115. Construction of Open Channels.—Inspection of the expression $V = C\sqrt{ms}$ shows that, for a given slope and degree of roughness, the velocity increases as m increases. This is also accentuated by the fact that the value of C also increases as m increases or, as shown by Eq. (106), V varies as $m^{2/3}$. Therefore for a given area of water cross-section the rate of discharge will be a maximum when m is a maximum. Or for a given rate of discharge the cross-section area will be a minimum when the design is such as to make m a maximum.

From Eq. (68) it may be seen that the value of m will be a maximum for a given area when the length of the wetted perimeter is a minimum. Now of all geometric figures, the circle has the least perimeter for a given area. Hence a semicircular open

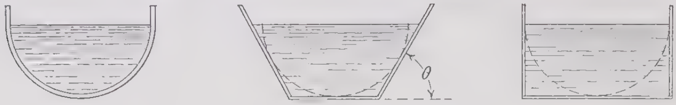
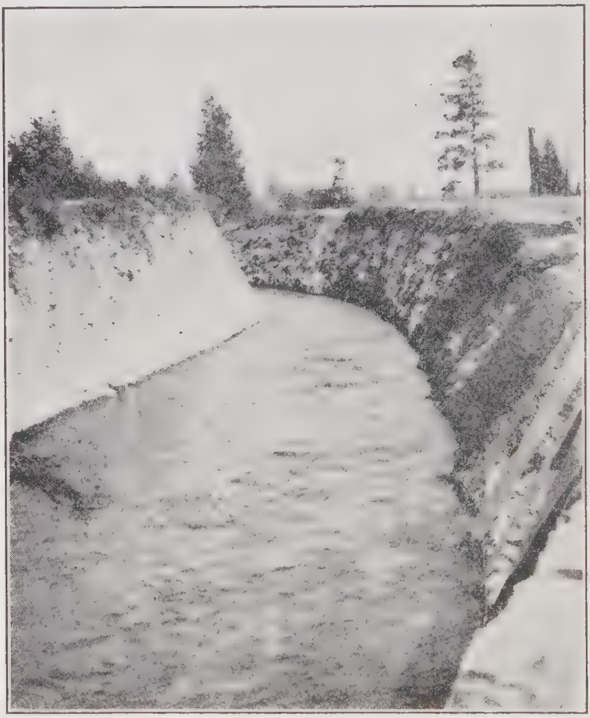


Fig. 139.

channel will discharge more water than one of any other shape, assuming that the area, slope, and roughness of surface are the same. Semicircular open channels are often built of pressed



From a photograph by the author.

Fig. 140.—Unlined canal with steep banks. In the Sierra Nevada Mts.

steel and other forms of metal, but for other types of construction such a shape is impractical.

For wooden flumes the rectangular shape is usually used. Of all rectangles the square has the least perimeter in proportion to

area and hence for an open channel the depth of the water should be half the width.

Canals excavated in earth must have a trapezoidal section, and of all trapezoids the half hexagon will have the largest value of m. But the angle θ cannot always be made equal to 60 deg. for other reasons. The slope of the sides must be such that the angle θ is less than the "angle of repose" of the material of which the banks are composed, otherwise the latter will cave in. In Fig. 140 the angle θ is made much greater than 60 deg. in order to save a considerable amount of excavation in a deep cut, the firm character of the soil permitting such steep sides.

Whatever the value of the angle, it will be found that the best proportions will be obtained when the sides are tangent to a semicircle whose center lies in the water surface.

But other forms of cross-section are often used either because they have certain advantages in construction or are desirable from other standpoints. Thus oval- or egg-shaped sections are common for sewers and similar channels where there may be large fluctuations in the rate of discharge. It is desirable that the velocity, when a small quantity is flowing, be kept high enough to prevent the deposit of sediment, and when the conduit is full the velocity should not be too high on account of wearing the lining of the channel.

116. Stream Gaging.—The determination of the rate of discharge of a stream for any given depth of water is termed *stream gaging*. It may be seen that the rate of discharge of a stream could be computed from the formula, $V = C\sqrt{ms}$, if the flow is uniform and the cross-section area, the hydraulic mean depth, and the slope of the water surface are known. But a more accurate determination of discharge can be made by measuring the velocity directly, by passing the water over a weir, or by other means.

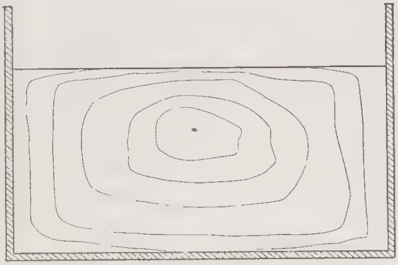


FIG. 141.

In a straight portion of an artificial channel the velocity might vary as shown in Fig. 141. These curves are velocity contours or curves of equal velocity. Within the area enclosed by the

curve the velocity is higher than that of a point on the curve. Outside the enclosed area the velocity is less than that on the curve. It may be seen that the velocity of the water varies from side to side and from top to bottom. If there is a bend in the channel, or if the bed is irregular, as in natural streams, these velocity curves are often very irregular and distorted from the forms shown here. It is, therefore, necessary to determine the velocity at a number of different points across the section of the stream.

The instrument that is commonly used for this purpose is the current meter, described in Art. 76. In using the current meter or any other device it is customary to divide the stream up into sections as in Fig. 142 and to determine the contour of the bed, so that the area may be computed. If, then, the average velocity

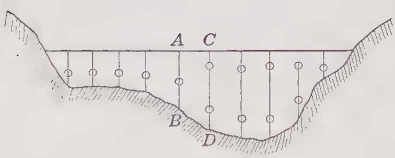


Fig. 142.

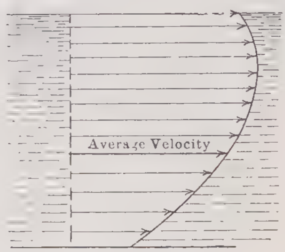


Fig. 143.

is determined for some section such as $ABCD$ the discharge through this section will be the product of this velocity and the area $ABCD$. The sum of all such partial discharges gives the total rate of discharge of the entire stream.

In finding the average velocity in the area $ABCD$ it is customary to take it as the average of the velocity measured in the line AB and the velocity measured in the line CD. But, as shown in Fig. 143, the velocity varies from A to B or from C to D, and hence the average velocity in each vertical line should be determined. This might be done by taking a number of observations so that curves similar to that in Fig. 143 could be plotted. But a study of a number of such curves has shown that in general the average velocity in a vertical line is found at about 0.6 the depth. Hence if the current meter be set at that depth, the velocity determined by it may be assumed to be the mean velocity. Of course this is only an approximation. To

insure a higher degree of accuracy than a single observation could give, measurements are often taken at 0.2 the depth and 0.8 the depth. The mean of these two values will be approximately the average velocity. Thus, in an actual stream gaging, observations would be made at the points indicated by the circles in Fig. 142. Further details of this topic are not within the scope of this text.[1]

Sometimes floats are used but such procedure is less accurate. They are, however, often applicable when other methods are not feasible, such as during floods. If surface floats are used, the average velocity may ordinarily be taken as about 0.9 that of the surface velocity. But the velocity at the surface is greatly affected by the wind.

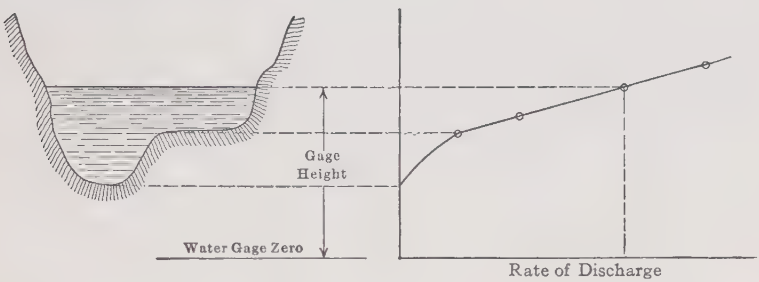


FIG. 144.—Rating curve.

117. Rating Curve.—If a natural stream is to be used for water supply or power purposes, it is necessary to determine the amount of water it can be depended upon to furnish. Since the flow will usually be subject to wide fluctuations during a long period of time it is necessary to make an extended series of observations upon it.

The level of the surface of the water in a stream is called the gage-height, and may be measured above any arbitrary point. Thus the gage-height does not necessarily coincide with the depth of the stream.

It is apparent that for a given stream, the rate of discharge will be a function of the gage-height. If the rate of discharge of the stream be determined for several gage-heights a curve, such as in Fig. 144, may be constructed. This curve is called the *rating curve*, and from it the value of q for any height of water can be obtained.

[1] See HOYT and GROVER, "River Discharge."

Thus in making a study of the stream it is necessary to make only a record of the gage-heights. From the rating curve the quantity of flow can then be determined. This gage-height might simply be read and recorded once a day by an observer, or by means of a float and clockwork a continuous record could be obtained which would show all the variations in the flow.

118. PROBLEMS

127. A rectangular flume of timber slopes 1 ft. per 1,000 ft. Compute the rate of discharge if the width is 6 ft. and the depth of water 3 ft. What would be the rate of discharge if the width were 3 ft. and the depth of water 6 ft.? Which of the two forms would require less lumber?

Ans. 114 sec. ft.

128. A rectangular channel of rubble masonry is 6 ft. wide, the depth of water is 3 ft., and the slope of 1 ft. per 1,000 ft. Compute the rate of discharge and compare with that in problem 127.

Ans. 65 sec. ft.

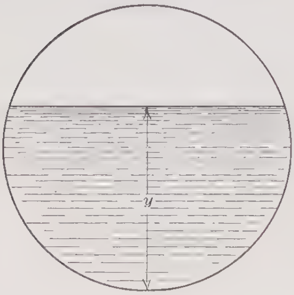


FIG. 145.

129. A semicircular channel of rubble masonry with a slope of 1 ft. per 1,000 ft. will give what discharge when flowing full if its diameter is 6.55 ft.? Compare the cross-section areas and amounts of lining required with that in problem 128.

Ans. 65 sec. ft.

130. A circular conduit of concrete (N = 0.012) is 10 ft. in diameter and slopes 1.6 ft. per 1,000 ft. (see Fig. 145). The following table gives values of wetted perimeter and area of water cross-section for various depths of water in the conduit. Find values of V and q for the various depths in the table. What value of y gives the highest velocity? What value of y gives the highest rate of discharge?

Depth, y	Wetted perimeter	Area, A	m	$\sqrt{m}$	C	V	q
1.0	6.44	4.09	0.635	0.797	115	3.67	15.0
3.0	11.59	19.82					
5.0	15.71	39.27					
8.0	22.14	67.36					
9.0	24.98	74.45					
9.5	26.91	77.07					
10.0	31.42	78.54					

CHAPTER X

NON-UNIFORM FLOW IN OPEN CHANNELS

119. Non-uniform Flow in Open Channels.—As a rule uniform flow is found only in artificial channels of regular shape and slope, although even under these conditions the flow for some distance may be non-uniform as shown in Fig. 135. But with a natural stream the slope of the bed and the form and size of the cross-section usually vary to such an extent that true uniform flow is rare. Hence the application of the equations given in Chap. IX to natural streams can be expected to yield results which are only rough approximations to the truth. In order to apply these equations at all it is necessary to divide the stream up into lengths within which the conditions are approximately the same. A satisfactory and reliable treatment of the problem of non-uniform flow in open channels is lacking.

In the case of pressure conduits, uniform and non-uniform flow have been dealt with without drawing any distinction between them. This is possible because in a closed pipe the area of the water cross-section, and hence the velocity, is fixed at every point. But in an open channel these conditions are unknown and the stream adjusts itself to the size of cross-section that the slope of the hydraulic gradient requires.

In either artificial or natural streams non-uniform flow may be produced in a variety of ways, each one of which leads to a different hydraulic phenomenon. When the slope of a channel is increased, as in Fig. 135, the water is accelerated in the portion *ABC* until the rate of frictional losses, which increase as the square of the velocity, are equal to the rate of decrease of potential energy, at which place equilibrium is attained and then the flow is uniform. When the slope is suddenly decreased a standing wave may be had, such as at *D* in Fig. 135. If the slope *DE* were steeper, however, the wave would not be so high as that shown in this figure and the surface curve would be flatter, as in Fig. 136.

When water flowing down a steep channel with a high velocity strikes an obstruction, there will be an abrupt increase in its depth, which is known as the "hydraulic jump" and is similar in appearance to the standing wave. Like the standing wave, the effect of this is confined to the immediate locality. If, on the other hand, the channel is relatively flat, there is the backwater curve which extends upstream for an infinite distance.

At entrance to an open channel from a body of water having a very low velocity, there will be an abrupt drop of level, as shown in Fig. 135, which is equal to the velocity head acquired plus the entrance losses. If the bed of the channel at this place is quite flat, the surface of the water will be concave downwards as in the portion AB. But if the slope is quite steep, the surface curve immediately beyond the entrance will be concave upwards like the portion BC. For one particular intermediate slope the surface curve will be a straight line parallel to the bed and thus the flow will be uniform immediately after the entrance drop. Unless the channel is very steep the departure from uniform condition is difficult to detect, since practically the curvature of the water surface is very slight.

A theoretical discussion of the various types of non-uniform flow is beyond the scope of this text, but a little space will be devoted to two types that are of the most practical importance.

120. General Equation for Non-uniform Flow.—Let Fig. 146 represent a case of non-uniform flow. In general the surface slope will not be a straight line and often the bed of the stream will not be either. If, however, the length l is not too great, they may be considered as straight lines with slopes equal to the average values. The water surface is the hydraulic gradient, while the energy gradient is the line passing through points which are located above the surface at distances corresponding to the velocity head at each section. Since the velocity is not uniform the slope of the hydraulic gradient, s', will be different from the slope of the energy gradient, s. For the case shown, s' is slightly less than s. If, on the other hand, the depth of the stream were diminishing downstream, the velocity would be increasing and the value of s' would be slightly more than s.

Since this is an open channel, the pressure is the same at all sections, and thus the general equation of energy becomes

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} + H'.$$

Noting from the figure that $z_1 - z_2 = il$, and since $H' = sl$ by definition, this may be written as

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + (s - i)l. \qquad (107)$$

This equation is equally true for the case as represented in the figure or for the case where the depth y diminishes on traveling

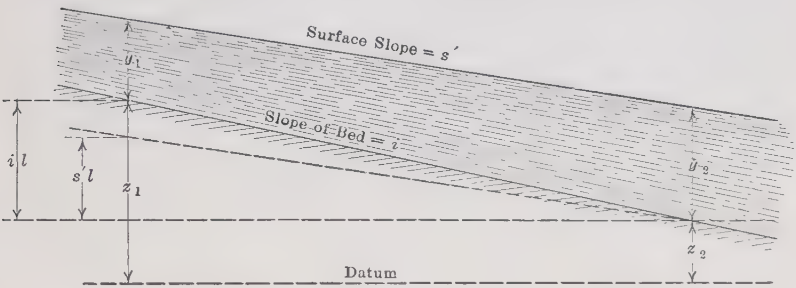


FIG. 146.

downstream. In the former case it will be found that s is less than i and in the latter case greater.

121. The Drop-down Curve.—As water flowing in a channel with a very flat slope approaches a free outfall, the surface curve may drop below the level for normal uniform flow. This is

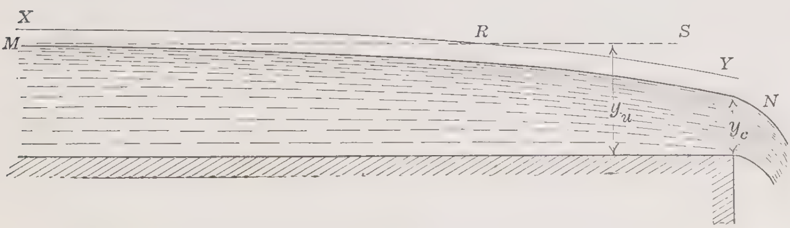


FIG. 147.—The drop-down curve.

called the drop-down curve and is shown by MN in Fig. 147, MS being the hydraulic gradient for uniform flow for the same rate of discharge. The drop-down curve approaches the line MS as an asymptote and the point M is common to the two

only at an infinite distance upstream. Practically, because of wave action and other irregularities, the distinction between the two curves disappears within a finite distance.

Proceeding downstream from M, the depth of water continually diminishes and the velocity increases, but, in accordance with the energy equation, the total energy must continually decrease. It may be shown that at the section of free outfall the depth y_c is such that $y_c + V^2/2g$ is a minimum. If A is the cross-section area at this point, this expression may be written $y_c + q^2/2gA^2$, and the value of y_c which makes this a minimum is best found by trial in general. It may be noted that this value of y_c for a given rate of discharge is independent of the slope or roughness of the channel. This, then, will be the value of the depth at this point for a flat slope provided the discharge takes place into the air or into a body of water whose surface is at a lower level. If the surface of the water into which discharge occurs is higher than this minimum value, then the depth of water at the end of the channel must rise until the two coincide.

It is impossible to have uniform flow in a channel whose bed is horizontal, and it is in this case that the drop-down curve is the most noticeable. With increasing values of the slope (or increasing smoothness of the channel) higher values of the velocity for uniform flow are found, and consequently lower values of y_u, until finally the slope and roughness of the channel are such that the depth for uniform flow is equal to or less than the computed y_c, and then uniform flow exists right up to the outlet. Thus the drop-down curve is found only where the conditions of slope and roughness are such that the depth for uniform flow is greater than this computed value of y_c, and the effect is greater the flatter the slope.

If the channel shown in Fig. 147 is very long and the depth must not exceed the value of y_u there indicated, it is seen that its capacity is determined by the depth required for uniform flow though its hydraulic gradient is MN. But if the length is only RS, it may be seen that it can carry a larger quantity of water corresponding to a hydraulic gradient XY. Thus short channels with very flat slopes have much higher capacities, because of the drop-down curve, than would be the case if the flow were uniform.

EXAMPLE

131. A portion of the Los Angeles outfall sewer is approximately a circular conduit 5 ft. in diameter and with a slope of 1 ft. in 1,100 ft. It is of brick for which $N = 0.013$. What would be its maximum capacity for uniform flow? If it discharges 120 sec. ft. with a free outfall, how far back from the end must it become a pressure conduit unless the size or the slope is changed?

The solution of example 130 shows that the discharge is a maximum when the depth is 0.95 times the diameter or 4.75 ft. in this case. The value of A is then 19.30 sq. ft. and m is 1.435 ft. For uniform flow the velocity may be found to be 4.37 ft. per second and the discharge 84.3 sec. ft.

For a discharge of 120 sec. ft. it may be found by trial that the depth at the mouth of the sewer is 3.15 ft. Proceeding upstream the depth increases until it becomes 4.75 ft. The problem is to find the distance to this section. This may be solved by a step-by-step method with Eq. (107). The procedure is to assume a depth of water and then compute the distance l to that section from some other section previously determined. Such a solution is greatly facilitated by a tabulation, such as here presented. Fill in the table and complete the solution.

Ans. 718.5 ft.

y	A	m	C	V	$V^2/2g$	$s = \frac{V^2}{C^2m}$	s average	$\Delta\left(y + \frac{V^2}{2g}\right)$	$s - i$	l	Σl
3.15	13.1	1.42	121	9.20	1.31	0.00406	....	000	....	0	0
3.50	14.7	1.48	122	8.17	1.04	0.00302	0.00354	0.080	0.00263	30.4	30.4
3.75	15.8	1.51	122	7.59	0.895	0.00257	0.00280	0.105	0.00189	55.5	85.9
4.00	16.9	1.53	123	7.13	0.790	0.00221	0.00239	0.145	0.00148	98.0	183.9
4.25	17.8	1.52	122	6.75	0.709						
4.50	18.6	1.49	122	6.45	0.645						
4.75	19.3	1.44	121	6.22	0.600						

122. The Back-water Curve.—When a dam or other obstruction is placed across a flowing stream, the water level is raised at the particular section and, if the slope of the channel is not too great, the effect will extend upstream for a considerable distance, producing what is known as the back-water curve.[1] The new water surface approaches the original water surface as an asymptote, coinciding with it only at an infinite distance. It is of considerable importance to know how far upstream the effect of the back-water is appreciable, or at a given location, how high the water will be raised.

[1] It may be proved, though it is not shown in this text, that the value of i must be less than g/C^2, in the case of a stream whose width is great compared with its depth, in order that the back-water curve may be obtained.

For an artificial channel where conditions are uniform, save for the variation in the depth of the water, this problem may be solved by the use of Eq. (107) in a manner identical with that shown in example 131. For a natural stream, such as that in Fig. 148, with varying slopes of bed and different cross-sections along its length, the solution is not so direct, because the form and dimensions of a cross-section of the stream cannot be assumed, and then the distance to its location computed. Since at different distances upstream there exist various slopes and

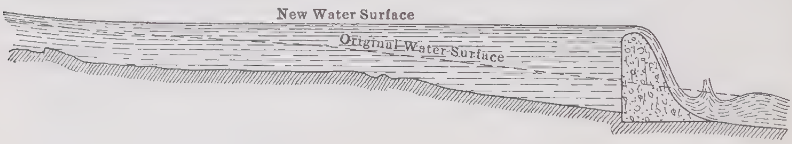


Fig. 148.—The back-water curve.

types of cross-sections, the value of l in Eq. (107) must be assumed, and then the depth of stream at this section must be computed by trial.

In view of the fact that for an irregular natural stream so many approximations must be made as to the actual conditions, the refinements of Eq. (107) are hardly justified, and it is fully as satisfactory to apply the simple equation for uniform flow, $V = C\sqrt{ms}$. In order to do this the stream must be divided up into various lengths within which the flow may be assumed to be fairly uniform. Then for each length average values of m and of V are used, and the value of s is determined by trial.

EXAMPLE

132. When the flow in a certain natural stream is 7,600 sec. ft., it is required to find the elevation of the water surface at different sections upstream from a certain initial point. Computations could be made downstream just as well, but it is customary to start at the dam, since the conditions there are usually assumed to be known. A survey of the channel shows that conditions are fairly similar for a length of 1,500 ft. upstream from the initial point, and then beyond that is another stretch of 2,200 ft., and so on. Assuming a rise in the water surface in the distance of 1,500 ft. to be 0.20 ft., a study of the stream bed shows the average value of the area to be 3,200 sq. ft. and the average perimeter to be 349 ft. This gives an average velocity of 2.45 ft. per second and an average m of 8.87 ft. The computed loss of head is then $sl = lV^2/C^2m = 0.283$ ft., which is greater

than that assumed. Hence assume a larger value and repeat. Complete the following table and find the rise in elevation in the first 1,500 ft.
Ans. 0.265 ft.

Assumed rise sl	A average	Perimeter average	m average	V average	Computed $sl = lV^2/C^2m$
0.20	3,100	350	8.86	2.45	0.283
0.25	3,180	359	8.86	2.39	0.269
0.26	3,190	360	8.86		
0.265	3,200	361	8.86		
0.27	3,220	363	8.86		
0.28	3,230	364	8.86		

In a similar manner the rises in other lengths may be computed and the sum of all of them up to the desired point will give the elevation at that point above the initial.

CHAPTER XI

HYDRODYNAMICS

123. Dynamic Force Exerted by a Stream.—Whenever the velocity of a stream of water is changed either in direction or in magnitude, a force is required. By the law of action and reaction an equal and opposite force is exerted by the water upon the body which produces this change. This is called a dynamic force in order to distinguish it from forces due to the hydrostatic pressure of the water.

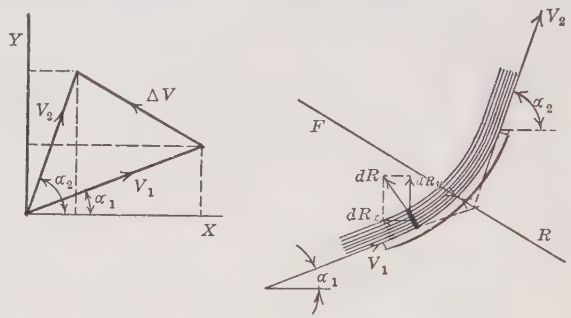


FIG. 149.

First Method.—Let the resultant force exerted by any body upon the water be denoted by R and its components by R_x and R_y. Let dR be the force exerted upon the elementary mass shown in Fig. 149 whose cross-section area is A and whose length along the path is ds. The weight of this elementary volume is $wAds$ and its mass is $wAds/g$. In general the velocity of a particle of water may vary as a function of both space and time. That is, at a given point the velocity may vary with the time, and it may also change from point to point along its path. Thus in general the acceleration is

$$a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

where V is a scalar and $\overline{V}$ a vector quantity. For steady flow, to which the discussion will be confined, $\partial \overline{V}/\partial t = 0$, and thus

$$dR = \frac{wAds}{g} V \frac{\partial \overline{V}}{\partial s} = \frac{wAV}{g} \frac{\partial \overline{V} ds}{\partial s}.$$

But $d\overline{V} = (\partial \overline{V}/\partial s)ds$, in this instance, and $wAV = W$, and, therefore,

$$dR = \frac{W}{g} d\overline{V}.$$

The summation of all such elementary forces along the path at any instant will give the total force exerted. In general these various elementary forces will not be parallel, and in order that the integration may be an algebraic and not a vector summation, components along the axes will be taken. Thus,

$$R_x = \frac{W}{g} \int_1^2 dV_x = \frac{W}{g} V_x \Big]_1^2.$$

Now at point (1) the value of V_x is $V_1 \cos \alpha_1$ and at (2) it is $V_2 \cos \alpha_2$. Inserting these limits and noting from Fig. 147 that $V_2 \cos \alpha_2 - V_1 \cos \alpha_1 = \Delta V_x$ the result is,

$$R_x = \frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g} \Delta V_x.$$

Second Method.—The preceding derivation pictures the total dynamic force exerted by a flowing stream to be the vector sum of all the elementary forces exerted along its path at any instant. The following derivation makes it clear that the total force depends solely upon the initial and terminal conditions and is independent of the path. (Of course the numerical value of the terminal velocity would be affected by friction losses which might be different for different paths.) The former method is based on the principle that resultant force equals mass times acceleration. The second method is based on the principle of force and momentum, which may be stated as follows: The time rate of change of the momentum of any system of particles is equal to the resultant of all external forces acting on the system. Thus instead of $R = mdV/dt$, it is $R = d(mV)/dt$.

Consider the portion of a filament of a stream in Fig. 150 which is between two cross-sections M and N at the beginning of a time interval dt, and between the cross-sections M' and N' at the end of the interval. Denote by ds_1 and ds_2 the distances moved during the interval by particles at M and N at the begin-

ning. Let A_1 be the cross-section area at M, V_1 the velocity of the particles, and α_1 the angle between the direction of V_1 and any convenient x axis. Let the same letters with subscript (2) apply to N.

At the beginning of the interval the momentum of the portion of the filament under consideration is the sum of the momentum of the part between M and M' and that of the part between M' and N. At the end of the interval its momentum is the sum of the momentum of the part between M' and N and that of the

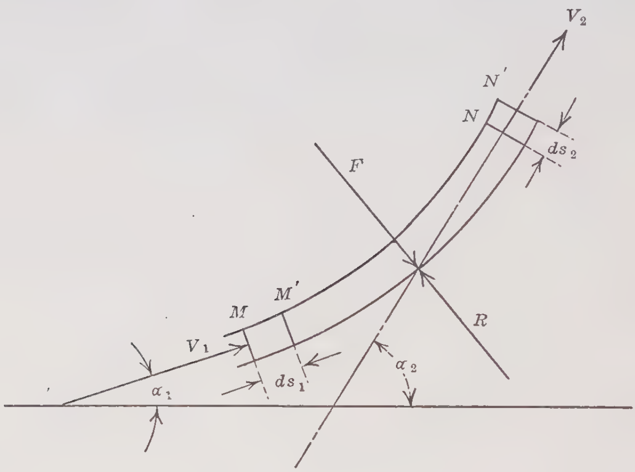


Fig. 150.

part between N and N'. In the case of steady flow the momentum of the part between M' and N is constant. Hence the change of momentum is the difference between the momentum of the part between N and N' and that of the part between M and M'. Noting that $wA_1ds_1 = wA_2ds_2$, since the flow is steady, the change in the x component of the momentum during dt is then

$$\frac{wA_1ds_1}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

If the rate of flow be denoted by W, then

$$wA_1ds_1 = Wdt$$

and the time rate of change of the x component of the momentum is

$$\frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1).$$

Denoting by R_x the x component of the resultant force which changes the momentum,

$$R_x = \frac{W}{g}(V_2 \cos \alpha_2 - V_1 \cos \alpha_1) = \frac{W}{g}\Delta V_x$$

which is identical with the expression derived by the first method.

If F indicates the value of the force exerted by the water, which is equal and opposite to R, then

$$F_x = \frac{W}{g}(V_1 \cos \alpha_1 - V_2 \cos \alpha_2) = -\frac{W}{g}\Delta V_x. \tag{108}$$

In similar manner the y component of F will be

$$F_y = \frac{W}{g}(V_1 \sin \alpha_1 - V_2 \sin \alpha_2) = -\frac{W}{g}\Delta V_y. \tag{109}$$

Since

$$F = \sqrt{F_x^2 + F_y^2} \text{ and } \Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2},$$

the value of the resultant force is

$$F = \frac{W}{g}\Delta V. \tag{110}$$

The direction of R will be the same as that of ΔV and the direction of F will be opposite to it. It is because F and ΔV are in opposite directions that the minus sign appears in the last terms of Eqs. (108) and (109). Note that ΔV is the *vector* difference between V_1 and V_2.

While in reality the total force, as shown by the first derivation, is made up of infinitesimal forces along the entire path, it is often convenient to regard it as made up of two single forces concentrated at points (1) and (2) as suggested by the second method. This is analogous to considering a distributed load on a beam, for instance, to be equivalent to one or more concentrated loads. From this viewpoint, the total force F may be considered equivalent to a force at inflow whose value is $(W/g)V_1$ and whose direction is the same as the velocity V_1, and a second force at outflow whose value is $(W/g)V_2$ and whose direction is opposite to that of the velocity V_2.

124. Dynamic Action upon Stationary Body.—In order to find the dynamic force exerted by a stream upon a stationary object, it is merely necessary to find the value of ΔV in Eq. (110), assuming the rate of discharge to be known. Thus referring to Fig. 151, let the angle θ be 30 deg. and suppose that the stream is a circular jet with a diameter of 2 in. and a velocity of 100 ft. per second, while the velocity of the water leaving at (2) is 80 ft.

per second. Taking the x axis as parallel to V_1, then $V_1 \cos \alpha_1 = 100$ and $V_2 \cos 30$ deg. $= 69.3$. Also $V_1 \sin \alpha_1 = 0$, and $V_2 \sin \alpha_2 = 40$. Since $q = 0.218 \times 100 = 21.8$ cu. ft. per second, $W/g = w \times 21.8/g = 4.22$. Therefore, $F_x = 4.22 \times (100 - 69.3) = 129.5$ lb. and $F_y = 4.22 \times (0 - 40) = -169$ lb. These two components may then be combined to give $F = 212.5$ lb.

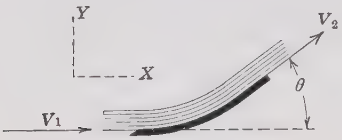


FIG. 151.

In some special cases, a stream may be equally divided in such a way that the value of F_y for one-half will be equal and opposite to that for the other half. Instances of this are where a stream strikes a flat plate normally or where it strikes in the center of a symmetrical object such as a Pelton bucket. Hence in these cases, which are often met with, $F = F_x$. Also in the special case where $\theta = 180$ deg., the value of F_y is zero, whether the flow is divided or not.

It may be noted in the example above that the reduction of the velocity at outflow by friction or otherwise increases the value of F_x. But if the angle θ is greater than 90 deg., the reduction of the value of V_2 diminishes the value of F_x. Hence in a Pelton water wheel, where the water is turned through an angle greater than 90 deg., the buckets are polished and made smooth.

EXAMPLES

133. If a jet of water is deflected through an angle θ without any change in the magnitude of the velocity, prove that $F = (2wA/g)V^2 \sin \theta/2$.

134. In Fig. 151 assume that $\theta = 60$ deg., and that the stream striking the body is a jet 2 in. in diameter with a velocity of 100 ft. per second. If the frictional loss is such as to reduce the velocity of the stream leaving the body to 80 ft. per second, find: (*a*) the component of the force in the same direction as the jet, (*b*) the component of the force normal to the jet, (*c*) the magnitude and direction of the resultant force exerted by the water.

Ans. (*a*) 254 lb. (*b*) 293 lb. (*c*) 388 lb. at 49 deg. 08 min. with direction of jet.

135 (*a*) Suppose the jet in problem 134 struck a flat plate normally, what would be the value of the force exerted upon the plate?

(*b*) Suppose the jet were completely reversed in direction, or that $\theta = 180$ deg. If V_2 were 100 ft. per second, what would be the component of the force in the same direction as the jet? What would be the component normal to the direction of the jet?

(*c*) Suppose that the value of V_2 were reduced to 80 ft. per second. What would be the value of the force exerted?

Ans. (*a*) 423 lb.
(*b*) 846 lb.
(*c*) 761 lb.

125. Force Exerted upon Pipe.—When a flowing stream is confined there may be forces due to static pressure as well as dynamic forces due to changes in velocity. Consider the water to be flowing to the right in Fig. 152. Since the velocity is increased from V_1 to V_2, the dynamic force exerted upon the water, according to Eq. (110), will be

$$R = \frac{W}{g}(V_2 - V_1).$$

This force, producing the acceleration of the water, must be the resultant of all the forces acting. The real forces acting upon the volume of water shown are the pressures upon the two ends

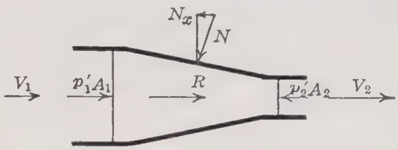


FIG. 152.

p'_1A_1 and p'_2A_2 exerted by the rest of the water, and the force N exerted by the pipe walls.[1] If there were no friction this force would be normal to the walls, but actually it will be inclined somewhat from the normal because it must have a frictional component. Let the component of N parallel to the axis of the pipe be denoted by N_x. It may be seen that R must be in the same direction as V_1 and V_2 in Fig. 152. Hence the sum of all the forces parallel to the axis of the pipe must equal R. Therefore

$$R = p'_1A_1 - p'_2A_2 - N_x.$$

Inserting the value of R given above, it follows that

$$N_x = p'_1A_1 - p'_2A_2 - \frac{W}{g}(V_2 - V_1). \tag{111}$$

It must be remembered that N_x is assumed to be the axial component of the force exerted upon the water by the conical portion of pipe. The force exerted by the water upon the pipe

[1] The N shown in Fig. 152 represents the force for an element only. For a pipe of circular cross-section the resultant force exerted by the walls must be axial.

is equal and opposite to this. That is, its magnitude is given by Eq. (111) but it acts toward the right.

If the velocity of the water in a closed passage undergoes a change in its direction, as in the pipe bend shown in Fig. 153, the procedure would be similar to that in the preceding case. The forces acting on the water in the bend are the pressures p'_1A_1 and p'_2A_2 and the pressure exerted by the walls of the pipe, designated by N. By Eq. (110) the resultant force acting upon this volume of water will be $R = W/g\Delta V$, but R is the resultant of the three forces just mentioned. Since these are vector quantities not in the same straight line, it will be better to take x and y components. Thus should be written

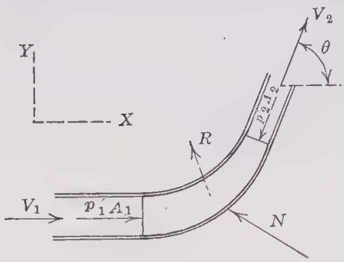


Fig. 153.

$$R_x = \frac{W}{g}(V_2 \cos \theta - V_1) = p'_1A_1 - p'_2A_2 \cos \theta - N_x$$

and

$$R_y = \frac{W}{g}V_2 \sin \theta = -p'_2A_2 \sin \theta + N_y.$$

Solving these equations it is found that

$$N_x = p'_1A_1 - p'_2A_2 \cos \theta + \frac{W}{g}(V_1 - V_2 \cos \theta) \qquad (112)$$

and

$$N_y = p'_2A_2 \sin \theta + \frac{W}{g}V_2 \sin \theta. \qquad (113)$$

But again N represents the force exerted by the pipe bend upon the water. The force exerted by the water upon the bend will be equal and opposite to this.

It may be seen that these forces tend to move the portion of pipe considered. Hence a pipe should be "anchored" where such changes in velocity occur.

EXAMPLES

136. On the end of a 6-in. pipe is a nozzle which discharges a jet 2 in. in diameter. The pressure in the pipe is 55 lb. per square inch and the pipe velocity is 10 ft. per second. The jet is discharged into the air. (*a*) What is the resultant force acting on the water within the nozzle? (*b*) What is the axial component of the force exerted on the nozzle?

Ans. (*a*) 304 lb. (*b*) 1,250 lb.

137. Water under a pressure of 40 lb. per square inch flows with a velocity of 8 ft. per second through a right-angle bend having a uniform diameter of 12 in. (*a*) What is the resultant force acting on the water? (*b*) What is the total force exerted on the bend?

Ans. (*a*) 137.8 lb. (*b*) 6,530 lb.

126. Theory of Pitot Tube.—The Pitot tube has been briefly described in Art. 75 and illustrated in Fig. 99. The dynamic action of the water upon it will now be considered. In Fig. 154 a Pitot tube is placed with its opening facing upstream, the velocity of the water being denoted by V. The dotted lines in the figure are intended to represent an imaginary cylinder of cross-section area A equal to that of the mouth of the tube and extending to point (1) as far upstream as the influence of the tube is felt.[1]

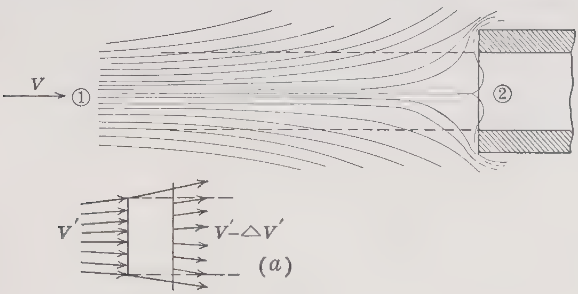


Fig. 154.

At point (1) the water within this cylinder has a velocity V and, as it approaches the Pitot tube, its velocity continually decreases until it becomes zero at point (2). But if water flows into this cylinder, bounded by the dotted lines, it must also flow out. It does this along the sides, for since the area A is constant while the velocity is a decreasing quantity, it follows that q (within the cylinder) must become less as the Pitot tube is approached. The conditions for a certain mass of water are therefore as shown in Fig. 154 (*a*). As the velocity of the water decreases the cross-section area must increase for the same value of q. Referring to Fig. 154 (*a*), consider that water flows into this portion of the stream with a velocity V' and leaves it with a velocity $V'-\Delta V'$. If the cross-section area of the stream

[1] The method of derivation of the Pitot tube formula given here, as well as some interesting experimental results, will be fo''*P*und Wap er by L. F. Moody, "Measurement of the Velocity of Flowing ineatr, a p*roc. Eng. Soc. Western Penn.*, vol. 30, p. 279, 1914.

entering the section is A, then $W = wq = wAV'$. If the two faces of this volume be taken at an infinitesimal distance apart the velocity will decrease by an amount dV', hence the dynamic force exerted upon this small mass of flowing water will be

$$dR = -\frac{W}{g}dV' = -\frac{wA}{g}V'dV'.$$

The value of V' varies from V at point (1) to 0 at point (2). Since A is constant then

$$R = -\frac{wA}{g}\int_V^0 V'dV' = \frac{wA}{g}\frac{V^2}{2}.$$

The dynamic force exerted by the flowing water upon the body of still water within the Pitot tube is equal and opposite to R. If the force be represented by F, then

$$F = wA\frac{V^2}{2g}. \tag{114}$$

This is the value of the total force distributed over the area A. The intensity of pressure is $p' = wV^2/2g$, or dividing by w the intensity of pressure in feet of water is obtained so that

$$h = \frac{V^2}{2g}. \tag{115}$$

That this is true has been amply demonstrated by experimental evidence. If the water is under pressure, the Pitot tube will read the sum of the pressure head and the dynamic head, given by Eq. (115).[1]

[1] The Pitot tube formula has often been derived by an incorrect application of the principles of Art. 123. If a jet of water with cross-section area A impinges normally upon a flat plate, the dynamic force will be

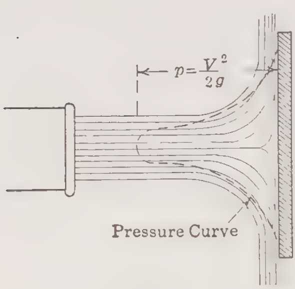


Fig. 155.

$$F = \frac{W}{g}\Delta V = \frac{wAV}{g}V = wA\frac{V^2}{g}.$$

This is twice the value given by Eq. (114), and dividing this by the area of the Pitot tube orifice, which is also assumed equal to A, the intensity of pressure in feet of water is apparently $h = V^2/g$. But this reasoning is incorrect; for a flat plate of an area the same as that of a jet would not be able to deflect all the water through an angle of 90 deg. Experiment shows that the dynamic pressure exerted by a circular jet is distributed over a circular area whose diameter is at least twice that of the jet. Therefore, if the entire stream of water is to be deflected through an angle of 90 deg. the area of the

127. Relation between Absolute and Relative Velocities.—In much of the work that follows it will be necessary to deal with both absolute and relative velocities of the water. The absolute velocity of a body is its velocity relative to the earth. The relative velocity of a body is its velocity relative to some second body which may in turn be in motion relative to the earth. The absolute velocity of the first body is the vector sum of its velocity

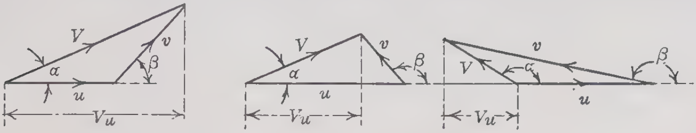


FIG. 156.—Relation between absolute and relative velocities.

relative to the second body and the absolute velocity of the latter. The relation between the three is shown in Fig. 156.[1]

128. Dynamic Action upon Moving Body.—The dynamic force exerted by a stream upon a moving object can be determined by a direct application of Eq. (110), provided the flow is steady. The principal difference between action upon a stationary and upon a moving object is that in the latter case it is necessary to deal with both absolute and relative velocities, and the determination of ΔV may be more difficult.

Another point of difference may be in regard to the amount of water which strikes the object. If the cross-section area of a stream is A_1 and its velocity is V_1, the rate of discharge is A_1V_1

---

plate must be at least four times that of the jet. Dividing F by $4A$ the average intensity of pressure should be $p' = wV^2/4g$. It is found experimentally that the maximum intensity of pressure at the center of the plate in feet of water is $V^2/2g$, and that this pressure diminishes in intensity as the outer margins of the area in question are approached, as shown in Fig. 155.

See GROAT, B. F., "Pitot Tube Formulas—Facts and Fallacies," *Proc. Eng. Soc. Western Penn.*, vol. 30, p. 324, 1914.

[1] A clearer idea of this relationship may be obtained from the illustration in Fig. 157. Suppose a raft is moving downstream with a uniform velocity u. A man on the raft at A walks over to the diagonally opposite corner at a uniform rate. But by the time he reaches B the latter point on the raft will have moved downstream to point C. Thus the path of the man relative to the raft is AB but relative to the earth it is AC. Since the velocities are all uniform they are all proportional to the distances traversed in this interval of time.

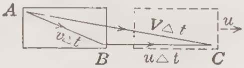


FIG. 157.

so that $W = wA_1V_1$. This is the weight of water per unit time that strikes a stationary body.

But in the case of a body having a motion of translation, the weight of water acting may be less than the above. As an extreme case, suppose the object to be moving in the same direction as the water and with a velocity equal to, or higher than, that of the stream; it is clear that none of the water will strike it. If the object is moving with a smaller velocity than that of the stream, then the amount of water which strikes it per unit of time will be proportional to the difference between the two velocities. Thus, if W' denotes the pounds of water per second striking a body moving with a velocity u the same direction as the stream, then

$$W' = wA_1(V_1 - u). \tag{116}$$

Our treatment here is necessarily restricted to the case where the body is moving in the same direction as the water, for, if it is otherwise, the case becomes one of unsteady flow, which is quite complex and beyond the scope of this text.

Thus for the case of steady flow, which requires the body to move with a uniform velocity in the same direction as the jet, the force exerted upon a single moving object is

$$F = \frac{W'}{g}\Delta V. \tag{117}$$

In general W' is less than W, and the difference between the two may be accounted for as follows: Suppose a jet issuing from a nozzle strikes an object moving with a velocity u in the same direction. In one second a volume of water A_1V_1 will issue from the nozzle, while only $A_1(V_1 - u)$ will strike the object. But in this same interval of time the object will have moved farther away from the nozzle by a distance u and thus the volume of water between the two will have been increased by an amount A_1u.

In Fig. 158, is shown a stream of water with a velocity V_1 striking a body moving with a constant velocity u. By the time a particle of water, which strikes the vane just at the instant it is in the position shown, has reached the point of outflow the vane will have reached the position indicated by the dotted outline. Thus two paths may be traced for the water: one the path relative to the moving vane, which is as it would appear to an observer who was moving with the vane; and the other relative

to the earth, termed the absolute path, and is as it would appear to an observer standing still with respect to the earth.

A study of Fig. 158 shows that the direction of the relative velocity at outflow from the vane is determined by the shape of the latter, but the relative velocity at entrance, just *before* the water is influenced by the vane, is determined solely by the relation between V_1 and u. Just *after* the water is influenced by the vane, its relative velocity must be tangent to the vane surface. To avoid excess energy loss, these two directions should be made to agree, otherwise there will be an abrupt

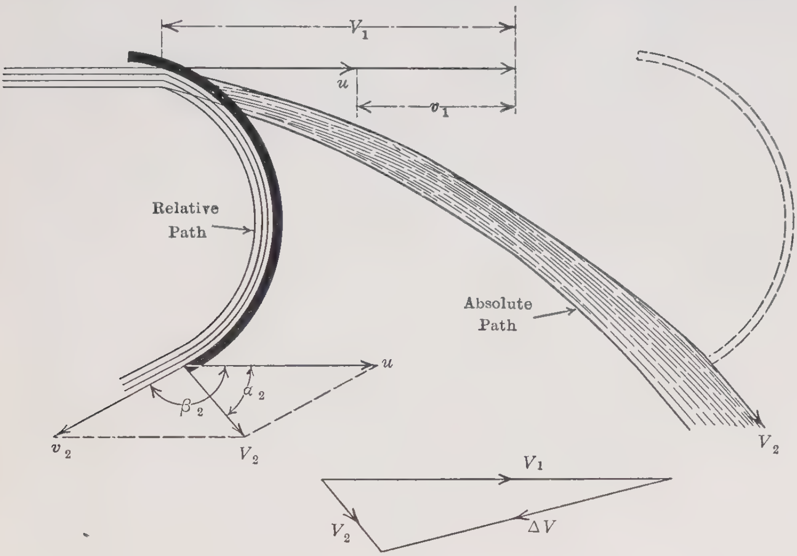


FIG. 158.—Action of water on moving vane.

change in the velocity and path of the water, as is shown in the figure.

As an illustration of the application of our fundamental principles, assume a jet with a diameter of 2 in. and a velocity of 100 ft. per second striking an object moving in the same direction with a velocity of 60 ft. per second. Suppose that $\beta_2 = 150$ deg. and that the friction losses in flow over the vane are such that $v_2 = 0.9v_1$. It is desired to find the force exerted. At entrance to the vane $v_1 = V_1 - u = 100 - 60 = 40$ ft. per second. (It must be noted that this is a special relationship because all the velocities are in the same straight line. In

general the relation between the three is as shown in Fig. 156.) At outflow $v_2 = 0.9v_1 = 0.9 \times 40 = 36$. The quantities V_2 and α_2 may be found by trigonometry, since two sides and an included angle are now known. Thus $\alpha_2 = 32$ deg. and $V_2 = 34$ ft. per second.

It is generally easier to deal with components, however, and practically, the component of the force in the direction of motion is usually all that is desired. Thus it is found that

$$V_2 \cos \alpha_2 = u + v_2 \cos \beta_2 = 60 - 36 \times 0.866 = 28.8$$
$$V_2 \sin \alpha_2 = v_2 \sin \beta_2 = 36 \times 0.500 = 18.$$

For a single moving object, $W' = w \times 0.0218 \times (100 - 60) = 54.4$ lb. per second and $W'/g = 1.69$. If the x axis is taken in the same direction as u and V_1, then

$$F_x = \frac{W'}{g}(V_1 - V_2 \cos \alpha_2) = 1.69(100 - 28.8) = 120.3 \text{ lb.}$$

and in a similar manner the value of F_y could be found, if desired.

It may be seen that the magnitude of the force exerted by a jet depends both upon the shape and the velocity of the object struck. In fact the same value of ΔV might be had with either a stationary or a moving object or with moving objects having different velocities provided only that their shapes, which in this case means their values of β_2, were suitable.

As another illustration of the foregoing the dynamic force exerted by a jet of water upon the moving body from which it issues may be considered. When a stream of water issues from any device, such as the vessel shown in Fig. 159, a force is required to accelerate the water and impart to it the velocity it has upon leaving. This force is exerted upon the particles of water flowing out the orifice by adjacent particles of water and ultimately by the walls of the vessel. By the law of action and reaction an equal and opposite force will be exerted upon the vessel. It is impossible to analyze this reaction in detail, but its total value will be given by an application of Eq. (110).

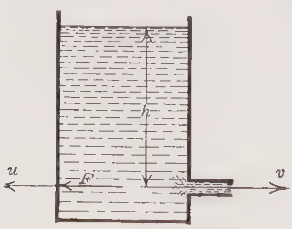


FIG. 159.

Assume that the vessel in Fig. 159 moves to the left with a uniform velocity u, and that the orifice is so small compared to the size of the vessel that the relative velocity of the water in

the latter may be neglected as may also the change in h. Then $V_1 = u$. If the jet issues from the orifice with a velocity v_2 the absolute velocity of the jet will be $V_2 = u - v_2$. Hence $\Delta V = V_1 - V_2 = u - (u - v_2) = v_2$. Therefore,

$$F = \frac{W}{g}v_2 = \frac{wA}{g}v_2^2. \tag{118}$$

This might have been determined more directly if another proposition had been previously established. That is that for any case whatever $\Delta V_x = \Delta v_x + \Delta u_x$, where the subscript x merely denotes a component along any axis. In this case, since u is constant, it may be seen that $\Delta V = \Delta v$ and is independent of the velocity of the vessel.

Since $v_2 = c_v\sqrt{2gh}$, then Eq. (118) may be written as

$$F = 2c_v^2wAh. \tag{119}$$

If losses of energy be neglected in both cases, it may be seen that the reaction of the jet in Fig. 159 is equal to the force of impact upon a flat plate, normal to the jet, providing the area of the plate be large enough to deflect the water through 90 deg.[1]

EXAMPLES

138. A jet of water 3 in. in diameter has a velocity of 120 ft. per second. It strikes a vane with an angle $\beta_2 = 90$ deg. which moves in the same direction as the jet with a velocity u. Assume that the loss in flow over this vane is such that $v_2 = 0.9v_1$. When u has values of 0, 40, 60, 80, 100, and 120 ft. per second, find values of: (*a*) W', (*b*) $V_2 \cos \alpha_2$, (*c*) F_x.

Ans. When $u = 40$, (*a*) 245, (*b*) 40, (*c*) 610.

139. If the jet in the preceding problem strikes a vane for which $\beta_2 =$ 180 deg., all other data remaining the same, find values of: (*a*) W', (*b*) v_2, (*c*) V_2, (*d*) F.

Ans. When $u = 40$, (*a*) 245, (*b*) 72, (*c*) -32, (*d*) 1,160.

129. Dynamic Action upon Rotating Wheel.—In the case of a water wheel, each bucket or vane may be considered as a body whose motion approximates that of translation for the short distance that it is in the line of action of the jet. Thus the

[1] The hydrostatic pressure on an area equal to that of the jet A at a depth h is given by wAh. The fact that this is only half the dynamic pressure considered is of no significance. As has already been pointed out, the dynamic pressure on a plate is distributed over an area much larger than that of the jet and the intensity of pressure has not increased in either case.

amount of water acting upon a single bucket is W', in accordance with the preceding article. But the wheel as a whole remains in the same position in space and must thus receive the entire rate of discharge W. The explanation is that more than one bucket may be acted upon at the same time, as may be seen in Fig. 215.

Thus to obtain the force acting upon a wheel, as a whole, the value of ΔV, or any desired component thereof, may be computed the same as in the preceding article, but it must be multiplied by W/g. Hence for this case Eq. (110) is used directly, that is

$$F = \frac{W}{g}\Delta V.$$

In reality it is physically impossible to have a series of vanes on a rotating wheel moving in a straight line in the same direction as the jet, though such an assumption is often made for the sake of simplicity. Thus in general the angle α_1 will not be 0 deg. and the three velocities at entrance will not be as shown in Fig. 158, but rather in one of the forms shown in Fig. 156. This means that the value of W' and also of ΔV for each individual vane will vary with time, and thus the condition of unsteady flow results. However, for the wheel as a whole, the effect upon all the vanes at any instant may be said to give an average which approximates a steady condition. Thus it is permissible to apply our equation to the entire wheel, even under the more general condition.

EXAMPLE

140. Solve example 139, applying the data there to a wheel instead of one single bucket.

Ans. When $u = 40$, $F = 1{,}740$ lb.

130. Torque Exerted.—When a stream flows through a turbine runner in such a way that its distance from the axis of rotation remains unchanged, the dynamic force can be computed by the methods shown. But when the radius varies, it is not feasible to compute a resultant force directly. Instead it is necessary to find the total torque produced by all the elementary forces. But it has been shown that the total of all the elementary forces may be considered as equivalent to two single forces concentrated at the points of entrance and exit. The torque may then be found by taking the moments of these two forces. Thus,

at entrance, the force may be assumed to be $(W/g)V_1$ and at outflow $(W/g)\ V_2$. Taking moments

$$T = \frac{W}{g}(r_1V_1 \cos \alpha_1 - r_2V_2 \cos \alpha_2). \tag{120}$$

It is immaterial in the application of this formula whether the water flows radially inward, as in Fig. 160, radially outward, or remains at a constant distance from the axis. In any case r_1 is the radius at entrance and r_2 is that at exit.

The solution for the perfectly general case illustrated in Fig. 160 is very similar to the illustrative problem in Art. 128. If the dimensions of the wheel, its speed, and the value of W are given, the procedure would be as follows: The value of V_1 would be determined by the equation $W = wA_1V_1$, where A_1 is the total

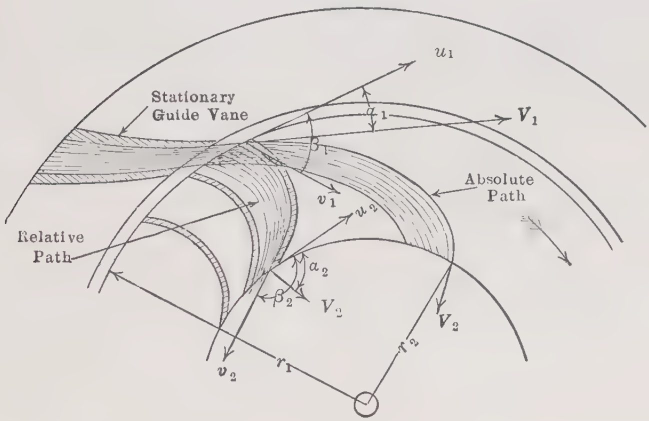


FIG. 160.—Hydraulic turbine.

cross-section area of all the guide passages measured normal to V_1. The angle α_1 is also determined by the position of these guide vanes. The vector triangle at entrance may then be solved by trigonometry for v_1 and β_1, if these quantities are desired. To avoid loss of energy the wheel vane at this point should be tangent to the relative velocity, as determined by the velocity triangle, that is it should also have the angle β_1. The value of v_2 at outflow may be determined by either one of the two methods which will be given in Art. 137, according to the type of turbine. Then $V_2 \cos \alpha_2$ may readily be computed.

In the case of a centrifugal pump the torque exerted by the impeller upon the water is given by Eq. (120) with the signs reversed. In Fig. 161 is shown the path of water through a pump impeller. If it enters radially, as shown, then $\alpha_1 = 0$ deg. Some pumps have guide vanes within the "eye" of the impeller which impart angular momentum to the water entering the impeller and for such the value of α_1 is used which is then fixed by construction. But for a pump without these vanes the water may enter the impeller at various angles depending upon the condition of operation. But any rotation of the water in such

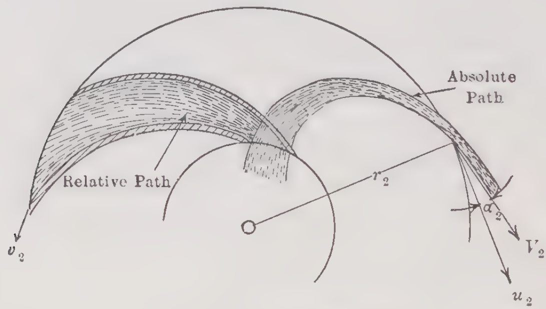


FIG. 161.—Centrifugal pump.

a case, even though it started back in the suction pipe, has in reality been derived from the impeller, for it is obvious that the impeller has supplied directly or indirectly all the angular momentum which the water ultimately acquires. Therefore, for the usual type of centrifugal pump without guides at entrance

$$T = \left(\frac{W}{g}\right) r_2 V_2 \cos \alpha_2. \qquad (121)$$

While Eqs. (120) and (121) are true, they are of little real service because the proper values to use in them are often not known with exactness. The precise values of velocities and directions of stream lines are difficult matters to determine. Since water does not fulfil the ideal conditions assumed, it will be found that these equations often yield numerical results that are considerably in error.

131. Power.—Power may be represented by several different combinations, each of which has certain uses, and usually certain

of the factors involved may be given various interpretations. Thus the horsepower may be represented by

$$\text{Horsepower} = \frac{Fu}{550} \tag{122}$$

$$= \frac{T\omega}{550} = \frac{T \times 2\pi N}{33{,}000} \tag{123}$$

$$= \frac{WH}{550} = \frac{wqH}{550} = \frac{qH}{8.81}. \tag{124}$$

In Eq. (122), F must be understood as the component of force in the direction of u. Now T, for instance, may be the torque measured by a brake, and its use in the above will give us brake horsepower. But if T has the value given by Eq. (120) the power will be greater than the power output of the turbine by an amount equal to the losses in mechanical friction. It will give the power that is actually delivered to the shaft by the water, which is analogous to the indicated horsepower of a steam engine. This is less than the power supplied in the water delivered to the turbine by an amount equal to the power lost in hydraulic friction within the turbine case, runner, and draft tube.

If T has the value given by Eq. (121) the power given by Eq. (123) will be less than that required to run the pump by an amount equal to the mechanical losses, and it will be greater than the power delivered in the water by the amount of the hydraulic losses. It represents the power actually expended by the impeller on the water and is analogous to the indicated power of a reciprocating pump.

In like manner, the quantity H in Eq. (124) may be interpreted as any head for which the corresponding power is desired.

In order to show the application of the above, the illustrative problem in Art. 128, may be considered, but the vane is to be considered as one of a series on a water wheel. Thus W will be dealt with rather than W'. It is seen that $W = wA_1V_1 = 62.4 \times 0.0218 \times 100 = 136$ lb. per second. Since the velocity is 100 ft. per second, the head in the jet is $H_1 = V_1^2/2g = 155$ ft. Thus the power in the jet is $136 \times 155/550 = 38.4$ hp. It was shown in Art. 128 that the component of ΔV in the direction of u, which for a wheel means the tangential component, was equal to $100 - 28.8 = 71.2$. Thus the component of the total force in the direction of motion is $F_x = (136/g)71.2 = 300$ lb. The power developed by the water upon the wheel is, therefore,

$F_x u/550 = 300 \times 60/550 = 32.8$ hp. The head lost in hydraulic friction in flow over the vanes will be proved later to be represented by $(v_1^2 - v_2^2)/2g = (1{,}600 - 1{,}296)/2g = 4.72$ ft. Thus the power lost in friction is $136 \times 4.72/550 = 1.17$ hp. The kinetic energy discharged from the wheel is represented by $V_2^2/2g = 34^2/2g = 18$ ft. and thus the power lost is $136 \times 18/550 = 4.43$ hp. It is now seen that $38.4 = 32.8 + 1.17 + 4.43$, which checks the calculations.

EXAMPLES

141. Find the power developed for each of the speeds given in example 138, and using the data there found except changing the angle β_2 to 180 deg. and considering an entire wheel and not a single vane.

Ans. When $u = 40$, hp. $= 126.7$.

142. The absolute velocity of water entering a turbine runner is 60 ft. per second and that leaving is 15 ft. per second. $\alpha_1 = 20$ deg., $\alpha_2 = 80$ deg., $r_1 = 2.5$ ft., $r_2 = 4.0$ ft. (*a*) If $W = 600$ lb. per second, find the torque on the wheel. (*b*) If $u_1 = 50$ ft. per second, find the power delivered to the wheel.

Ans. (*a*) 2,430 ft.-lb. (*b*) 88.5 hp.

132. Head Utilized.—In turbine and pump practice the word "head" is used to express several different physical quantities. The head h under which a turbine or pump operates is explained in Arts. 103 and 104. But the actual head converted into mechanical work in the turbine is less than this by the amount of the hydraulic friction losses, including the energy lost at discharge. The energy per pound of water which is converted into mechanical work may be called the head utilized, and may be designated by h''. Thus the total mechanical work done per second is Wh''. But $Wh'' = T\omega$, where T has the value given by Eq. (120) and h'' replaces H in Eq. (124). Since the angular velocity $\omega = u/r = u_1/r_1 = u_2/r_2$, $(r_1V_1 \cos \alpha_1 - r_2V_2 \cos \alpha_2)\omega = (u_1V_1 \cos \alpha_1 - u_2V_2 \cos \alpha_2)$. Therefore, $Wh'' = T\omega = (W/g)(u_1V_1 \cos \alpha_1 - u_2V_2 \cos \alpha_2)$ or the head utilized is

$$h'' = \frac{u_1V_1 \cos \alpha_1 - u_2V_2 \cos \alpha_2}{g}. \tag{125}$$

In the case of the centrifugal pump the mechanical energy that is actually imparted to the water must be greater than the net head h by an amount equal to all the hydraulic losses, and may be expressed by the above equation with the signs reversed. As has been explained, in many cases the angle α_1 may be con-

sidered as 0 deg. so that $h'' = u_2 V_2 \cos \alpha_2/g$, in the case of the usual centrifugal pump.

EXAMPLE

143. Find the head utilized in example 142.
Ans. 81 ft.

133. Definitions of Turbine Efficiencies.—The word "efficiency" without any qualifying adjective is always understood to mean gross or total efficiency. It is the ratio of the developed or brake horsepower to the power delivered in the water to the turbine. That is

$$e = \frac{\text{b.hp.}}{\text{w.hp.}} \tag{126}$$

Mechanical efficiency is the ratio between the power delivered by the machine and the power delivered to its shaft by the water. If q represents the total turbine discharge while q' equals the amount of leakage through the clearance spaces, the actual amount of water doing work is $q - q'$. Hence

$$e_m = \text{b.hp.}/\frac{w(q - q')h''}{550}. \tag{127}$$

Hydraulic efficiency is the ratio of the power actually delivered to the shaft to that supplied in the useful water. That is

$$e_h = \frac{w(q - q')h''}{w(q - q')h} = \frac{h''}{h}. \tag{128}$$

Volumetric efficiency is the ratio of the water actually used by the runner to total amount discharged. Thus

$$e_v = \frac{(q - q')}{q}. \tag{129}$$

The total efficiency is the product of these three separate factors. That is

$$e = e_m \times e_h \times e_v. \tag{130}$$

134. Definitions of Pump Efficiencies.—The various pump efficiencies are similar to those for the turbine. The total efficiency is

$$e = \frac{\text{w.hp.}}{\text{b.hp.}} \tag{131}$$

The mechanical efficiency is

$$e_m = \frac{w(q + q')h''}{550}/\text{b.hp.} \tag{132}$$

The hydraulic efficiency is

$$e_h = \frac{w(q + q')h}{w(q + q')h''} = \frac{h}{h''}. \tag{133}$$

The volumetric efficiency is

$$e_v = \frac{q}{(q + q')}. \tag{134}$$

As in Eq. (130) the total efficiency is the product of these three.

135. Centrifugal Action or Forced Vortex.—If a vessel containing a liquid is rotated about its axis, the liquid will tend to rotate at the same speed. If the vessel is open the free surface of the water will assume the curve shown in Fig. 162 (*a*). If the water is confined within a closed vessel, which it completely fills so that it cannot change its position, the pressure along a horizontal line will vary in the same way as in the preceding case. In fact if piezometer tubes were connected to the vessel the water would rise in them as shown in Fig. 162 (*b*). Since the hydraulic gradient represents the free surface that corresponds to the actual pressure conditions, it is seen that the two cases are equivalent. Such a rotation is sometimes described as a forced vortex because the water is forced to rotate by external forces.

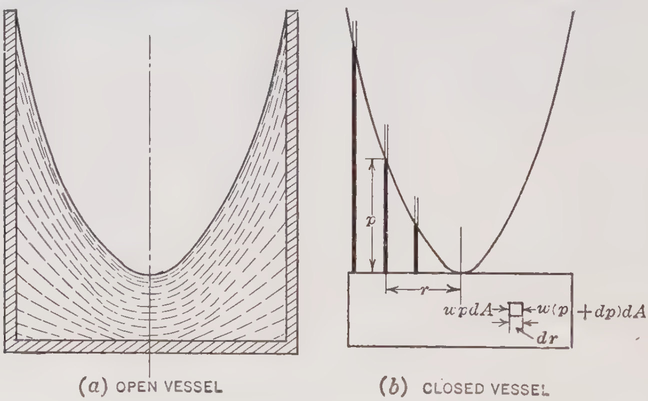


FIG. 162.—Forced vortex.

The variation in pressure in such a body of water may be found in the following manner. If an elementary volume in Fig. 162 is taken whose length along the radius is dr and whose area normal to the radius is dA, an elementary mass $wdAdr/g$ moving in a circular path is obtained. This mass has an acceleration u^2/r or $\omega^2 r$, directed toward the axis of rotation. Conse-

quently the accelerating or resultant force is $(wdAdr/g)\omega^2r$ directed toward the axis. The intensity of pressure on the two faces of the elementary volume differs by $dp' = wdp_r$. The value of the resultant force is therefore wdp_rdA. Consequently,

$$wdp_rdA = \left(\frac{wdAdr}{g}\right)\omega^2r$$

$$dp_r = \left(\frac{\omega^2}{g}\right)rdr.$$

But this expression shows only the difference of pressure along the radius and in the same horizontal plane. By moving along a path parallel to the vertical axis of rotation so that the radius is constant, the pressure decreases directly as the elevation increases. Thus

$$dp_z = -dz.$$

The variation of the intensity of pressure in *any* direction may be found by combining the two preceding equations. Thus, in general, when both r and z vary,

$$dp = -dz + \left(\frac{\omega^2}{g}\right)rdr. \tag{135}$$

To find the equation of the free surface or any surface of equal pressure it is necessary only to place dp equal to zero. Then

$$\int dz = \left(\frac{\omega^2}{g}\right)\int rdr$$

$$z = \frac{r^2\omega^2}{2g} + \text{constant}.$$

To determine the constant it may be assumed that $z = 0$ when $r = 0$. Thus the constant $= 0$ so that

$$z = \frac{r^2\omega^2}{2g}. \tag{136}$$

From this it may be seen that the free surface or any surface of equal pressure is a paraboloid.

To find the variation of pressure in the same horizontal plane it is necessary only to assume $dz = 0$, and integrating between limits the following is obtained

$$p_2 - p_1 = \frac{(r_2^2 - r_1^2)\omega^2}{2g} = \frac{(u_2^2 - u_1^2)}{2g}. \tag{137}$$

For the difference in pressure between any two points it is necessary to integrate Eq. (135) which gives

$$p_2 - p_1 = z_1 - z_2 + \frac{u_2^2}{2g} - \frac{u_1^2}{2g} \tag{138}$$

EXAMPLES

144. An open cylindrical vessel is rotated about its axis, which is vertical. If the vessel is partially filled with water, what speed would be necessary to cause the water surface at a radius of 1.5 to be 4.0 ft. higher than the surface at the center of rotation?

Ans. 102 r.p.m.

145. A closed vessel completely filled with water is rotated about its axis at a speed of 2,000 r.p.m. If the pressure at the center of rotation is 2 ft. of water, what will it be at a radius of 6 in.?

Ans. 172 ft.

136. Free Vortex.—Where external forces are applied to the water, as in the preceding case, we have a *forced* vortex. Where no external forces are applied but the water rotates by virtue of its own angular momentum, previously derived from some source, we have a *free* vortex.

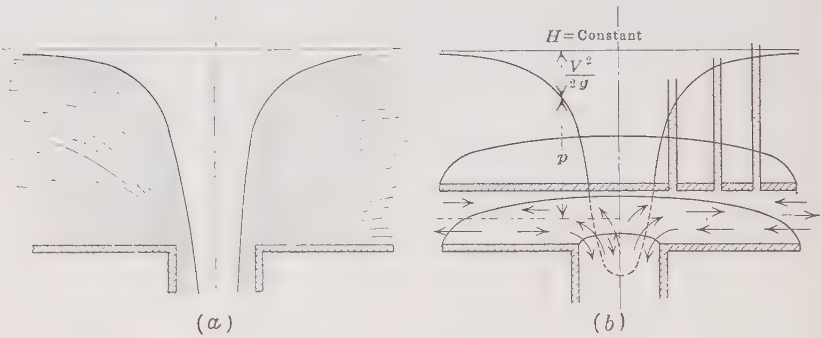


Fig. 163.—Free vortex.

A free *circular* vortex consists of a body of water in rotation without any appreciable flow so that the stream lines are concentric circles. Since no torque is exerted on the water, neglecting friction, it follows that there can be no change in angular momentum. Since angular momentum is proportional to $rV \cos \alpha$, it is apparent that $V \cos \alpha$ varies as $1/r$, as the angular momentum is constant. Since the stream lines are circles $V \cos \alpha$ is the value of V itself. Since no energy is imparted to the water, if friction is neglected, the result will be

$$H = p + z + \frac{V^2}{2g} = \text{constant}.$$

The free surface of such a vortex is shown in Fig. 163 (*a*.i A familiar example of such a surface is when water enter) ang

vertical pipe sets up a rotation and sucks air down the center, though, of course, that velocity then has a radial component. Since $V \times r =$ constant, it is seen that when $r = 0$, the value of V is infinity and $p + z = -$infinity. Since this is impossible, the free vortex never exists with extremely small values of r. (If p is constant or equal to zero as in the case of the free surface, values of z will give the elevation of the surface. If z is constant values of p will give the hydraulic gradient, which is the same curve.)

Considering a pure *radial* flow between two circular plates, either inward or outward, as in Fig. 163 (b) and letting b equal the distance between the two plates, then $q = 2\pi rbV$. For steady flow q is constant and hence rbV is constant. And if the plates are parallel rV is constant. Thus V varies as $1/r$, as in the preceding case.

A free *spiral* vortex is a combination of radial flow and circular flow. The velocities in the two cases above are then merely components of the velocity in the latter case. Since each component varies as $1/r$ it follows that the velocity in a spiral flow also varies as $1/r$. Also since both components vary at the same rate, the angle α, which the velocity makes with the tangent to the circle, remains constant. Thus the *free* stream line is the equiangular or logarithmic spiral. Since the total head is also constant here, neglecting friction, the free surface or the hydraulic gradient, as the case may be, is the same as shown in Fig. 163.

Since H is constant, considering the two points to be at the same elevation,

$$H = p_1 + \frac{V_1^2}{2g} = p_2 + \frac{V_2^2}{2g}$$

$$p_2 - p_1 = \left[1 - \left(\frac{r_1}{r_2}\right)^2\right]\frac{V_1^2}{2g}. \qquad (139)$$

Of course the effect of friction is always to make p_2 smaller than would be given by the above, since H is not constant. (Note that the flow may be either inward or outward.) It may be seen that as r increases, V decreases, and p approaches H as a limit.

The principal application of Arts. 135 and 136 is in the case of the centrifugal pump. In Fig. 229 may be seen a forced vortex in the impeller from (1) to (2) and a free vortex in the casing between (2) and (3). It may be added that the foregoing treat-

ment may readily be extended to the case where the width b is variable.

EXAMPLE

146. If the inner and outer radii of the "whirlpool chamber" in Fig. 163 are 8 and 16 in. respectively, what will be the values of V and α at the outer diameter when water enters the inner diameter with a velocity of 80 ft. per second at an angle of 15 deg. with the tangent? What will be the gain of pressure, neglecting losses?

Ans. 74.6 ft.

137. Flow through Rotating Channel.—The treatment of the forced vortex (Art. 135) will now be extended to the more general case where the water flows through the rotating vessel. It has been seen that with the free vortex the hydraulic gradient or the resulting Eq. (139) is the same whether the water merely rotates in concentric circles or flows in spiral paths, but with the forced vortex the equation will be found to be somewhat different when flow occurs. The reason is that in the free vortex no energy, save that lost by friction, is imparted to, or taken from, the water; but in case water flows through a rotating vessel, energy is delivered either to it or by it.

In Art. 132 it was shown that the value of h'' given by Eq. (125) represents head utilized. That is h'' is the head given up by the water and converted into mechanical work. But if h'' is found to have a negative value it signifies that energy is being delivered to the water by the vessel instead of being abstracted from it. In practice the former action takes place in a turbine and the latter in a centrifugal pump.

The general equation of energy may be applied to this case as well as any other, provided that in addition to the head lost in hydraulic friction, that lost (or gained) in mechanical work is also considered. Hence

$$H_1 - H_2 = h' + h'',$$

where h' represents the head lost in hydraulic friction. This may be expanded by substituting $p + z + V^2/2g$ for H and the value given by Eq. (125) for h''. Noting that by trigonometry $V^2 = v^2 + u^2 + 2uv \cos \beta$ and $V \cos \alpha = u + v \cos \beta$, the absolute velocities may be replaced by the relative velocities so that the above readily becomes

$$\left(p_1 + z_1 + \frac{v_1^2 - u_1^2}{2g}\right) - \left(p_2 + z_2 + \frac{v_2^2 - u_2^2}{2g}\right) = h'. \quad (140)$$

If there is no flow, both v_1 and v_2 become zero and the equation reduces to that of the forced vortex, Eq. (138). If there is no rotation, both u_1 and u_2 become zero, the relative velocity v becomes the same as the absolute velocity V, and the result is the general equation of energy in its usual form.

The head lost in hydraulic friction is proportional to the square of the velocity of flow and is commonly taken as

$$h' = k\frac{v_2^2}{2g}. \tag{141}$$

It may be seen that Eq. (140) is much broader than Bernoulli's theorem in that the latter is only a special case. Its chief use, however, is to fix certain relations between conditions at inflow and outflow from the runner in turbine and centrifugal pump theory. Thus where the water in flowing through the wheel is open to the atmosphere, or any constant pressure, $p_1 = p_2$. Usually $z_1 = z_2$ also, and thus the equation reduces to

$$v_1^2 - u_1^2 = v_2^2 - u_2^2 + kv_2^2. \tag{142}$$

And in some special cases, where the flow takes place at a constant distance from the axis of rotation, $u_1 = u_2$, which simplifies the equation still further. In this special case it may be seen that the loss of head in friction is proportional to $v_1^2 - v_2^2$. Equation (142) is usually used to find v_2 when v_1 is known.

In a type of turbine where the water completely fills the entire passages, the various velocities are determined by the equation of continuity, which now takes the form

$$q = AV = av \tag{143}$$

which may be applied to specific sections, so that

$$q = A_1V_1 = a_1v_1 = a_2v_2 = A_2V_2, \text{ etc.}$$

Note that in each case the cross-section area must be measured normal to the direction of the velocity concerned. In this type of turbine, the pressure will not be constant throughout and Eq. (140) could then be used to find the drop (or gain) in pressure, $p_1 - p_2$.

138. Water Hammer and Surges in Unsteady Flow.—In all the rest of this book the treatment is restricted to cases of steady flow, but in the present article a brief description will be given of the problems of unsteady flow that are of the most practical importance. An adequate mathematical treatment of unsteady flow would occupy too much space to warrant its

inclusion here and no attempt will be made to do more than record some accepted results.

Instantaneous Closure.—In the event of a valve at C in Fig. 164 (a) being instantaneously closed, the velocity of the water in the pipe will be abruptly reduced to zero. But in so doing there will be a considerable rise in pressure within the pipe, which may be much greater than any static pressure that could possibly exist in the given pipe. This high pressure lasts for an instant only, and then follows a periodic fluctuation of pressure which finally dies out, if the pipe does not burst in the meantime. This is known as water hammer.

What happens is that the lamina of water next to the valve at C is brought to rest and is then compressed by the rest of the column of water flowing up against it. At the same time the walls of the pipe surrounding this lamina will be stretched by the excess pressure. The next lamina of water will be brought to rest by the first and so on. It is seen that the volume

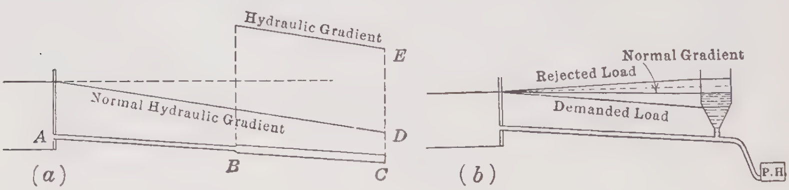


FIG. 164.

of water in the pipe does not behave as a rigid body but that the phenomena are affected by the elasticity of the water and the pipe. Thus the cessation of flow and the increase of pressure progresses along the pipe as a wave action. After a short interval of time the volume of water BC will have been brought to rest, while the water in the length AB will still be flowing with its initial velocity, and with its initial pressure. But the volume of water in BC will be under a much higher pressure due to the compression it is under, and the pipe walls will be stretched. The excess pressure DE is the same for all portions of the pipe and is independent of the length of the pipe.

Finally the pressure wave will have reached the reservoir and the entire volume of water will be at rest. But, a pressure at A higher than that due to the depth of water in the reservoir cannot be maintained, and it instantly drops again to the normal

value. A wave of unloading now traverses the pipe from A to C and by the time it reaches C the entire pipe is under normal pressure once more. But in the meantime, due to the compression of the water and the tension of the pipe, the flow has reversed and water is being returned to the reservoir. The stopping of this reversed velocity now causes the pressure at the valve to drop below the normal value, and thus a wave of rarefaction travels back up the pipe from C to A. This cycle is repeated over and over again with diminishing amplitude due to frictional effects until it dies away. If the valve were to be alternately opened and closed at just the proper intervals of time it would be possible to add one pressure wave on top of another, so that there is no limit to the maximum pressure that might be attained.

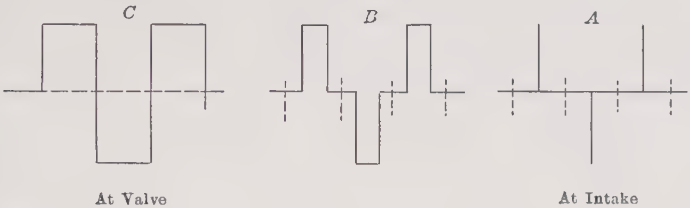


FIG. 165.—Pressure variation with instantaneous closure in ideal case.

In Fig. 165 is shown the variation of pressure as a function of time for three points in the pipe, at the valve C, at some intermediate point B, and at the intake A. The appearance of the diagram for the intermediate point will approach that of A or C according to the location of B in the pipe line. Figure 165 is purely ideal in that it neglects frictional and other factors, but it serves to give a general idea of the phenomena.

The velocity with which this pressure wave progresses along the pipe is the velocity of a sound wave and will be given by the following formula[1]:

$$S = 4{,}700\sqrt{\frac{E}{E + 300{,}000\dfrac{d}{t}}} \tag{144}$$

where S equals velocity of pressure wave in feet per second, E equals modulus of elasticity in tension of the material com-

[1] The complete expression involves the volume modulus of elasticity of the water, the density of the water, and the value of g. Using average values of these quantities the above is obtained.

posing the pipe in pounds per square inch, and d/t is the ratio of the diameter of the pipe to the thickness of the walls, which means that both d and t must be in the same units. The values of E for steel, cast iron, and wood are about 30,000,000 lb. per square inch, 15,000,000 lb. per square inch, and 1,500,000 lb. per sq. inch respectively. For pipes of ordinary dimensions the velocity of this pressure wave will be about 3,300 ft. per second. In any event it will be less than 4,700 ft. per second, which is the velocity of sound in water or the velocity with which a pressure wave would be propagated in water in a rigid pipe.

The time required for a pressure wave to travel the length of the pipe, or the time that it takes for the entire mass of water to be brought to rest will be

$$T = \frac{L}{S}. \tag{145}$$

where L is the length of the pipe AC.

The total force exerted may be determined by applying the principle that force equals mass times acceleration. Since the volume of water is a non-rigid body, the acceleration of the mass center must be dealt with. The pressure wave travels at a uniform rate; hence the velocity of the mass center is uniformly retarded. Therefore, the acceleration may be determined by dividing the change in velocity by the time required for the change to occur. The velocity of all the water, and hence that of the mass center, decreases from V to O in the time T. Thus may be written:

$$F = \frac{wAL}{g}\frac{V}{T} = \frac{wAL}{g}\frac{VS}{L} = wA\frac{VS}{g}.$$

Dividing by the area A and also by w, intensity of pressure in feet of water is obtained. If this excess pressure, due to water hammer, be denoted by y then

$$y = \frac{VS}{g}. \tag{146}$$

It will be observed that this pressure increase is independent of the length of the pipe line.

For the sake of clearness in explanation it has been assumed in the preceding discussion that the velocity of the water has been reduced to zero. But if the valve closure is only partial and not complete, the results are equally true if for V the value

ΔV or $V' - V''$ is substituted, where V' is the initial and V'' the final velocity.[1]

Rapid Closure.—While a valve may be closed very rapidly, it is physically impossible for the closure to be instantaneous, as assumed in the preceding discussion. Therefore, the case where an appreciable time interval is occupied will be considered. Such a closure may be conceived to be a series of instantaneous movements each one of which has started a pressure wave proportional to the small change of velocity involved. Thus from Eq. (146) should be obtained $dy = (S/g)\, dV$, as the value of the pressure produced by a change in velocity dV. Suppose that a valve movement takes place during a time T_v, and that in Fig. 166

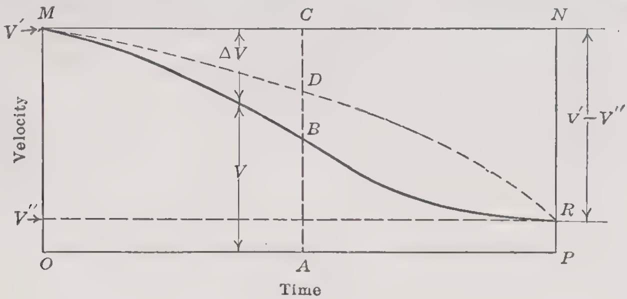


FIG. 166.

there is a representation of the variation in velocity during this time interval, within which the velocity in the pipe decreases from the initial value of V' to the final value of V''. It may be seen that at the end of any time interval, such as OA, the velocity is AB, and that the decrease from the initial velocity is BC. Thus the pressure produced at the valve at this instant has attained the value $(S/g)BC$. Hence while ordinates to the curve measured up from OP indicate velocity as a function of time, the ordinates when measured down from MN indicate values of ΔV, and thus by a suitable scale will give the time history of the pressure at the valve. It may be noted that the pressure may vary during this time according to MBR or MDR or any other curve, but that its maximum value is proportional

[1] The subject of water hammer has been experimentally investigated by Joukovsky of Moscow on pipes of 2-, 4-, 6-, and 24-in. diameter and with lengths ranging from 1,050 to 7,007 ft. He found the results to agree with the formulas given. For a résumé of his work see SIMIN, "Water Hammer," *Trans. Amer. Water Works Assoc.*, 1904.

to NR or to $(V' - V'')$, the same as if the valve movement had been made instantaneously. If the closure is complete, the value of V'' becomes zero. The essential difference, therefore, between this and instantaneous closure is not in the magnitude of the maximum pressure attained, but rather that the vertical lines in Fig. 165 (C) are replaced with curves, such as MBR in Fig. 166, and thus the time during which the maximum pressure is maintained is decreased.

Since any pressure at the valve is transmitted along the pipe with the acoustic velocity S, the curve in Fig. 166 not only indicates the history of the pressure at the valve C, but also the history of the pressure at any other portion of the pipe, the distance to which is X, the only difference being a time lag X/S. Furthermore, by the time the valve movement is completed and the pressure at the valve has reached the maximum value corresponding to NR, the first pressure wave set up will have traveled a distance ST_v. Thus the curve, with ordinates measured down from MN, also indicates the distribution of excess pressure along the pipe for a distance $MN = ST_v$. It may now be seen that with the departure from instantaneous closure, the "vertical front" of the pressure wave in Fig. 164 (a) is replaced with some form of curve.

Slow Closure.—The preceding discussion has assumed a closure so rapid, or a pipe line so long, that there was an insufficient length of time for a pressure wave to make the round trip before the valve motion was completed. The time required for a pressure wave to make the trip from the valve to the reservoir and back again is twice the value given by Eq. (145), that is $2T$ or $2L/S$. If the valve movement is completed within a time interval less than this, the maximum pressure attained is the same as if it had been made instantly. But if the time interval for the valve movement is greater than $2L/S$, then the maximum pressure attained is less than given by Eq. (146). Thus in Fig. 166, suppose that the time $2L/S$ is equal to OA. Then it may be seen that the wave of pressure "unloading" will reach the valve before the motion is completed and hence the pressure curve MBR is stopped at the point B. The subsequent pressure history is quite complex, but generally the pressure is not greater than the value corresponding to CB and it is always less than that represented by NR. But if the velocity curve had been MDR, the maximum pressure for this case would be a function of

CD. Hence as many values of the maximum pressure for slow closure may be found, as curves can be drawn betweeen M and R. Thus it is impossible to formulate any simple, general equation for the maximum pressure produced with a slow gate movement. The various formulas that have been proposed, but which are not given in this text, are necessarily restricted to certain special cases, and differ in their results because based upon different assumed conditions.[1]

It may be observed that the use of the terms rapid and slow with reference to the gate movement is purely relative. The criterion is as to whether T_v is less than, or greater than, $2L/S$. In the case of a very short pipe the value of $2L/S$ is so small that it is nearly impossible to close the valve quickly enough to produce water hammer of maximum intensity, while in a very long pipe line care must be taken not to do so. In most practical cases the time of valve movement is greater than $2L/S$.

For any valve closure, which is not instantaneous, the effect of the return "pressure unloading" wave is to cause the maximum rise in pressure to assume diminishing values as the intake is approached. Thus the hydraulic gradient for the excess pressure, for the actual case, is changed from that in Fig. 164 (*a*) so that it more nearly resembles the one designated "rejected load" in Fig. 164 (*b*).

It might be well to add that during a valve closure, the excess pressure produced in the line prevents the rate of discharge, and hence the velocity in the pipe, from diminishing directly with the area of the opening. Thus it is not easy to determine the law of variation of the pipe velocity during this period. It may be seen that the normal head in the pipe also has an influence on this rate of discharge and its variation, and is involved in this indirect way.

Water hammer may be prevented by the use of slow closing valves, or its effects diminished by the use of automatic relief valves which permit water to escape when the pressure exceeds a certain value. Also air chambers of suitable size provide cushions which absorb a great portion of the shock. But for water power plants a standpipe or surge chamber such as is shown in Fig. 164 (*b*) has certain marked advantages.

[1] For a more adequate treatment of this entire subject see Durand, W. F., "Hydraulics of Pipe Lines," and Gibson, N. R., *Trans. A. S. C. E.*, vol. 83, p. 707, 1919–1920.

In the event of a sudden decrease in load on a water power plant it would be necessary for the governors rapidly to reduce the amount of water supplied to the wheels, if the speed of the latter is to be maintained constant. A surge chamber provides a place into which this excess water may flow and thus avoids water hammer in the supply pipe. The inertia of the mass of water flowing down this supply pipe may be such as to carry the water level above the static level and produce an ascending hydraulic gradient. But this excess pressure acts as a retarding force on the mass of water in the pipe line and thus reduces its velocity. In any event the temporary water level in this surge chamber will be higher than the normal value, and hence it will reduce the velocity of flow too much. This will result in fluctuations of velocity in the pipe line accompanied by "surges" of the water level in the chamber until a condition of equilibrium is finally reached. The phenomenon is very similar to that of water hammer as there are periodic alternations in pressure and velocity, but the pressure variations are much less severe.

The surge chamber fulfils another valuable function in that it not only takes care of excess water in case of a sudden reduction of flow, but it also provides a source of water supply in the event of a sudden demand. When the load on the plant increases it is necessary to supply more water to the wheels at once. If the pipe line is long it may take some time to accelerate the entire mass of water, and in the meantime the head at the plant has dropped considerably in order to provide an accelerating force. But the surge chamber permits a certain amount of water to flow out during that period. To be sure, enough flows so that the hydraulic gradient drops below its normal level for the new load, but the effect is not as serious as if the surge chamber were absent.

In Fig. 209 is shown a surge chamber of large size. It is at the end of a pressure tunnel which is approximately 7.76 miles in length, with an average cross-section area of 100 sq. ft. and in which is a maximum velocity of flow of 10 ft. per second.[1]

EXAMPLES

147. A cast-iron pipe line is 24 in. in diameter and the metal is 0.75 in. thick. If the velocity of water in it is 6 ft. per second, find the pressure that

[1] Durand, W. F., "Control of Surges in Water Conduits," *Jour., A. S. M. E.*, June, 1911. See also, Johnson, R. D., "The Differential Surge Tank," *Trans. A. S. C. E.*, vol. 78, p. 760, 1915.

would be created by the instantaneous closure of a valve.

Ans. 296.5 lb. per square inch.

148. If the pipe line in the above were 500 ft. long, within what length of time must the valve be closed to produce the same pressure as an instantaneous closure? What would the length of time be if it were 5,000 ft. long?

Ans. $T = 0.27$ sec., 2.7 sec.

139. PROBLEMS

149. If a jet of water strikes a body moving in the same direction and flows over it without friction loss, prove that $F = (wA_1/g)(1 - \cos \beta_2)(V_1 - u)^2$. If it acts upon a wheel under similar conditions, prove that $F = (wA_1/g)(1 - \cos \beta_2)V_1(V_1 - u)$.

150. Using the latter expression in the preceding problem for the force exerted upon a wheel, prove that the power developed is a maximum when $u = 0.5V_1$.

151. A water wheel is used under a net head of 450 ft. and runs at 350 r.p.m. The rate of discharge is 138 sec. ft. The angle $\alpha_1 = 15$ deg., $\beta_2 = 162$ deg., $r_1 = 2.0$ ft., $r_2 = 2.5$ ft., and $k = 0.2$. The wheel is of such a type that $p_1 = p_2$ and $z_1 = z_2$. Find v_1, v_2, V_2, α_2, torque, head utilized, hydraulic efficiency, and power delivered by water.

Ans. $v_1 = 101$, $v_2 = 105$, $V_2 = 33.4$, $\alpha_2 = 103$ deg. 40 min., $T = 93{,}300$, $h'' = 396$ ft., $e_h = 0.88$, hp. $= 6{,}220$.

152. A turbine runs at 660 r.p.m. under a head of 50 ft. and discharges 30 cu. ft. of water per second. It is of such a type that the water completely fills the passages and also $z_1 = z_2$. Its dimensions are $\alpha_1 = 35$ deg., $\beta_2 = 155$ deg., $r_1 = 0.70$ ft., $r_2 = 0.42$ ft., $A_1 = 0.83$ sq. ft., $a_2 = 0.88$ sq. ft., $k = 0.3$. If $p_1 = 30$ ft., find the value of p_2. Find the head utilized.

Ans. $p_2 = -2.9$ ft., $h'' = 42.6$ ft.

CHAPTER XII

DESCRIPTION OF THE IMPULSE WHEEL

140. Impulse and Reaction of a Jet.—When a stream of water strikes any object, the dynamic force exerted, due to the impact, is often termed the *impulse* of the jet. The dynamic force exerted by the jet upon the vessel from which it issues is often called the *reaction* of the jet. But in both cases the force is due to the change that is produced in the velocity of the water.

141. Distinction between Impulse and Reaction Turbine.—The distinction between these two fundamental types of turbines according to the action of the water as defined in the preceding article was proper in primitive wheels. But in modern turbines the so-called impulse at entrance and reaction at exit may both be effective in either type. A better classification is as to "pressureless" and "pressure" turbines.

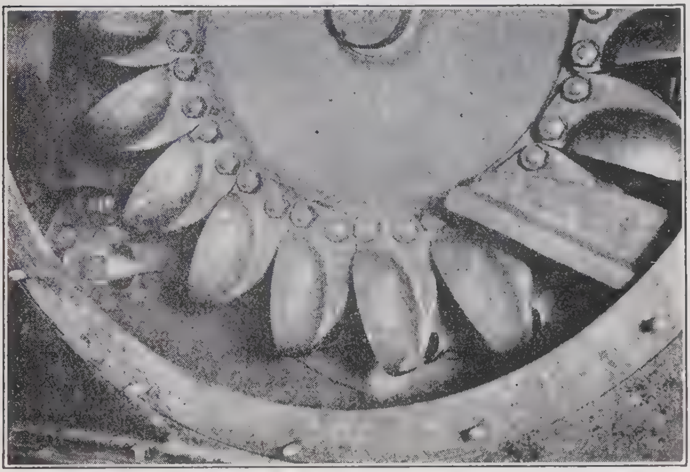
Thus the water within the impulse wheel is not confined but is open to the air, while in the reaction turbine the wheel passages must be completely filled with water. In the former the pressure remains unchanged in flowing over the buckets, while with the latter the pressure decreases during flow through the runner. The energy delivered to the impulse turbine is all kinetic, while that delivered to the reaction turbine is partly kinetic and partly "pressure energy."

But it is well to bear in mind that in both types the essential thing is that the velocity of the water must be altered in order that a dynamic force may be exerted upon the wheel. And in both types it is necessary if high efficiency is attained that the absolute velocity of the water as it leaves the wheel be low, since this velocity represents so much kinetic energy that is not utilized.

142. The Impulse Wheel.—There have been several types of impulse turbines produced, but the only one that has survived in this country is of the kind shown in Fig. 167. This is the impulse wheel or the Pelton wheel, so called in honor of L. A. Pelton who contributed to its early development. It may be

also designated by the name of the tangential water wheel, from the fact that the center line of the jet is tangent to the path of the center of the buckets.

The wheel in Fig. 167 is operated by a jet of water from the nozzle at the left. This same wheel in action may be seen in Fig. 217. A view of another wheel showing the relation of the nozzle to the buckets is shown in Fig. 168. The jet strikes the dividing ridge, or "splitter," of the buckets, is divided into two parts, flows over the face of the bucket, and is finally discharged at both sides of the latter.



From a photograph by the author.

Fig. 167.—Impulse water wheel with needle nozzle. (Needle is drawn back and nozzle is wide open.)

In Fig. 169 there may be seen a view of an assembled wheel with the "chain type" of construction. That is, each bolt is instrumental in holding two buckets, so that the latter are fastened together as a chain. This permits of a compact construction and enables the buckets to be placed closer together than in the type shown in Fig. 167.

The device shown at the right in Fig. 167 is the "stripper," its function being to prevent water being carried around with the wheel and thus adding to the windage losses. The buckets pass through an opening in this with a clearance of about 0.5 in.

143. Buckets.—Typical styles of buckets now in use for impulse wheels are seen in Figs. 170 and 171. The theory shows that the face of the bucket should be a surface of double curvature, and it is also found that the shape of the back of the bucket may be as important as that of the face. The reason for this is that the back of the bucket may interfere with the water

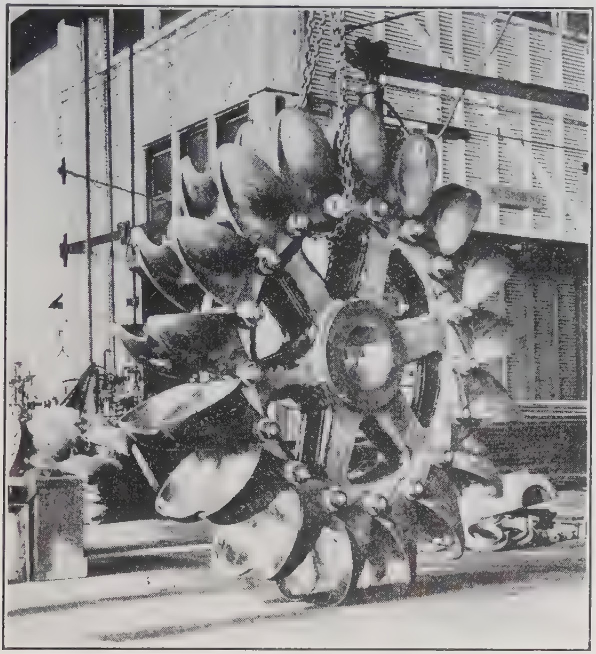
From a photograph by the author.

FIG. 168.—Impulse wheel viewed from below. (Nozzle closed by needle.) $D = 84''$; $h = 134'$; $N = 124$; $Hp = 280$.

which is acting upon the bucket ahead, for when a bucket swings down into the jet it merely cuts off the jet from the preceding bucket and leaves a "slug" of water to complete its work on the one ahead. If the back of the bucket is not properly shaped it may not leave sufficient clearance for the water. The "notch" is cut out of the Pelton bucket so that it may reach a posi-

tion where its path is more nearly tangent to that of the jet before the latter strikes it.[1]

For service under moderate heads these buckets may be made of cast iron, though the better ones are of bronze or steel. For very high heads only the latter may be employed. The working face of the bucket should be smoothed up or polished and the dividing edge, or "splitter," ground to a knife-edge in order to reduce hydraulic friction losses.

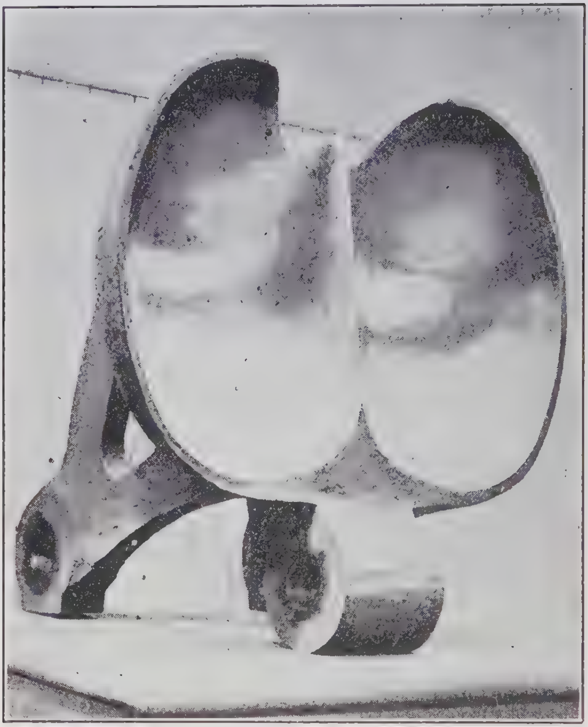


From a photograph by the author.

Fig. 169.—Pelton wheel, in shop of Pelton Water Wheel Co. $D = 76''$; $h = 540'$; $N = 257$; $Hp = 2{,}100$.

For high efficiency it is desirable that the bucket reverse the relative velocity of the jet as nearly as is feasible. But a complete reversal of 180 deg. is not permissible, a the water must be

[1] For impulse wheels of high specific speeds there are other reasons for this construction which space forbids taking up here in detail. In brief, it is so that every bit of water may complete its work upon the bucket before the latter leaves the line of action of the jet, in which event some of the water would not be utilized (see Fig. 215).



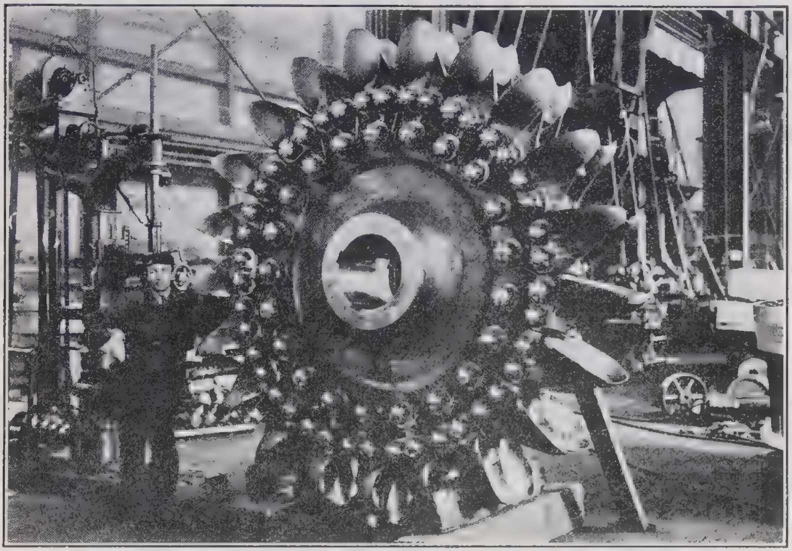
From a photograph by the author.

FIG. 170.—Pelton ellipsoidal bucket.

Courtesy Allis-Chalmers Mfg. Co.

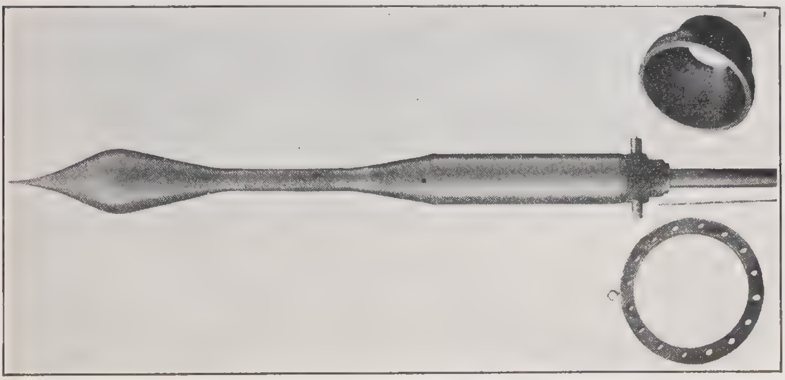
FIG. 171.—Allis-Chalmers buckets.

thrown to one side so as to clear the following bucket. An angle of about 165 deg. is usually employed, though even 170 deg. may



Courtesy Allis-Chalmers Mfg. Co.

FIG. 172.—Allis-Chalmers impulse wheel for Pacific Light & Power Co. $D = 94''$; $h = 1{,}860'$; $N = 375$; $Hp = 10{,}000$.



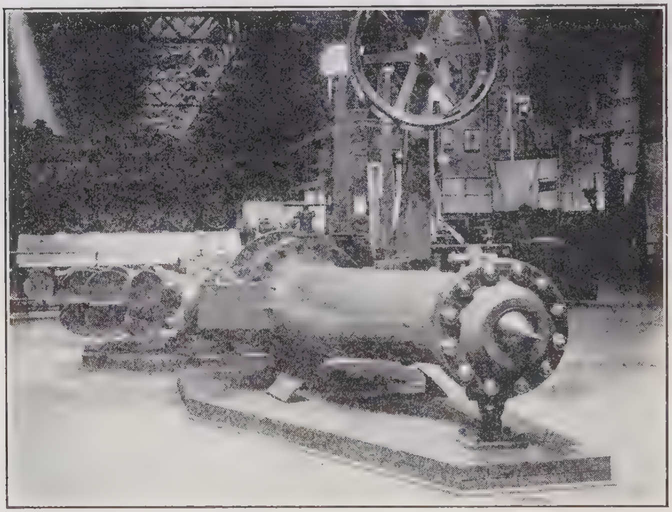
Courtesy of Pelton Water Wheel Co.

FIG. 173.—Pelton needle and nozzle tip.

frequently be used. Due to surface tension the actual direction of the water will always be somewhat less than the bucket angle,

the difference between the two decreasing as higher heads are used. For good efficiency the width of the bucket should be at least three times the diameter of the jet, and the diameter of the wheel should be at least nine times that of the jet. (The usual ratio is 12 in the latter case.) Since jets 10 in. or more in diameter are in use, buckets of at least 30 in. in width are sometimes seen.

144. Nozzles and Governing.—The jets used in impulse wheels are almost always furnished by needle nozzles such as is shown in Figs. 174 and 175. The needle itself is shown in Fig. 173.



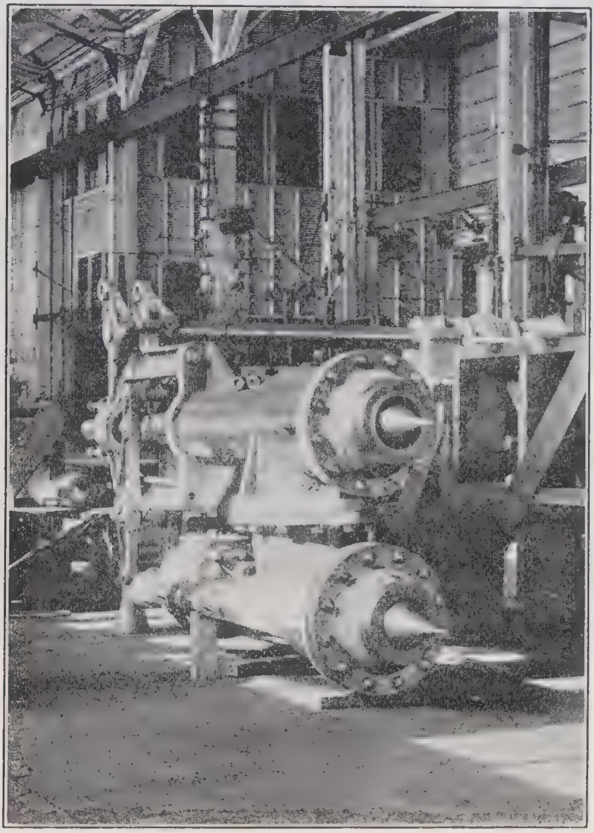
From a photograph by the author.

Fig. 174.—Deflecting needle nozzle for a 10,000 hp. jet.

As it is moved back and forth in the nozzle it varies the size of the nozzle opening and hence varies the amount of water discharged. But fortunately it does not involve any serious loss of head until the nozzle is nearly closed. The efficiency of a needle nozzle when it is wide open may be about 97 or 98 per cent, the velocity coefficient being about 0.99 or a little less. The nozzle efficiency would not fall below 90 per cent until the needle was closed so far that about half the maximum amount of water was being discharged. Thus it is a very efficient regulating device.

In order to keep the speed of a wheel constant under different loads it is necessary to vary the amount of water so that the

power supplied to the turbine will be proportional to the power demanded. This can sometimes be done by changing the position of the needle in accordance with the power the wheel must deliver. Under certain conditions the governor may control the position of the needle for this purpose. But if the changes of



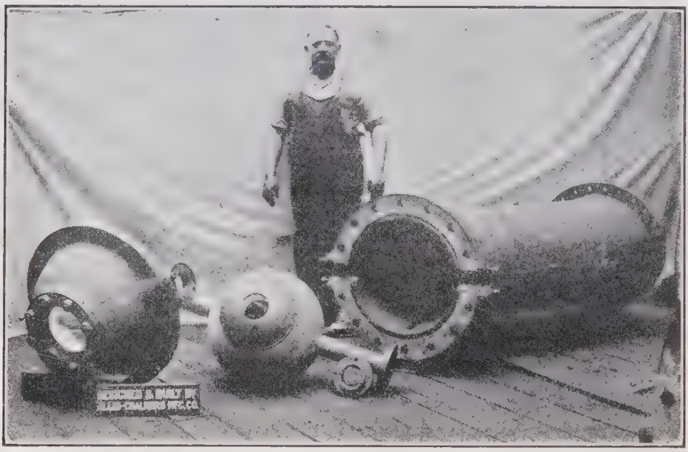
From a photograph by the author.

Fig. 175.—The needle nozzle with auxiliary relief.

load are rapid and the pipe line is long, this procedure would involve serious water hammer, if close speed regulation were attempted.

In order to secure close speed regulation and yet be free from the danger of water hammer, the deflecting nozzle is often used. The entire nozzle is movable about a ball and socket joint near

the base and swings on trunnions. In case of a sudden drop of load on the machine the governor could lower the end of the nozzle so that only a small part of the jet struck the buckets, the rest of the water being wasted. As the load increased the nozzle could be raised so that a larger amount of water would strike the wheel. As this would be wasteful of water, such nozzles are almost always equipped with needles as well, which can be set by the station attendant in accordance with the load the wheel carries. Thus water would be wasted for a short time only, but the needle would be closed so slowly that no damage would be done to the pipe line. But the nozzle may be deflected



Courtesy of Allis-Chalmers Mfg. Co.

Fig. 176.—Needle nozzle with deflecting tip.

with any degree of rapidity so that close speed regulation may be secured. Of course, in case of an increase in load it would be necessary for the operator to open the nozzle, as the governor is powerless there. But the experience is that increases of load come on gradually enough for this to be done. The chief function of the governor is to prevent racing in cases of abrupt decreases in load. Occasionally the nozzle is so made that the governor deflects it first and then slowly closes the needle.

The needle nozzle with an auxiliary relief, as shown in Fig. 175, is frequently used. In this type the jet from the upper nozzle strikes the wheel while that from the lower nozzle goes below it. It is so arranged that when the governor closes the upper nozzle

it opens the lower one. Thus there is no abrupt change in flow in the pipe line as the surplus water simply flows out through another place. But in order to prevent waste of water, the connection between the governor and the auxiliary nozzle is a dashpot arrangement which permits the needle to be moved only when the governor movement is rapid, and when the relief

From a photograph by F. H. Fowler.

FIG. 177.—DeSabla power plant in normal operation. Under head of 1,531 ft.

has been opened this arrangement permits it to be gradually closed again. Thus close speed regulation has been accomplished and also economy in the use of water has been secured.

The nozzle shown in Fig. 176 is similar in principle to the deflecting nozzle in that the jet is deflected below the wheel. But it is so constructed that only the tip of the nozzle has to be moved rather than the entire nozzle. This has certain advantages.

It will be noted that all of these devices may prevent rapid changes in flow in the pipe line in the case of decreasing loads. But only a surge chamber located near the wheels will be able to supply water in the case of a sudden demand.

145. Conditions of Service.—The impulse wheel is well adapted for service under high heads, though it may also be employed under low heads if the power is light. In fact the choice of the type of turbine is a function of power as well as head.

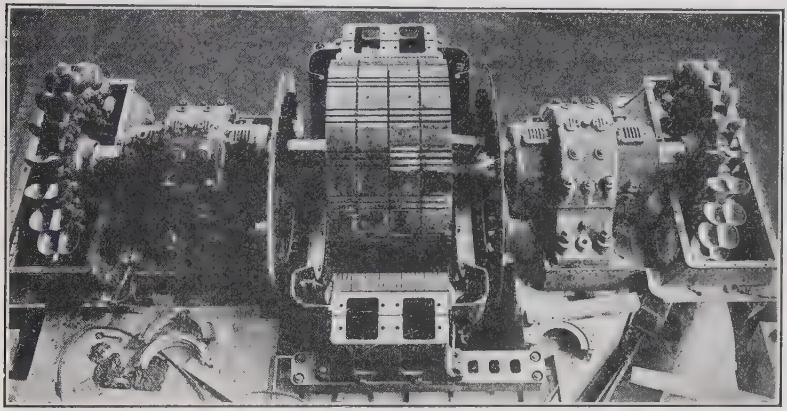
From a photograph by F. H. Fowler.

FIG. 178.—DeSabla power plant with nozzles deflected.

The highest head that has ever been developed is in Switzerland where 15,000 hp. is generated under a head of 5,412 ft. The power of each wheel in the plant is 3,000 hp., its diameter is 11.5 ft. and it runs at 500 r.p.m. The diameter of the jet is 1.5 in.

In this country the highest head that has been used is 2,100 ft., and heads ranging from 1,000 to 2,000 ft. are not uncommon. A plant under 2,100 ft. static head is shown in Fig. 212.

The jets used upon impulse wheels are of all sizes up to about 10 in. or a little over. Ordinarily only one jet is used with a single wheel, but occasionally two or more nozzles may be



Courtesy of Allis-Chalmers Mfg. Co.

Fig. 179.—Double overhung Allis-Chalmers wheels for Pacific Light and Power Co. $D = 94''$; $h = 1,860'$; $N = 375$; $Hp = 20,000$ (for unit).

employed, though at a slight sacrifice of efficiency. In order to increase the power of a single unit two separate wheels are often used on the same shaft, as in Fig. 179.

The largest power developed by a single impulse wheel with one jet upon it is 15,000 hp. A wheel of 10,000 hp. is shown in Fig. 172, though there are several other cases where the power of a single wheel has approached such a value.

CHAPTER XIII

DESCRIPTION OF THE REACTION TURBINE

146. The Reaction Turbine.—With the impulse turbine, in general, water may be admitted by a series of nozzles around the circumference of the wheel, but with the usual type of Pelton wheel one nozzle only is employed, though occasionally two or more may be used. Thus only a small portion of the buckets on the wheel are in service at a time. But by definition the passages of a reaction turbine must be completely filled with water, and in order that this condition may be fulfilled it is necessary that water be admitted around the entire circumference. Since every vane or bucket is in continuous use, it may be seen that more power may be developed upon a reaction turbine of a given diameter than is the case with a Pelton wheel.

The general arrangement of one type of reaction turbine is shown in Fig. 180. Water under pressure enters a spiral casing which encircles the unit. Since water is flowing into the runner around its entire circumference the cross-section area of this casing may decrease in proportion to the decreasing volume of water to be handled. From the casing the water flows through passageways between guide vanes which serve to give it a direction as near tangential as is practicable, and then it flows through the runner and into the draft tube. A cross-section of the runner showing the buckets or vanes may be seen in the figure. It may be noted, however, that the vane, being a warped surface, will not lie in the plane of the paper and hence must be portrayed by circular projection. That is, points along both edges may be rotated about the axis until they fall in the plane of the paper. In most cases, the entrance edge may lie in one plane containing the axis of rotation, and the discharge edge may lie in another plane also containing the axis, but this is not always true. The view of the vane or bucket shown is thus not a true cross-section and is more properly termed a "profile."

Additional features shown in Fig. 180 are the thrust bearing which carries the entire rotating weight of turbine and generator,

the servo-motor, and the stay vanes. The servo-motor is a cylinder containing a piston actuated by oil under pressure and controlled by the governor. It serves to operate a mechanism which rotates each guide vane about its own axis for the purpose of regulating the water supplied to the wheel. In the case of a large unit, such as shown in this figure, the stay vanes are introduced as columns to assist in sustaining the load above. But

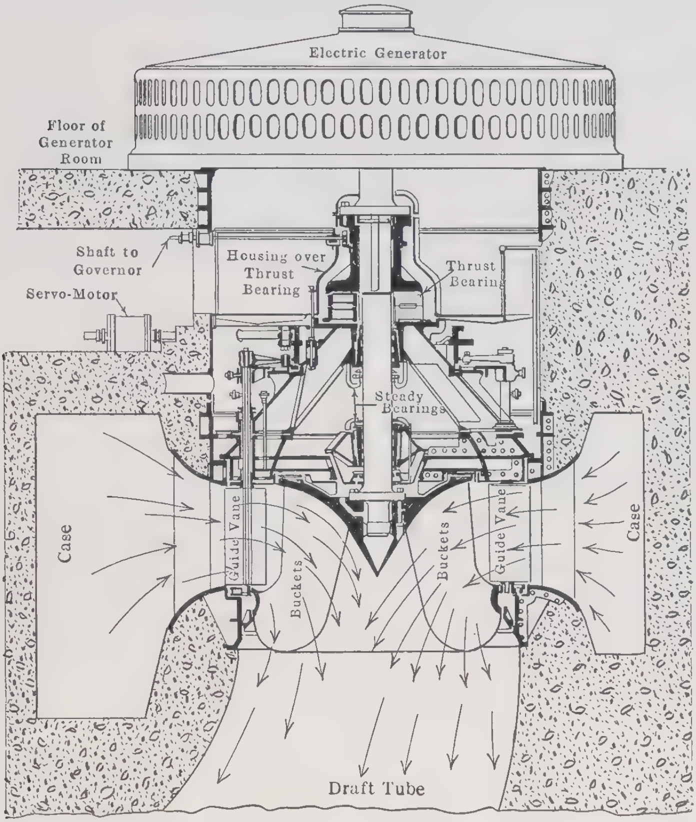


FIG. 180.—Reaction turbine.

instead of being round columns, which would produce eddy currents, they are made as vanes with a curvature conforming to the natural stream lines. These stay vanes are outside of the guide vanes, and the assembly, surrounding the guide vanes, is sometimes called a "speed-ring," though for no apparent reason.

The shaft need not necessarily be vertical, but may be horizontal, as in Fig. 181. The latter also shows a casing of metal instead of one formed in concrete, but the relative arrangements of case, guides, and runner, and the conditions of the flow of water through it, are the same whatever the setting. The difference is purely mechanical.

The leakage of water from the casing to the draft tube past the runner cannot be prevented, but may be kept within reasonable limits by having a small clearance between the rotating runner

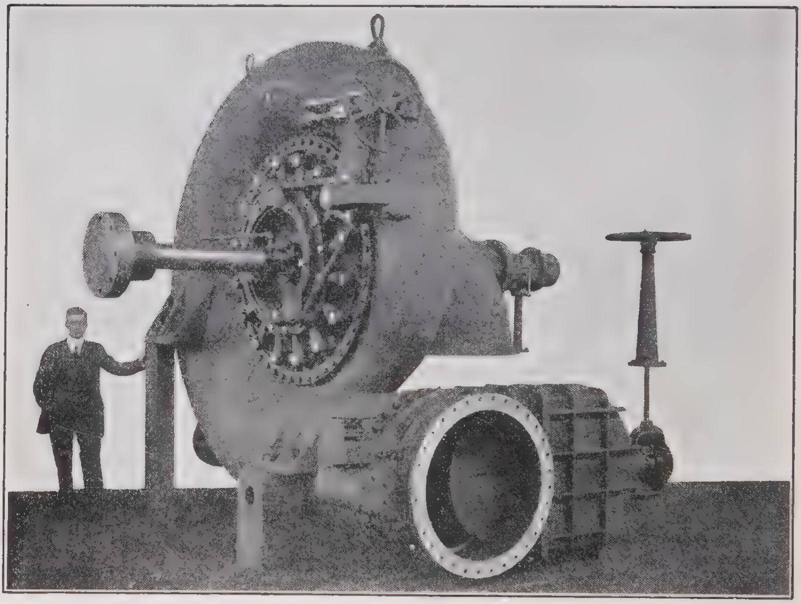


Fig. 181.—Spiral case turbine showing main gate valve, shifting ring and links for guide vanes, and draft elbow.

and the case. The construction in Fig. 182 minimizes this leakage by introducing a tortuous labyrinth passage for the water to traverse.

The arrangement of another reaction turbine with a horizontal shaft may be seen in Fig. 183. This particular one is of the open-flume type and is set so as to be completely surrounded by water in a manner similar to the vertical-shaft turbine shown in Fig. 198. The water flows through the stationary guide vanes and enters the runner, which is in the center. In Fig. 183 there are

two runners set on the same shaft and discharging into a common draft chest, from which the water flows down to the tail race through a draft tube.

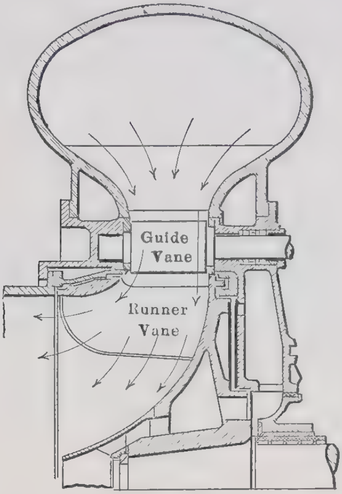
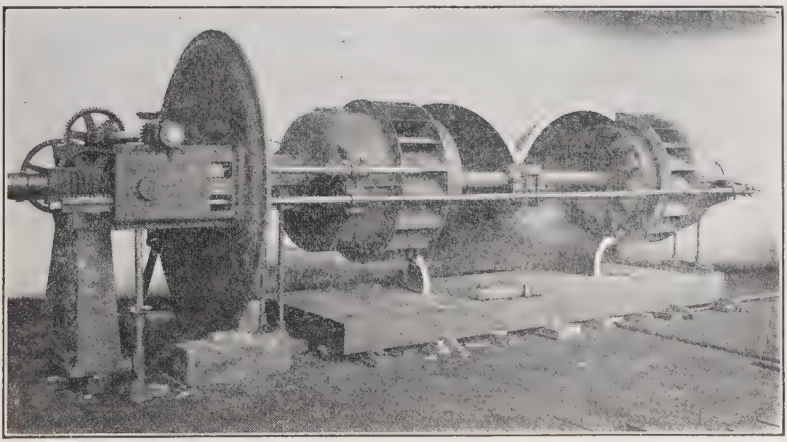


FIG. 182.—Labyrinth seal.



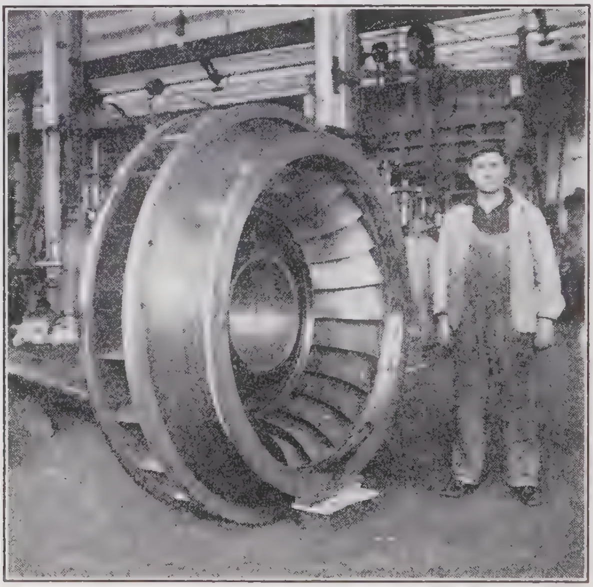
Courtesy of Platt Iron Wks. Co.

FIG. 183.—Turbine with cylinder gates for open flume.

147. Runners.—The part of the turbine upon which the water does its work is called the runner. Runners may be built up of separate pieces of metal which are welded together, but they

are usually cast in one piece. Occasionally they are built in sections and the sections bolted together. For large sizes and low heads cast iron is employed. Better runners are made of bronze and occasionally cast steel is used for high heads.

Runners differ considerably in their proportions and appearance. One extreme is shown in Fig. 184 while the other extreme is shown in Figs. 186, 187, and 188. It may be noted that the runners in Figs. 184 and 187 develop the same amount of power though differing widely in size. This is due to the fact that the



Courtesy of Pelton Water Wheel Co.

FIG. 184.—Low-speed turbine runner. $D = 74''$; $h = 487'$; $N = 360$; $Hp = 20{,}000$.

smaller runner operates under a much higher head and consequently needs to discharge less water for the same amount of power. And a still larger runner, which is shown in Fig. 186, develops less power than either of the others because it is under a still lower head.

It may be noted that the width of the runner parallel to the shaft in Fig. 184 is a very much smaller proportion of the diameter of the runner than in the type shown in Fig. 186.

Sometimes runners are of the double discharge type as in Fig. 185 which is equivalent to placing two single discharge runners

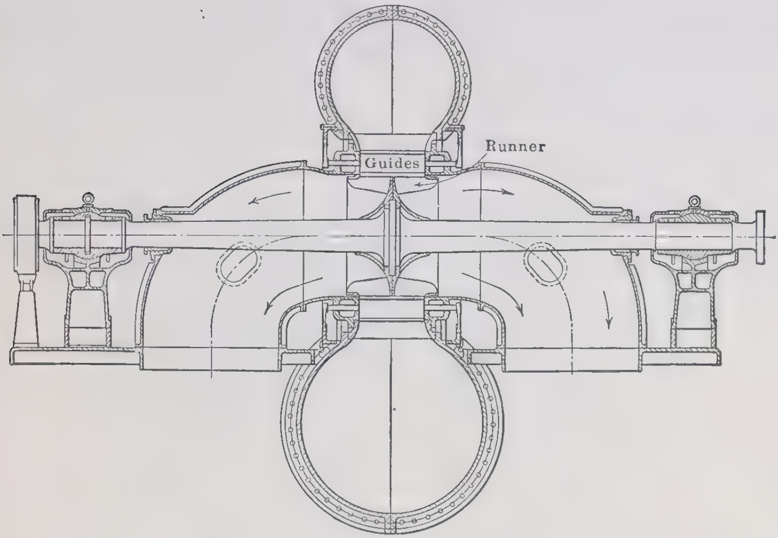


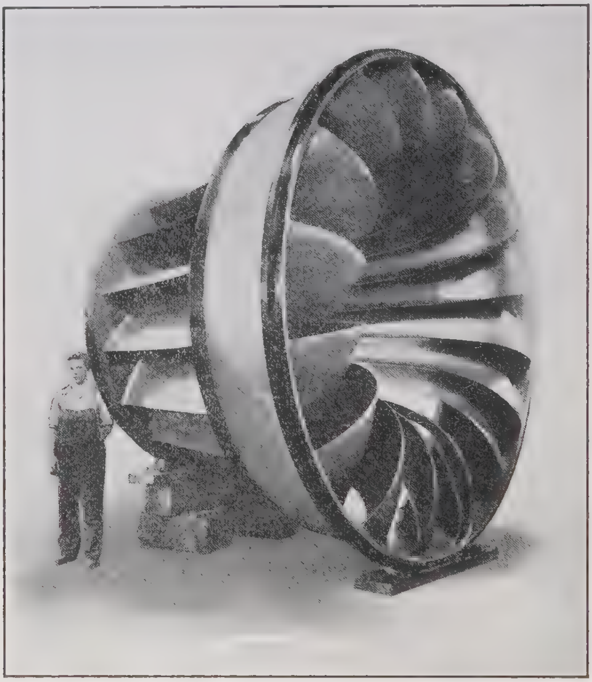
Fig. 185.—Double discharge turbine.

From a photograph by the author.

Fig. 186.—One of the world's large turbine runners. For Cedars Rapids Mfg. and Power Co. $D = 143''$; $h = 30'$; $N = 55.6$; $Hp = 10{,}800$.

back to back. Such a turbine must have two separate draft elbows.

The nominal diameter D of a turbine runner is that shown in Fig. 190 (a) and (b), and is not necessarily the maximum value. This is the dimension that will be found in the title of Fig. 186, for instance, but the maximum diameter is 17 ft. 7 in.



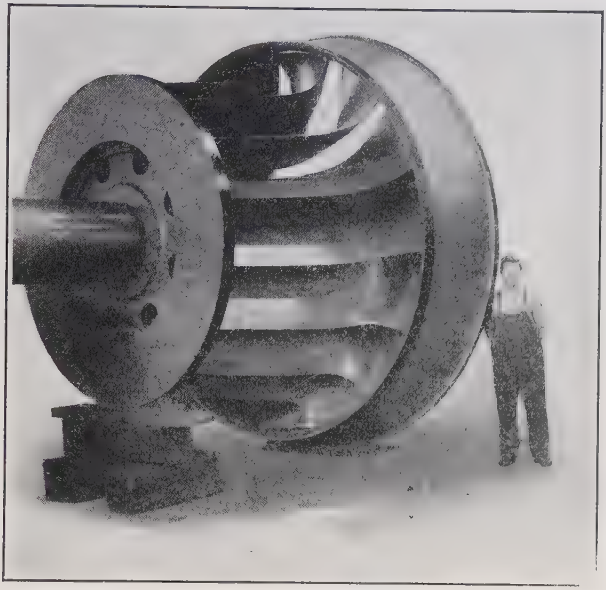
Courtesy of I. P. Morris Co.

FIG. 187.—High-speed turbine runner. $D = 102''$; $h = 76'$; $N = 120$; $Hp = 20{,}000$.

Since the runner always moves in a direction opposite to that of the relative velocity at discharge, it may be seen that the runners in Figs. 184 and 187 would turn counterclockwise, as viewed in the picture, while the runner shown in Fig. 186 would turn clockwise if viewed from above, as does likewise the one in Fig. 188.

The first reaction turbines were of the outward-flow type, that is the water enters the runner on the inside and flows towards

the outer radius. But the inward-flow type is the only one that is of any importance at the present time, all others having been eliminated because of certain disadvantages, mostly of mechanical construction.



Courtesy of I. P. Morris Co.

FIG. 188.—Turbine runner for Laurentide Co.

An inward-flow turbine runner was proposed by Poncelot in 1826, but the first one was built and patented by Howd in 1838. In 1849 J. B. Francis constructed a pair of pure radial inward-flow turbines from the Howd patent but his wheels were of much better design and workmanship. Because of the publicity given to these wheels due to the very precise tests which he conducted on them, his name became attached to them and today modern inward-flow turbines are known as Francis turbines, even though they may differ considerably from his original design, which is shown in Fig. 189, and was of a pure radial-flow type.

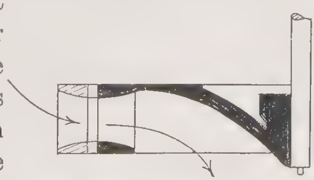


FIG. 189.—Francis turbine.

By radial flow is meant that a particle of water, during its flow through the rotating runner, remains in a plane which is normal to the axis of rotation, so that its position changes only with respect to its distance from the axis of rotation. In the evolution of the modern turbine it became desirable to have the water enter the runner with a "radial" flow and then to turn and flow in such a manner that a component of its velocity might be parallel to the shaft. In fact some of the particles of water, at

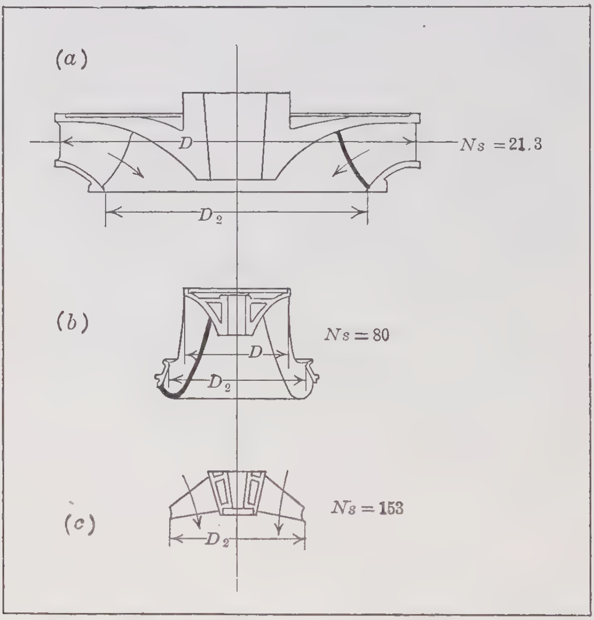


Fig. 190.—Relative size of Francis and propellor type runners for same power under same head.

least, before they reached the discharge edge of the bucket or vane might be following paths which lay on the surfaces of cylinders concentric with the axis. This is known as mixed flow, and such a type of turbine is sometimes called the American turbine, though the name Francis is generally extended to cover all inward-flow wheels. In Figs. 184 and 190 (*a*) may be seen the nearest approach in present practice to a radial-flow runner, while in Figs. 187 and 190 (*b*) may be seen the mixed-flow type.

The evolution of the type in Fig. 190 (*b*) was due to the demand for both higher power and speed. For a given head, a higher

power requires a larger diameter, but a higher rotative speed requires a smaller diameter. In order to meet these conflicting requirements, it was therefore necessary to modify the type shown in Fig. 190 (*a*) by decreasing the diameter to secure higher rotative speed, and at the same tiime extending the dimension parallel to the axis in order to maintain the capacity. Certain other alterations which do not appear in this profile view accompanied these changes, so that the same power under the same head may be obtained with runners which differ from each other very materially in appearance and in regard to their proper rotative speed. With a continued demand for still higher power and speed under low heads, a new type indicated in Fig. 190 (*c*) has now been produced, called the propeller type, because it resembles, to some extent, a ship propeller. The flow through it is practically axial, that is each particle of water moves in a path which is at about a constant radius from the axis of rotation of the wheel.

In Fig. 190 each runner will develop the same power under the same head, but the size will vary as shown. The actual rotative speed is proportional to the numerical values there designated by N_s. The latter is not, however, the actual speed of the runner, but rather a factor indicating the type of the turbine. Although called "specific speed" it involves the head and power as well as the speed of rotation, as may be seen in Art. 171.

The runner in Fig. 184 is described as a low-speed runner though its actual speed is 360 r.p.m. while that in Fig. 186 may be called a high-speed runner though it runs at only 56.6 r.p.m. But the use of these terms is relative. Thus if a turbine of the first type were to be built to deliver the same power under the same head as the second, it would run at only 17 r.p.m. Hence our terms of "low-speed" for the former and "high-speed" for the latter are seen to be justified. If it were physically possible to use a Pelton wheel for the same power and head as the turbine in Fig. 186 it would run at only 3 or 4 r.p.m. Thus the Pelton wheel may be seen to be a lower speed wheel even than any reaction turbine. If actual rotative speeds encountered in practice are higher, it is because such wheels are used under higher heads.

148. Gates and Governing.—The quantity of water passed through the turbine is regulated by means of gates, of which there are several kinds. In Fig. 183, the cylinder gate is used. In

that class of turbine the guide vanes surrounding the runner are absolutely fixed. Between the ends of these vanes and the runner is a metal cylinder which may slide along parallel to the shaft. If moved in one direction it admits water to the runner and may be so far withdrawn as to offer no obstruction whatever between the guides and the wheel. And if it is moved in the other direction it is possible to shut off the water altogether.

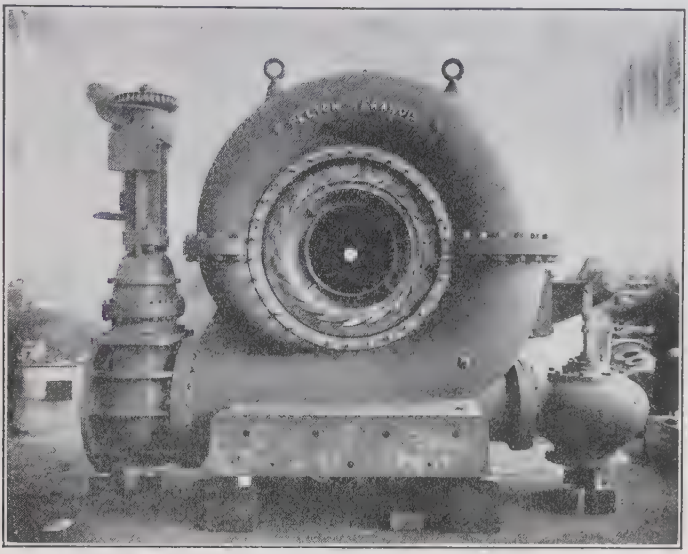
Courtesy of Pelton Water Wheel Co.

FIG. 191.—Wicket gates or swing gates.

This style of regulation causes the turbine to have a poor efficiency on "part gate," which is the term used when the turbine is running under less than full load. But such a style of gate permits a turbine to be constructed at less cost.

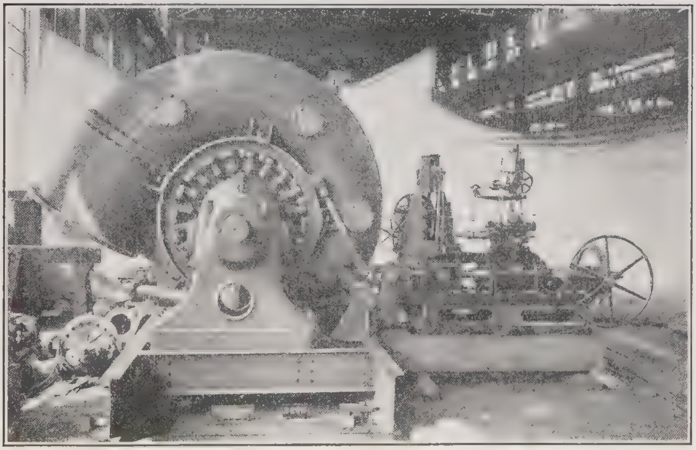
The better type of gate so far as efficiency is concerned is the kind shown in Fig. 191. Here the guide vanes themselves are

movable and by rotating about their axes they may vary the size of the area through which water may flow. This means that



Courtesy of Pelton Water Wheel Co.

FIG. 192.—Spiral-case turbine showing swing gates.



Courtesy of Platt Iron Wks. Co.

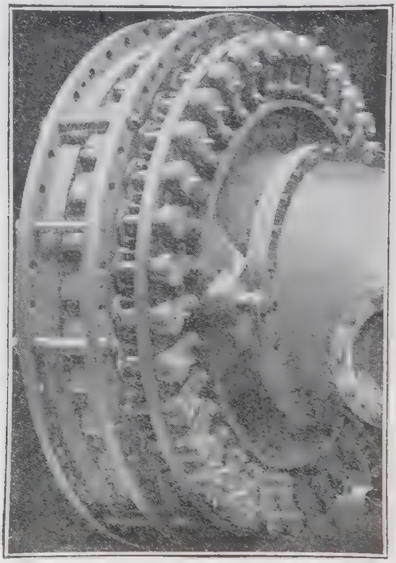
FIG. 193.—Shifting ring for operating gates.

the angle α_1 changes. These gates are known either as swing gates, wicket gates, or pivoted guide vanes. They involve more

expensive construction than the cylinder gate but are vastly better if economy of water is any object.

In Fig. 192 may also be seen some movable gates as they are installed in the turbine. The runner is to go into the space in the center.

The swing gates are operated by moving a "shifting ring" to which each gate is attached by links. In Fig. 193 may be seen the rods from the governor connected to this ring so that, when it is moved slightly with the turbine shaft as a center of rotation.



Courtesy of Allis-Chalmers Mfg. Co.

FIG. 194.—Swing gates.

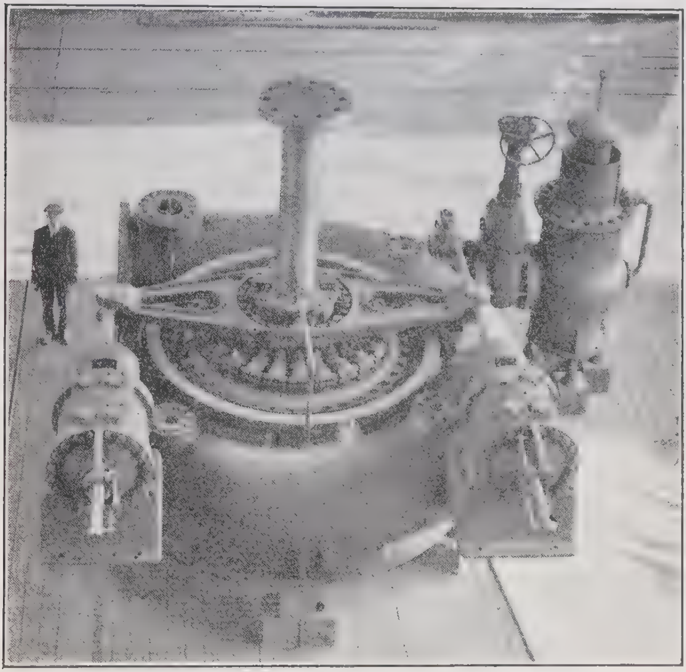
each gate will be turned through some angle. The links which connect the gates to this ring can be seen more clearly in Fig. 194.

The problem of governing a reaction turbine is similar to that of the impulse wheel. When the governor closes the gates and thus reduces the discharge through the turbine it is necessary to provide some by-pass for the water in order to prevent water hammer in the pipe line. The usual practice is to use a relief valve such as that shown over at the right in Fig. 195. When the governor closes the gates it opens the relief valve at the same time, and the water coming down the pipe is then discharged

through this into the tail race alongside the draft tube. The action of such relief valve may be seen in Fig. 196.

The connection between the governor and the relief valve is usually not a rigid connection, in order that the relief valve may slowly close and prevent the waste of water.

149. The Draft Tube.—The water is conducted from the turbine to the tail race through a draft tube, which may be constructed of riveted-steel plates as in Fig. 196 or may be molded in

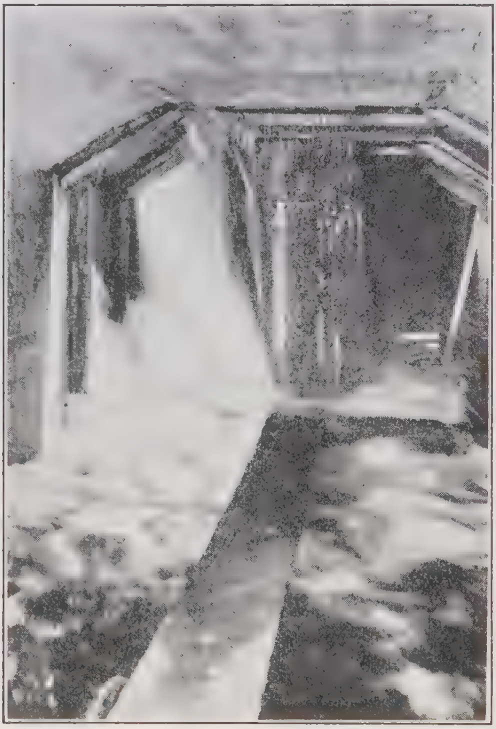


Courtesy of S. Morgan Smith Co.

Fig. 195.—Tallulah Falls turbine showing gate mechanism and relief valve $h = 580'$; $N = 514$; $Hp = 19{,}000$.

concrete as in Fig. 199. The draft tube should be made airtight so that a partial vacuum can exist within it and thus there may be a "suction" produced on the discharge side of the runner which will compensate for the elevation of the latter above the tail water level. By the use of the draft tube it is possible to set the turbine at a convenient distance above the water level without losing any head thereby.

But this is not the sole function of the draft tube. The velocity with which the water is discharged from the runner represents kinetic energy that is not utilized and such a loss cuts down the efficiency of the wheel. If the draft tube is made to diverge, the velocity at its mouth will be much less than that with which the water enters it from the runner and hence the kinetic energy finally lost may be much reduced. With some types of turbine



From a photograph by the author.

FIG. 196.—Discharge from relief valve when opened.

runners it is necessary to allow the water to be discharged with a relatively high velocity and such wheels would not possess favorable efficiencies if it were not for the use of suitable draft tubes. The usual rate of diffusion provided for is such that a circular tube will be made a frustum of a cone the vertex angle of which is 8 deg. Some experiments by the author indicate that

a larger angle than this might be permissible. For a given rate of diffusion the longer the tube the greater the reduction of the kinetic energy of the water. Therefore, in some cases it is desirable to have a long draft tube even though the runner might be set very near the water level.

On account of the function which is fulfilled by the draft tube it is properly regarded as an integral part of the turbine. Considering the turbine and the draft tube as a unit, it may be seen that the less the kinetic energy lost from the mouth of the tube the higher the efficiency of the wheel, thus justifying the statement of the preceding paragraph. But it is not yet clear just how this saving in the draft tube enables the turbine itself to deliver more power until the effect of the draft tube upon the pressure at the exit from the runner is considered. The less the losses within the draft tube and the less the discharge loss from the draft tube the less the pressure will be at the point of discharge from the runner, thus increasing the effective head on the runner.

With a low-speed type of turbine, the water discharging from the runner at full load may have very little, if any, rotation, but at any other gate opening it may be found that the water is rotating in the draft tube as well as flowing parallel to the axis of the latter. And in the case of modern high-speed wheels, there is always a condition of whirl to be found in the draft tube at all loads. A condition is thus found approaching that of the free vortex described in Art. 136. In order to conserve and efficiently transform the whirl component of the velocity, the type of draft tube shown in Fig. 197 may be used. In some cases, where mechanical construction permits, it would be desirable to have this inner cone extend clear up to the runner itself. The reason for this is that, according to Art. 136, as the radius diminishes the velocity of whirl increases, and consequently the pressure decreases. But it cannot fall below the vapor pressure of the water, and consequently there is an unstable and turbulent condition established, which the introduction of a solid core prevents. It may be seen that as the water spreads out at the bottom of the tube, there exists the same case as pictured in Art. 136.

A similar effect to the Moody "spreading tube" is secured by the "hydraucone" draft tube of W. M. White where the cone is assumed to be formed of "dead" water.

150. Cases and Settings.—The turbine, draft tube, and all parts intimately connected with it comprise what is called the setting. Impulse wheels are almost always set with horizontal shafts, but reaction turbines may have either horizontal or vertical shafts. For large units under low heads the vertical shaft is the most recent practice, as it permits several desirable features to be attained.[1] Occasionally several runners may be mounted on the same shaft but the tendency is to eliminate such construction and have larger runners and fewer of them, and two on the

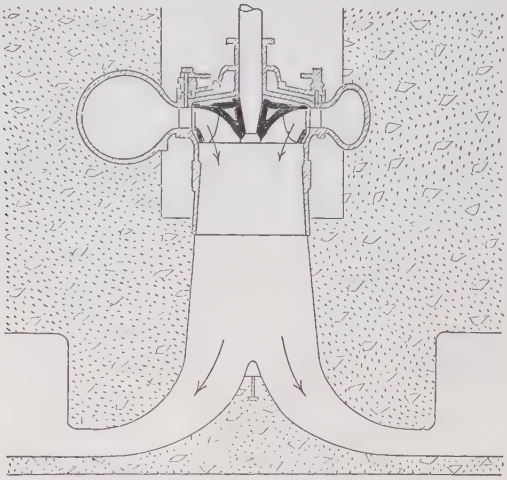


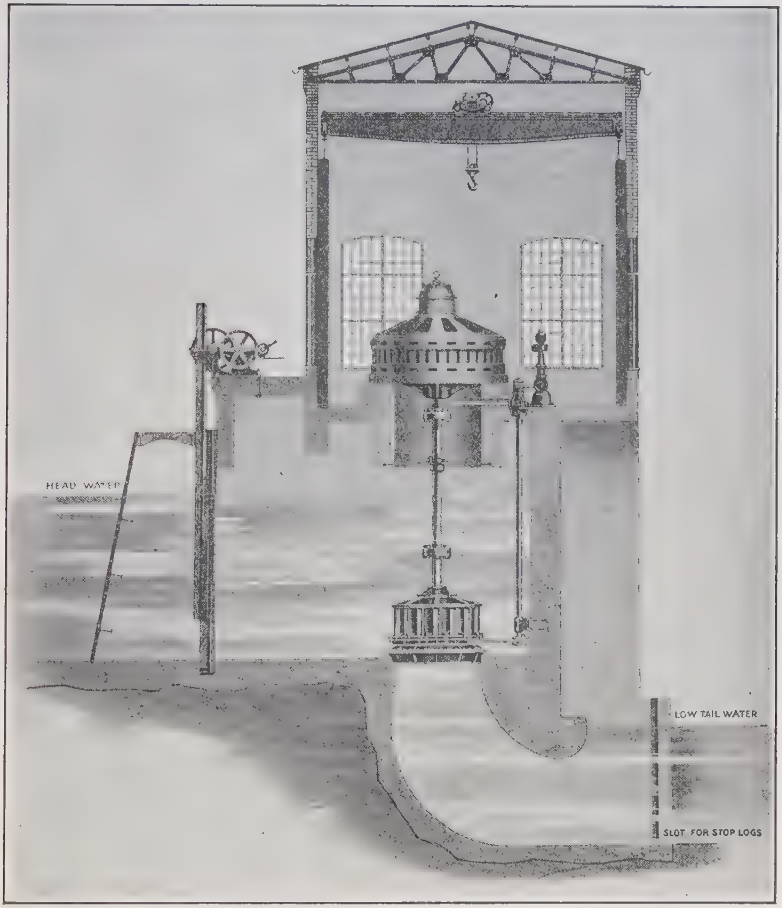
Fig. 197.—Moody spreading draft tube.

same shaft as in Fig. 183, is as many as are desirable. As in the case of the impulse wheel, there may be two separate turbines connected to a single generator.

Under low heads of not more than 20 or 30 ft., the open flume setting such as is shown in Fig. 198 is sufficient, but for higher heads this is not practicable. For either low or moderate heads the wheel may be enclosed within a concrete case as in Fig. 199, but the action of the turbine is no different from that in the preceding case. The water has no free surface immediately above the turbine but it is under practically the same pressure as if it did have. The only difference is that, since the area of the water passage is less than before, the velocity with which the water

[1] Taylor, H. B., "Present Practice in Design and Construction of Hydraulic Turbines," *Can. Soc. Civil Eng.*, Jan. 15, 1914.

approaches the turbine will be somewhat higher, and thus there will be a lesser rate of acceleration as the water enters the guide vanes. For still higher heads a concrete case would be unsuitable and then the guide vanes are surrounded by a metal case as shown in Fig. 200. The only difference between this

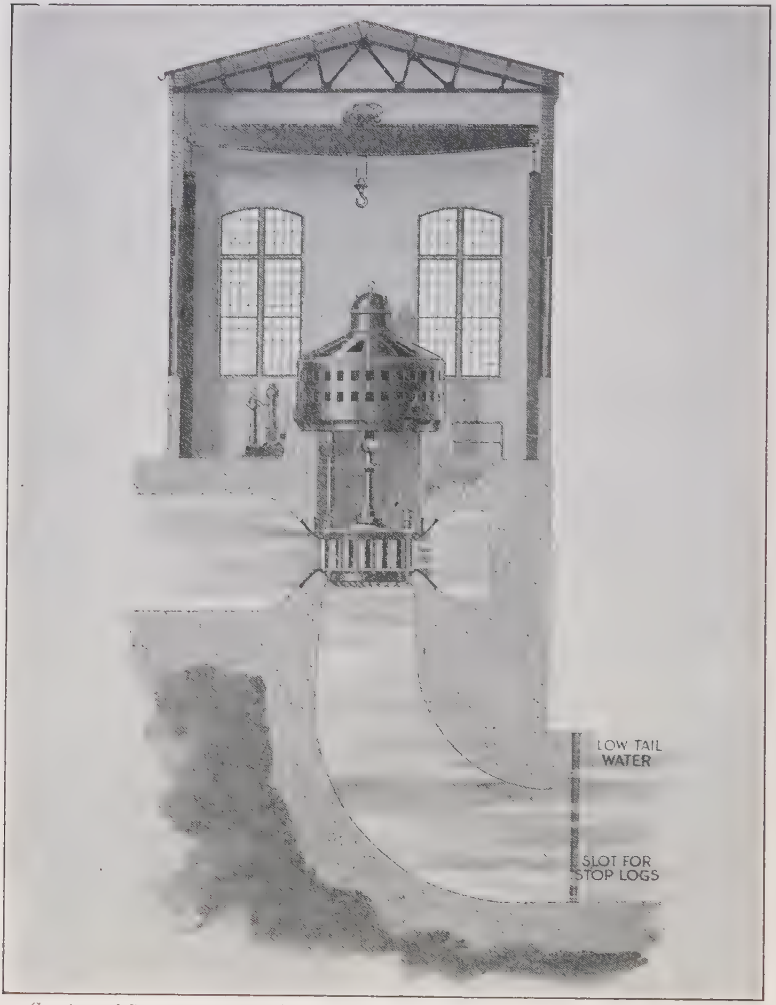


Courtesy of S. Morgan Smith Co.

Fig. 198.—Reaction turbine in open flume.

and the other two cases, so far as the hydraulics is concerned, is largely one of appearances, save that the velocity of the water as it approaches the guides may be somewhat higher owing to the smaller area.

In order that the water may have the same velocity of approach to the guides all around the circumference, the spiral case is frequently used. Cases of this type are illustrated in Figs. 181,

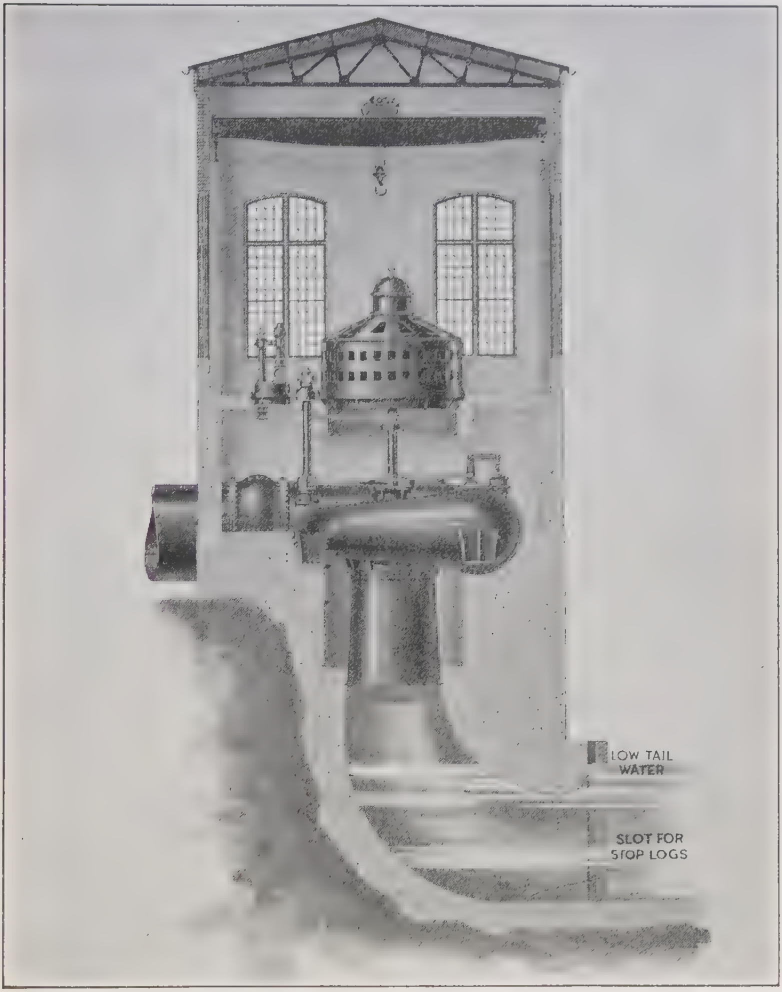


Courtesy of S. Morgan Smith Co.

Fig. 199.—Reaction turbine in concrete case.

192, 200, and 201. In Fig. 181 may also be seen the main gate valve which may be used to shut off the water more completely than is possible with the wicket gates, and on the right-hand

side may be seen a portion of the draft elbow. Very large cases are built in sections as shown in Fig. 201. The spiral case is considered the most desirable type though other less expensive ones are sometimes used.

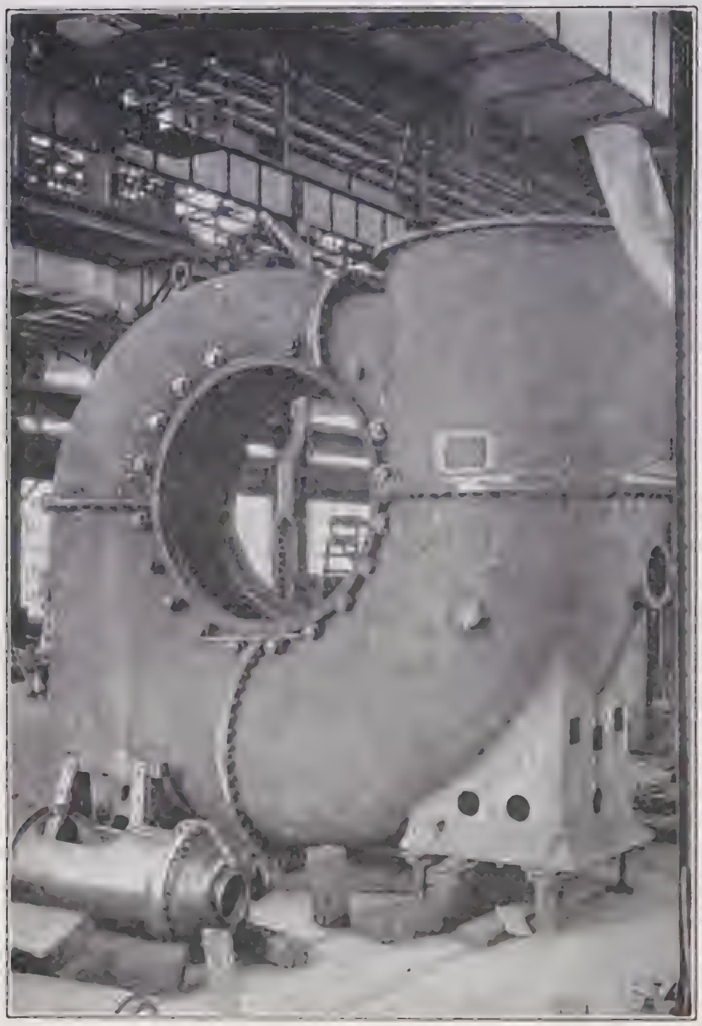


Courtesy of S. Morgan Smith Co.

Fig. 200.—Reaction turbine in metal case.

In Fig. 202, a glimpse is given into the intake of a large turbine set as in Fig. 199. In such a setting the runner and guide vanes

may be surrounded by a set of stay vanes, also called a "speed ring," such as shown in Fig. 203. The columns which support



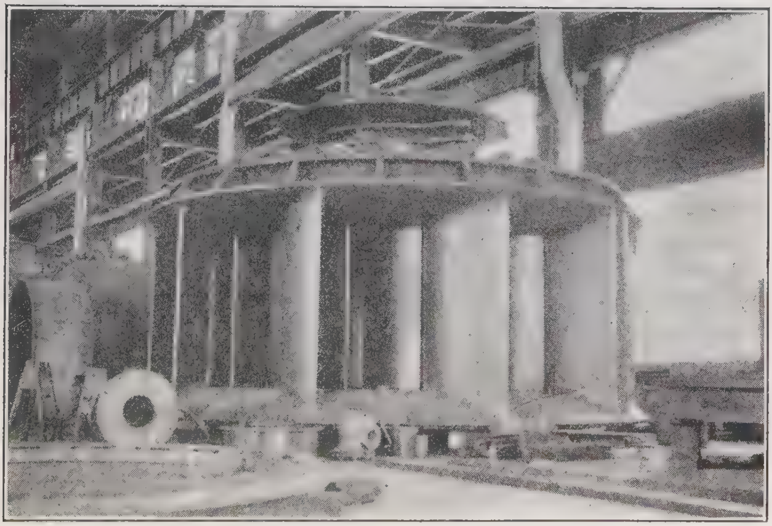
From a photograph by the author.

Fig. 20[illegible].—Large spiral case for Canadian Light and Power Co. in shop of I. P. Morris Co.

the upper crown plate and its load are made of a shape similar to guide vanes so as to reduce eddy losses and also to give the

Courtesy of Mississippi River Power Co.

FIG. 202.—Intake for turbine at Keokuk.

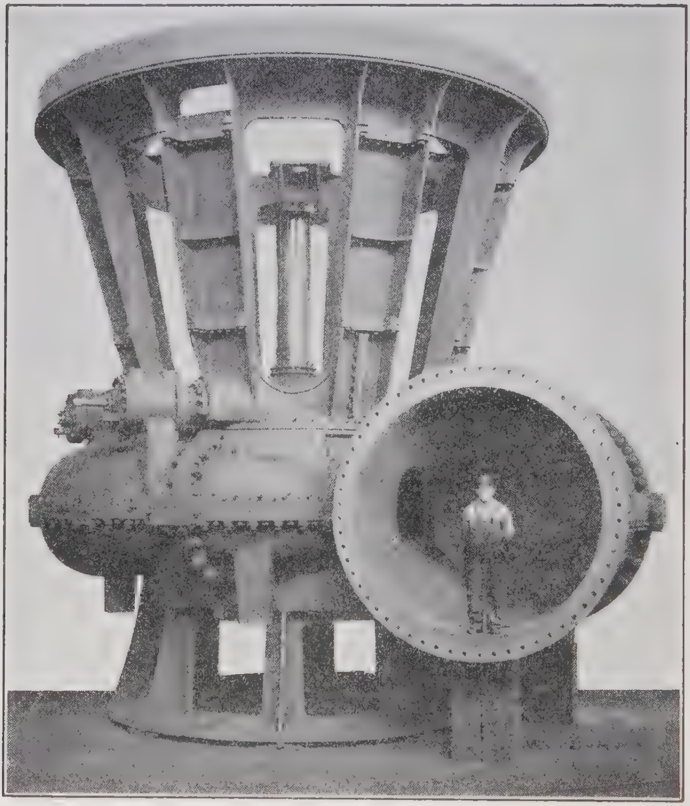


From a photograph by the author.

FIG. 203.—Speed ring for Canadian Light & Power Co. in shop of I. P. Morris Co.

water the proper direction as it enters the real guide vanes. In Fig. 204 is shown a vertical shaft turbine for a higher head.

151. Conditions of Service.—The reaction turbine is well adapted for service under low heads especially for large powers. They may also be used very satisfactorily for heads of several hundred feet. The highest head that has ever been employed



Courtesy of S. Morgan Smith Co.

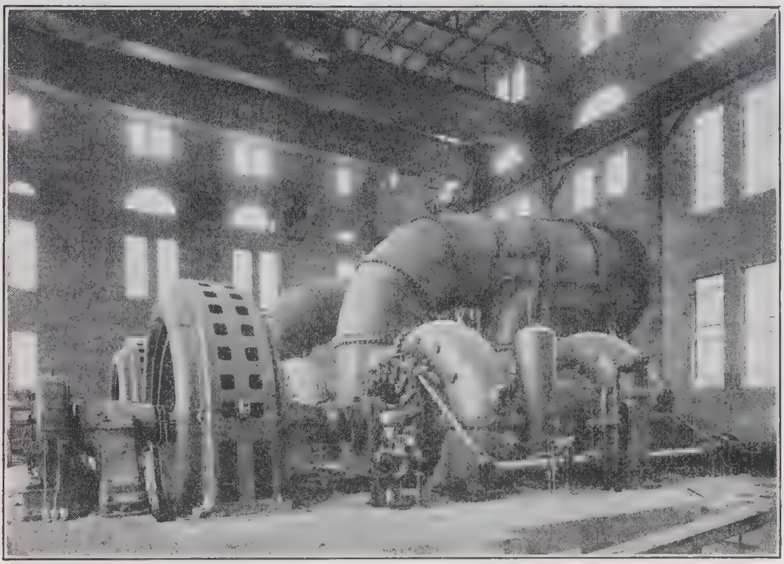
FIG. 204.—Vertical shaft spiral case turbine for Great Falls, Mont. $h = 150'$; $N = 200$; $Hp = 15{,}000$.

for a reaction turbine is 860 ft. and several have been used for heads in the neighborhood of 800 ft.

The most powerful turbines yet built will develop 70,000 hp. under a head of 213 ft. These are single runner, vertical shaft units for the Niagara Falls Power Co. There are at present quite

a number of turbines in operation whose power ranges from 20,000 to 30,000 hp.

But the power of a turbine depends not only upon its size but also upon the head under which it operates. Thus the most powerful turbines may not be as big in size as others which



Courtesy of I. P. Morris Co.

FIG. 205.—Washington Water Power Co. Two 22,500 hp. units at 200 r.p.m. under head of 168 ft.

develop less power because they run under lower heads. Until recent years the largest runners in size were those at Cedars Rapids, one of which is shown in Fig. 186. They slightly exceed in size those at Keokuk. At the present time the largest runners in the world in size, as well as in power are the 70,000 hp. runners at Niagara Falls. These runners are 184.75 in. in diameter, weigh 5 tons, and are about 6 ft. in height at entrance.

CHAPTER XIV

WATER POWER PLANTS

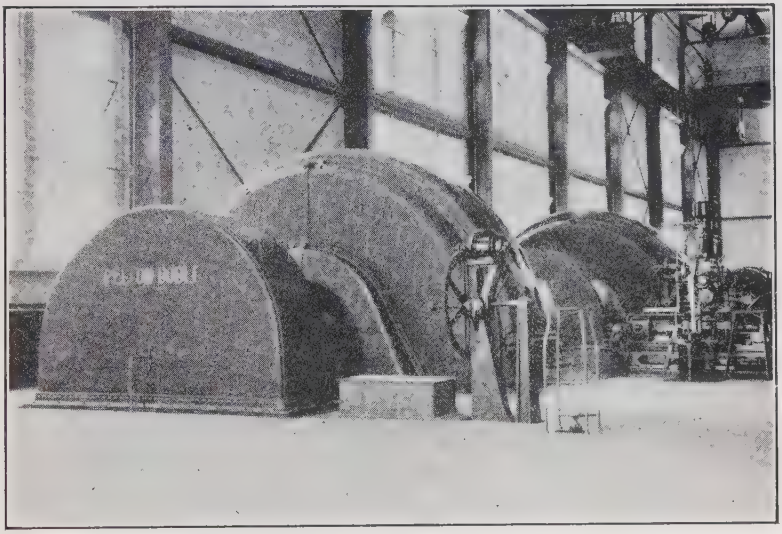
152. Elements of a Water Power Plant.—A complete water power development may comprise a great deal of construction and equipment aside from the power house and contents, so

From a photograph by the author.

FIG. 206.—Penstock leading to Drum power house of Pacific Gas & Elec. Co. under 1,375 ft. head.

much so that the cost of the latter is often a small proportion of the total investment. For a complete plant some or all of the following details may be required according to the physical situation.

A dam of some sort is usually essential. It may be nothing more than a wing wall extending a short way into the river to divert a small portion of the flow, or it may extend clear across the stream. In the latter case the water level will be raised above its former height and also a certain amount of water will be stored up by it. If the contour of the land permits, a dam may create an artificial lake or storage reservoir. In some cases the power plant draws water directly from this body and in other cases it would be used merely as a "feeder."



From a photograph by the author.

Fig. 207.—Pelton impulse wheels in Drum power house of Pacific Gas & Elec. Co. $h = 1{,}375'$ static or $1{,}300'$ under normal load; $N = 360$; $Hp = 8{,}500$ per wheel.

The water is conducted to the power house through canals, flumes, pressure tunnels, or pipe lines, as the case may be. It is not uncommon for the water to be carried from 5 to 10 miles or more in order to permit the utilization of a higher fall than could be obtained near the intake. It is desirable that the water be kept at as high an elevation as possible during the first portion of its course, as this permits the use of open channels or low-pressure pipes, which is cheaper than if the water had to be carried under high pressure all the way. This portion of the conduit is often called the "flow-line" from the fact that its main function is to deliver water and not to transmit pressure.

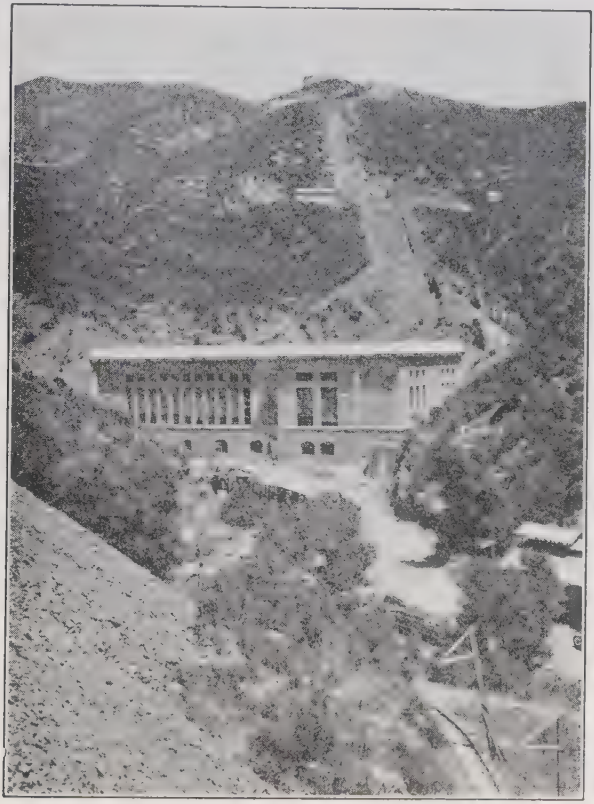
At the end of such a flow-line the water will be abruptly dropped down the hillside as shown in Fig. 206. This portion of the pipe line is the penstock.

From a photograph by F. H. Fowler.

FIG. 208.—Las Plumas plant at Big Bend on the Feather River in California, containing six reaction turbines of 18,000 hp. each under a head of 465 ft.

Where the distance from the intake to the power plant is a number of miles, it is desirable that there be some break in the

continuity of flow, on account of speed regulation. If conditions permit, a forebay may be constructed at the head of the penstock. The forebay is a reservoir of limited capacity whose function is to equalize the flow. Into it the water may be delivered at a uniform rate, while from it the water may be drawn by the penstock at varying rates according to the demands upon the tur-



From a photograph by the author.

Fig. 209.—San Francisquito Power Plant No. 1 on the Los Angeles Aqueduct. Static head from maximum water level in surge chamber on crest of hill to the nozzles is 941 ft.

bines. Thus the fluctuations in the flow of water through the turbines need not extend back all the way to the source.

Where a forebay is impossible or not really necessary, it is desirable to provide surge chambers or other means of relieving the abnormal conditions attendant upon changes of flow. In

the upper left-hand corner of Fig. 208 is seen a small surge chamber, and an overflow. The five penstocks receive water from a pressure tunnel 3 miles in length. In case of a sudden decrease in discharge through the turbines, the excess water could surge up the large pipe line running up the hillside, and if the surge was

From a photograph by the author.

Fig. 210.—Cornell University hydro-electric plant. Head = 140 ft., 1–550 hp. turbine, 2–280 hp. impulse wheels, 2–50 hp. impulse wheels.

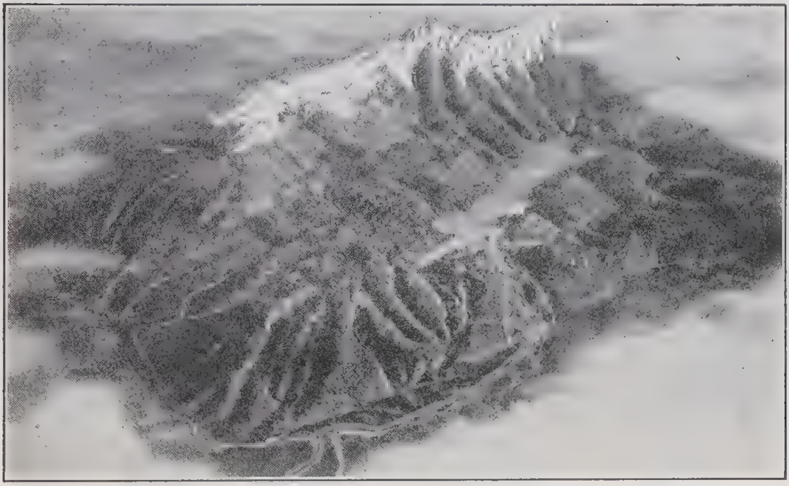
great enough some water would overflow, thus preventing any excessive increase in pressure.

In Fig. 209 is shown a power plant with a large surge chamber at the end of a pressure tunnel which is 7.76 miles in length.

It is 100 ft. in diameter at the top and the maximum water level is 150 ft. above the pressure tunnel. Only 35 ft. project above the ground. This is also provided with a spillway so that it may overflow if the surge is violent.

The power plant shown in Fig. 210 receives water through a conduit 1,711 ft. in length, and is equipped with a large air chamber within the power house to absorb shocks.

The water from a power plant may be discharged directly into some natural stream or it may be necessary to construct an artificial channel for a tail race, as in Fig. 213. In other cases,



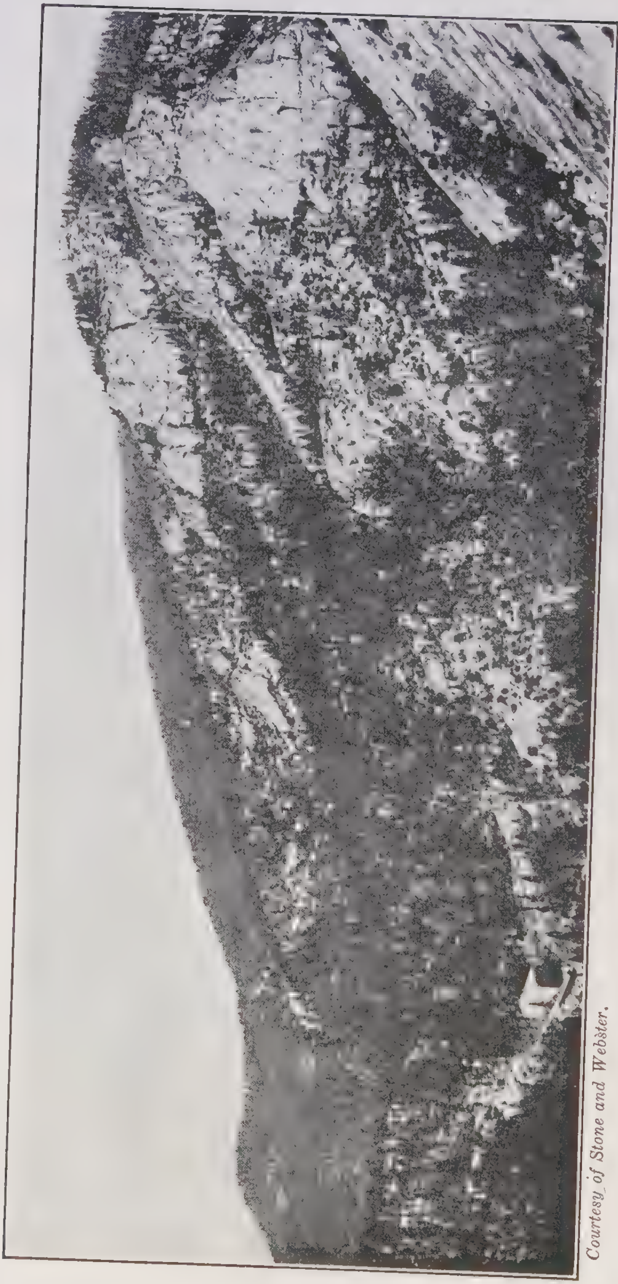
Courtesy of Stone and Webster.

Fig. 211.—Big Creek development of Southern California Edison Co. The fall from the lake to the first power house is 2,100 ft., and from that to the second power house is 1,900 ft.

as with some of the plants at Niagara Falls, the tail race may be a long tunnel.

153. High-head Plants.—It is impossible to establish any definite number of feet which is required to differentiate a high-head from a medium- or low-head plant. A high-head is one of several hundred feet or more, while a low-head plant would doubtless be under 50 ft. But one type shades very gradually into the other.

In Fig. 211 is shown a high-head development, where a fall of 4,000 ft. is divided between two power houses in series. In this one view may be seen a complete plant with many of the features



Courtesy of Stone and Webster.

FIG. 212.—Power Plant No. 1 at Big Creek near Fresno, Calif. Static head = 2,100 ft; Hp = 40,000.

that have been described, except that a forebay is not required. The mountain ranges, which rise to a height of 11,000 ft., provide a watershed, the runoff from which is gathered by a lake about 4 miles long, and with an elevation of 6,000 ft. The lake is created by the erection of three dams which can be seen placed in gaps in the hills. From the lake the water flows down the penstocks to the first plant. The discharge from this supplemented by some water from a little stream then flows through a tunnel for a way until it takes another drop to the second power house which can be seen in the lower part of the picture, a little to the left of the center. In Fig. 212 is a closer view of the first

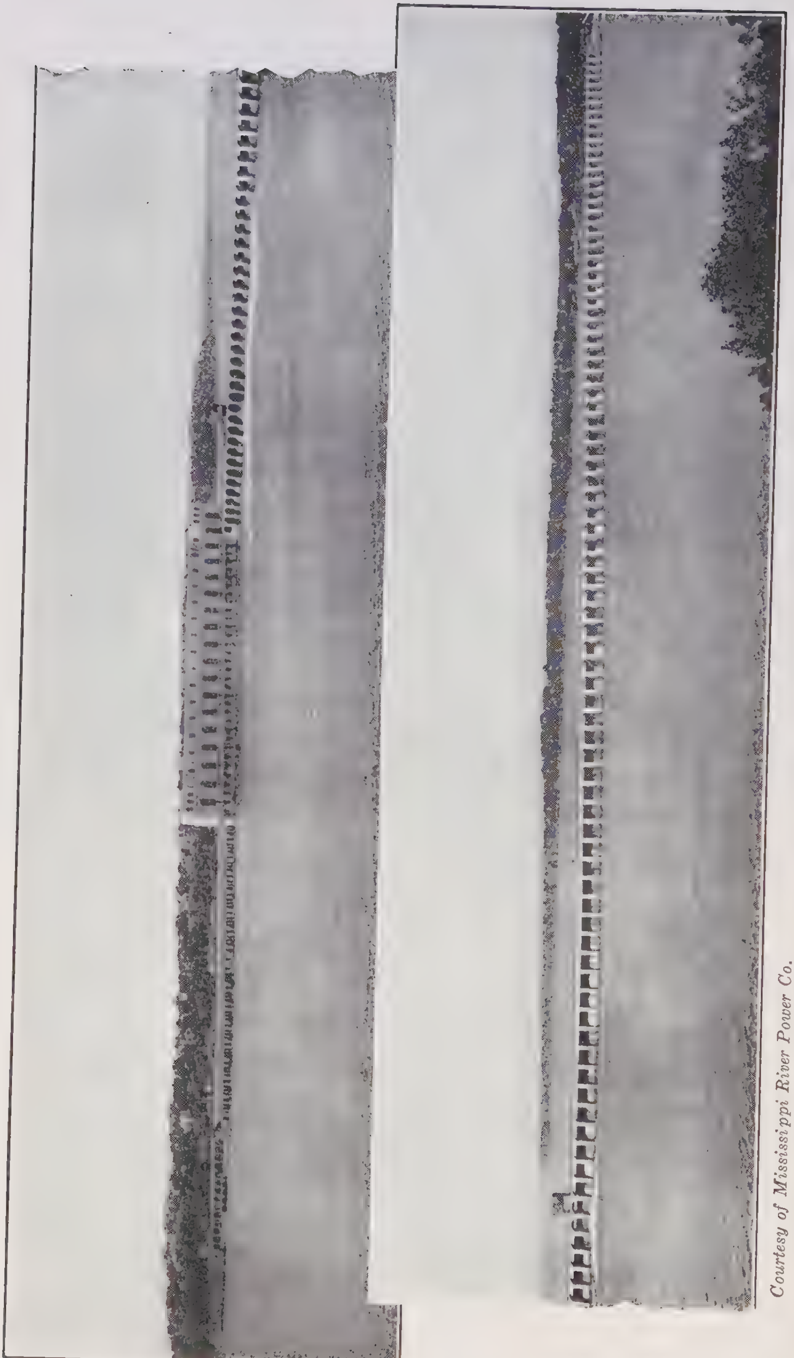
Courtesy of I. P. Morris Co.

Fig. 213.—Appalachian Power Co. development No. 2. Head = 49 ft., four turbines of 6,000 hp. each at 116 r.p.m.

plant. At the upper right-hand corner of Fig. 212 may be seen two standpipes, the water level in which will be nearly as high as that in the lake so that the entire 2,100 ft. drop is shown here.

A high-head plant requires but little water for a given amount of power and it is usually so situated that a storage reservoir is to be had. Consequently it may be able to run for a long time merely on the water that is conserved by the creation of such reservoirs. It is always necessary to have a penstock and many of the other details that have been enumerated.

154. Low-head Plants.—A typical low-head plant is shown in Fig. 213. The head, under which the turbines operate, has been practically created by the erection of a dam. There are no pipe



Courtesy of Mississippi River Power Co.

FIG. 214.—The Mississippi River Power Co. at Keokuk, Ia. Head = 32 ft., capacity is 15 units of 10,000 hp. each at 57.7 r.p.m. Maximum capacity of plant is 200,000 hp.

lines and the body of water produced by the dam now becomes the forebay. The turbines in such a plant may have any one of the three types of settings shown in Figs. 198, 199 and 200.

It may be seen that fluctuations in the flow of the river, with consequent changes in water level, cause variations in the head under which the turbines operate. This is something that scarcely exists in a high-head plant. Also low heads are usually found in fairly flat countries, where the nature of the topography renders it impractical to store up large quantities of water, and, furthermore, under a low head a large amount of water is required to develop a given power. This makes it impossible to run very long on storage and hence the plant is dependent upon a regular stream flow.

The differences between the high- and low-head plants are such as to require turbines of different characteristics in order to meet the conditions most satisfactorily.[1]

Another typical low-head plant is shown in Fig. 214. The length of the dam across the river is nearly a mile. While a low-head plant is often free from many of the items that are required in a high-head development, it must be remembered that it must be built to handle large volumes of water, and much massive construction is required.

[1] Daugherty, R. L., "Hydraulic Turbines," chap. XIII.

CHAPTER XV

THEORY OF THE IMPULSE WHEEL

155. Action of the Water.—The impulse wheel is more accurately described as a tangential water wheel from the fact that the center line of the jet is tangent to the path described by the center of the buckets. The latter is called the "impulse circle" and computations are based upon the linear velocity of the wheel at this radius. For an impulse wheel the nominal value of D is the diameter of this circle.

It is often stated that the jet at impact is also tangent to this circle, but this is not a true representation of the facts as Fig.

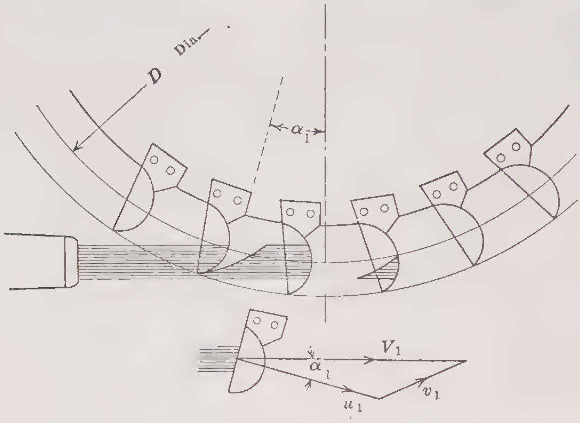


FIG. 215.

215 will show. The jet strikes the buckets before they arrive at a point directly under the center of rotation and hence the angle α_1 is not zero. Observation of various Pelton wheels in action has convinced the author that average values of α_1 may vary from 5 to 20 deg., according to the design of the wheel. The value of α_1 must be an average value for a given wheel from the fact that the bucket moves through a certain angle from the time it first enters the jet until the last drop of water has struck it.

The illustration also shows that when a bucket first enters the jet it cuts off the water from the preceding bucket and leaves a "slug" of water to catch up with the latter and to complete its work upon it. Thus the water may be acting upon several buckets at the same time. This explains why there is a difference between the W' of Art. 128 and W. The former is the amount of water acting upon a single bucket, the latter is the total acting upon all the buckets. It is not necessary to know how many buckets are in action at a time for, since the wheel does not move away from the nozzle, it follows that all of the water discharged by the nozzle may act upon it.

But it is not necessarily true that the wheel utilizes all of the water under all circumstances. Suppose, for instance, that the buckets were to move as fast as the jet; it would then be seen that none of the water could overtake them, but that all would go right on through. And for speeds somewhat less than this a portion of the water would deliver its energy to the buckets and the latter portions of the intercepted "slugs" would not be able to overtake the buckets before they had swung up above the line of action of the jet. The problem is so to design the buckets and the wheel that all of the water in the jet will be able to do its work upon the wheel, when running at the proper speed. For speeds much above the normal speed a certain amount of water must necessarily go right on through without having had a chance to do work.

By the proper speed or normal speed is meant the one that the wheel should have for the jet velocity in question. A high jet velocity would require a high wheel speed and *vice versa*. In fact the chief concern is with the relation between the various velocities rather than with their actual values, and hence it is desirable to introduce factors which will express this relationship and be independent of the head. Thus if the jet velocity be denoted by V_1 and the linear velocity of the bucket at the impulse circle by u_1, c_v and ϕ may be used so that,

$$V_1 = c_v\sqrt{2gh} \tag{147}$$

$$u_1 = \phi\sqrt{2gh}. \tag{148}$$

It may be seen that c_v is the velocity coefficient of the nozzle, the value of which is constant for any setting of the needle. Thus for any given value of ϕ the relation between V_1 and u_1 is known at once regardless of the value of h.

The absolute path of the water and the velocity vectors at discharge from the buckets may be seen in Fig. 216. For different wheel speeds under the same head, which means different values of ϕ, there should be such diagrams as are shown in Fig. 217. As the speed of the wheel increases from zero, the angle of deflection of the jet continually decreases. It may also be seen from the diagrams that the value of V_2 is relatively high when the wheel is at rest; that it becomes a minimum at such a speed; that α_2 is approximately equal to 90 deg.; and then increases again.

The action of the water as just described is illustrated by some rather unusual photographs taken of a 42-in. wheel in action. The side of the casing was removed for the purpose. The needle was withdrawn as far as possible so that the maximum size jet which the design permitted is shown in the photographs.

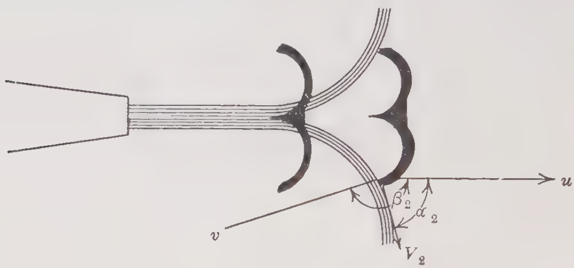


FIG. 216.

In Fig. 217, when $\phi = 0$, the wheel was prevented from rotating by applying a sufficient torque to the shaft. The jet cannot be seen, but the water leaving the bucket is shown. In Fig. 217, when $\phi = 0.45$, the wheel is running at its most efficient speed. The water leaving the buckets drops down into the tailrace with most of its energy abstracted. In Fig. 217, when $\varphi = 0.80$, the wheel is shown at runaway speed, all load having been removed save its own friction and windage and that of the generator to which it is direct-connected.

156. Force Exerted by Jet.—The tangential water wheel, Pelton wheel, or impulse wheel, as it is variously called, is really an impulse turbine with approximately "axial" flow. By this is meant that $r_1 = r_2$. The latter is not strictly true but is sufficiently close for all practical purposes.

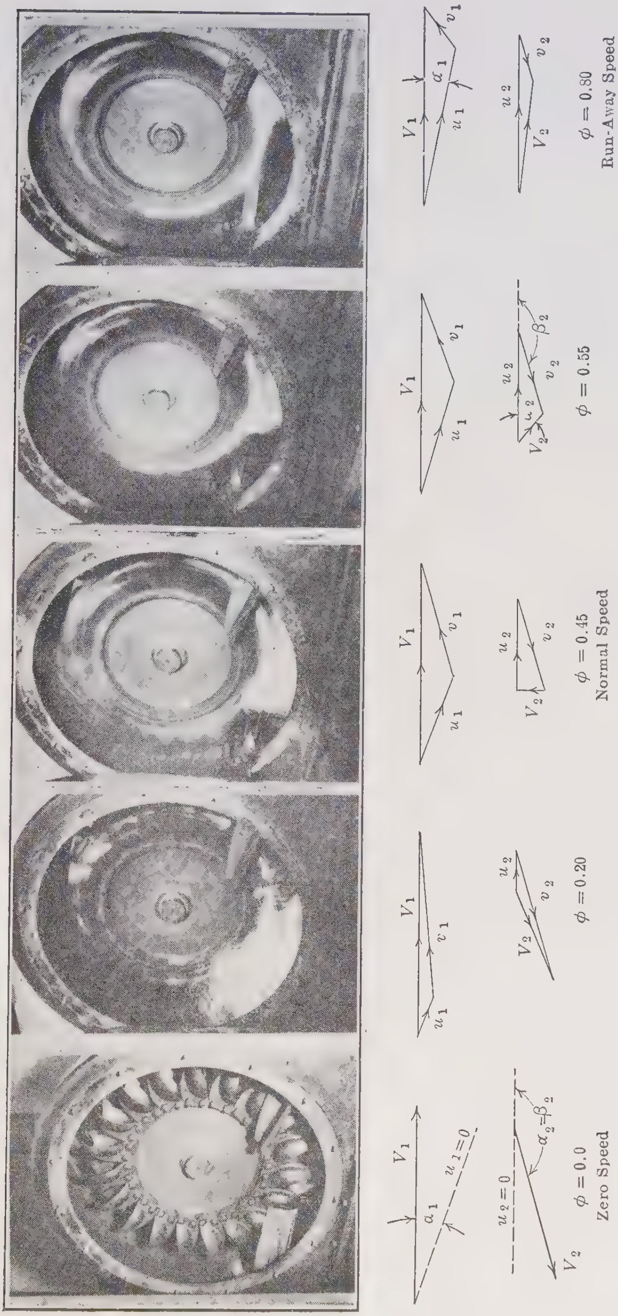


FIG. 217.—42-in. impulse wheel at different speeds under same head.

The force desired is really the tangential component of the resultant force. This may be obtained by computing the tangential component of the ΔV in Art. 123, or, since r_1 and r_2 are equal, it may be obtained from Art. 130. The results are identical. The desired component is

$$F = \frac{W}{g}(V_1 \cos \alpha_1 - V_2 \cos \alpha_2). \tag{149}$$

The values of V_2 and α_2 depend upon the jet velocity and the speed of the wheel, and are therefore variable and unknown. It is desired to replace them in terms of V_1 and u_1 and wheel dimensions which may be supposed to be known. It is thus necessary to find some relation between the velocities at entrance and those at discharge. The equation of flow between these two points will be found in Art. 137, and for the impulse turbine it becomes

$$\frac{v_1^2}{2g} - \frac{v_2^2}{2g} + \frac{u_2^2}{2g} - \frac{u_1^2}{2g} = k\frac{v_2^2}{2g}$$

where k is a coefficient of loss in flow over the buckets, such that the head lost is $kv_2^2/2g$. Since it is assumed that $u_1 = u_2$, then, for the Pelton wheel, is obtained the special relation

$$v_2 = \frac{v_1}{\sqrt{1 + k}}.$$

In a numerical case the value of v_1 can be computed by trigonometry according to whichever one of the methods given on page 322 is deemed to be more convenient or more accurate. Having v_1 the value of v_2 may be found by the equation just given. The velocity diagram at outflow is now determined since v_2, u_2, and β_2 are known. Thus $V_2 \cos \alpha_2$ may be found and the value of F computed by Eq. (149). The procedure is identical with that illustrated in Arts. 124 and 130.

The numerical computation of the value of F is greatly simplified, if it is assumed that $\alpha_1 = 0$ deg., but the result is merely a close approximation to the truth.

A study of Fig. 217 shows that the value of ΔV decreases as the wheel speed increases. Assuming that $\alpha_1 = 0$ deg., it may be shown that ΔV is proportional to $V_1 - u_1$, and thus $F = B(V_1 - u_1)$, where B represents all the other factors. This is apparently the equation of a straight line between F and u_1 for any constant jet velocity V_1. Actually it is not a straight line because B is not a constant. This is due to the fact that not

only is α_1 not equal to 0 deg. but it varies with the wheel speed, the factor k is not a constant for all speeds, and above all the amount of water acting upon the wheel decreases as the speed increases above normal, as has previously been explained.

The torque exerted by the water upon the wheel may be obtained by multiplying F by r_1, the radius of the impulse circle. The torque which the wheel can deliver is somewhat less than this because of bearing friction and windage.

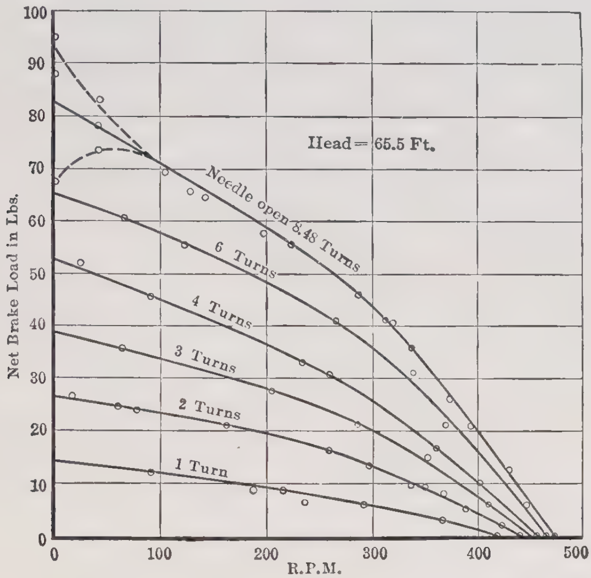


FIG. 218.—Relation between torque and speed.[1]

Figure 218 shows the performance of a certain wheel at different speeds under a constant head. The variation in the force at zero speed is due to changes in the angle α_1 and r_2/r_1 with different positions of the buckets. This is shown for one nozzle setting only, though it exists in all.

157. Power of Wheel.—Since power is the product of F and u_1 or T and ω, it may be seen that it is zero when the wheel is at rest, though the torque is then a maximum, and it is also zero when the speed is a maximum, for the torque is then zero. The maximum power will be obtained for some speed between these two extremes, as shown by Fig. 219.

[1] From the test of a 24-in. wheel by F. G. Switzer and the author.

Since for a given head and nozzle opening the power input is constant regardless of the speed of the wheel, it follows that the efficiency is directly proportional to the power developed. But it should be noted that the power delivered in the water

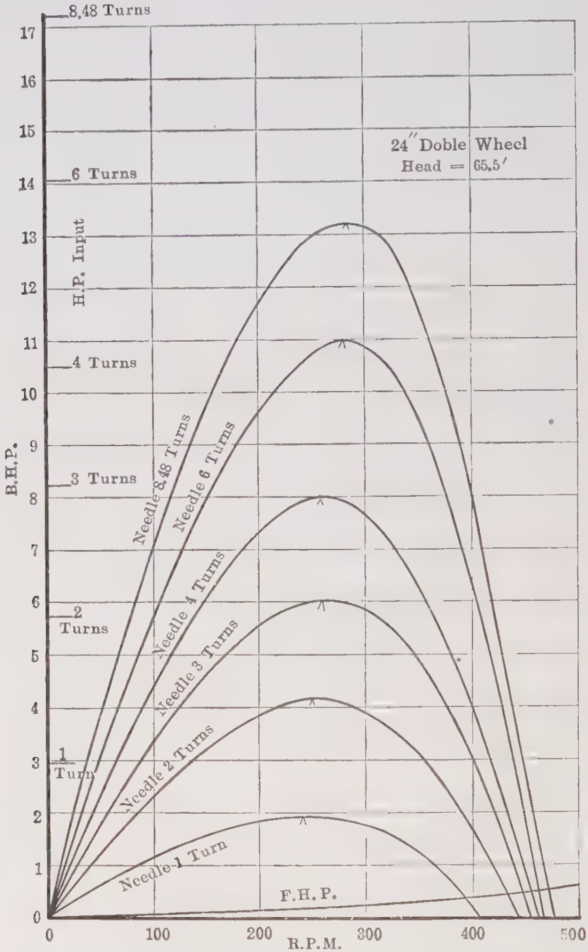


Fig. 219.—Relation between power and speed.

increases with the nozzle opening so that the needle setting that gives the largest power is not necessarily the most efficient.

158. Speed.—It may be well to emphasize that in the present discussion the word speed is being used in a relative rather than in an absolute sense. Any observation on the effect of a change

in the speed of the wheel u_1 is based on the assumption that the jet velocity V_1 remains the same. And by a high peripheral speed, for instance, is meant merely high as compared with that of the jet. Since, then, the principal concern is with ratios of speeds rather than with their absolute values, the factor ϕ could better be used, as defined in Eq. (148).

Using the expression $F = B(V_1 - u_1)$ it should be concluded that F would become zero when $u_1 = V_1$, or when $\phi = c_v$, the value of which would be about 0.98. Also if u_1 should be used as multiplier, the value of the power would be $P = Fu_1 = B(V_1 - u_1)u_1$, and from this the power would appear to be a maximum when $u = 0.5V_1$. But, as has been stated, the factor B is not a constant. Because of the large amount of water that is not effective upon the wheel at high values of ϕ, and also because the bearing friction and windage prevent the torque from ever being reduced to zero, the actual maximum speed attained by the tangential water wheel is such that ϕ is approximately equal to 0.80.

In like manner the maximum power, and hence the maximum efficiency for a given nozzle opening, is also attained when the wheel speed is something less than $0.5V_1$. Thus in actual practice for the best efficiency

$$\phi_e = 0.43 \text{ to } 0.48.$$

In practical applications the performance of a wheel at a constant speed under a constant head is usually of most interest. Values for this may be obtained from Figs. 218 and 219 by following along any vertical line. Generally the vertical line should be the one for the speed at which the maximum efficiency is found. The resulting curves for the impulse wheel would be very similar to those for the reaction turbine shown in Fig. 223.

159. Head on Impulse Wheel.—The nozzle is considered an integral part of the impulse wheel and hence the head under which the wheel is said to operate must include it. If C in Fig. 220 indicates a point at the base of the nozzle,

$$h = H_c = p_c + \frac{V_c^2}{2g}. \qquad (150)$$

It is this value of h that should be used in determining the efficiency of the wheel.

This value of h is the total fall from headwater to nozzle minus the head lost in the pipe line. The energy supplied at this point

is expended in four ways. A small amount is lost in flow through the nozzle, a portion is expended in hydraulic friction and eddy losses within the buckets, kinetic energy is carried away in the water discharged from the buckets, while the rest is delivered to the wheel to do useful work and overcome mechanical friction and windage losses. Calling h'' the head delivered to the buckets,

$$h = \left(\frac{1}{c_v^2} - 1\right)\frac{V_1^2}{2g} + k\frac{v_2^2}{2g} + \frac{V_2^2}{2g} + h''.$$

The head utilized h'' may be determined as shown in Art. 137. The procedure is similar to that for the computation of a value of F.

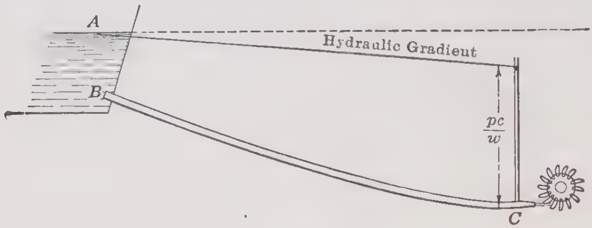


FIG. 220.

160. PROBLEMS

153. A nozzle having a velocity coefficient of 0.98 discharges a jet 6 in. in diameter under a head of 800 ft. This jet acts upon a wheel with the following dimensions: diameter 6 ft., $\alpha_1 = 10$ deg., $\beta_2 = 165$ deg., and it is assumed $k = 0.70$. Find the force exerted upon the buckets when $\phi = 0.45$.

Ans. $F = 17{,}600$ lb.

154. Solve the above, assuming that $\alpha_1 = 0$ deg., and find the power developed upon the buckets of the wheel. What is the hydraulic efficiency of the wheel? If the mechanical efficiency of the wheel is 0.97, what is the gross efficiency?

Ans. $F = 17{,}750$ lb., 3,290 hp., 0.833, 0.807.

155. Assuming $\alpha_1 = 0$ deg. in problem 153, find the power lost in hydraulic friction within the buckets. Find the value of V_2 and determine the power carried away in the water discharged from the turbine.

Ans. 460 hp., 57.5 hp.

156. A good proportion between jet and wheel is that the diameter of the wheel in feet should equal the diameter of the jet in inches. Using this ratio, what size wheel would be required to deliver 5,000 hp. under a head of 1,400 ft., assuming an efficiency of 82 per cent? What would be the speed of the wheel?

Ans. 4.48 ft., 563 r.p.m.

CHAPTER XVI

THEORY OF THE REACTION TURBINE

161. Introductory Illustration.—The reaction turbine is so called because an important factor in its operation is the reaction of the streams of water discharged from the runner. It is well to bear in mind, however, that the total dynamic effort is due to the entire change in the momentum of the water just as in the impulse turbine.

As an illustration, consider the vessel ABC of Fig. 221 into which water enters across AB with a velocity V_1 and is discharged at C with a velocity V_2. Now the reaction of the jet alone could be determined by an application of Art. 123. But the total force is due not only to this reaction but also to the impulse of the water entering at AB. It is not feasible to separate the effect of impulse from that of reaction, neither is it necessary to do so. The horizontal component of the total dynamic force is obtained directly by

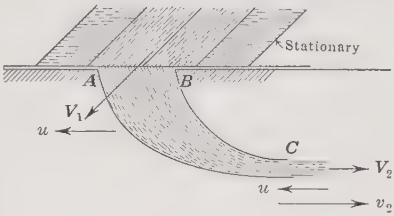


FIG. 221.

$$F = \frac{W}{g}(V_1 \cos \alpha_1 + V_2).$$

Suppose now that this vessel moves to the left with a uniform translation u. Assume that in some way the water is still supplied to it with a velocity V_1. This might be the case if the vessel passed under a series of stationary passages each of which in turn was permitted to discharge water into it. The value of the absolute velocity of discharge is now

$$V_2 = v_2 - u.$$

Inserting this value above,

$$F = \frac{W}{g}(V_1 \cos \alpha_1 + v_2 - u).$$

This equation indicates that F decreases as the speed increases, just as in the case of the impulse turbine. Also the water entering the vessel at AB is under pressure and is not a free jet. Therefore, V_1 must be less than $\sqrt{2gh}$. Since all the passages are completely filled with water the equation of continuity can be applied, and it will show that V_1 and v_2 are inversely proportional to the areas of their respective streams. But the value of v_2 depends upon the losses of head, and in a real turbine these hydraulic losses vary with the speed. Since V_1 is proportional to v_2, it follows that V_1 varies with the speed of the wheel.

Thus some fundamental differences between impulse and reaction turbines are that in the former $V_1 = c_v\sqrt{2gh}$, where c_v is a velocity coefficient nearly equal to unity. This velocity, and hence the amount of water discharged by the nozzle, is entirely independent of the design of the wheel and its operation. But for the reaction turbine

$$V_1 = c\sqrt{2gh} \tag{151}$$

where c is not a velocity coefficient but a factor whose value varies from about 0.6 to 0.8 for ordinary designs. The value of c depends upon the design of the wheel and the speed at which it is run under a given head. This means that c is also a function of ϕ, where ϕ has the meaning given by Eq. (148).

With the radial-flow type of turbine centrifugal force also causes the value of c to vary with the speed of the wheel, the head remaining constant. The centrifugal force opposes the flow of water in the case of the inward-flow turbine so that, as the speed increases under a constant head, the discharge tends to decrease as shown in Fig. 222. But there are other influences at work also, so that for some inward-flow turbines the value of q actually increases somewhat as the speed increases above zero, but after a certain speed is exceeded the rate of discharge falls off again.

162. Torque Exerted.—The preceding article merely illustrates a few fundamental points regarding the reaction turbine. Since with the real machine the radii of the water at inflow and outflow differ materially, it is not feasible to compute the force exerted by the water and the torque must be obtained instead. Before proceeding any further with the theory it should be noted that, while our equations are rational, they must assume that all particles of water move in similar paths with equal velocities.

Actually, average values have to be dealt with. But these average values are not known with any precision. For example, there is no assurance that the angles α_1 and β_2 are the same as the angles of the guide vanes and the runner vanes respectively. In fact there is some evidence to indicate that they differ by as much as 5 or 10 deg. The same condition exists with regard to the areas of the streams and all other dimensions used. Thus

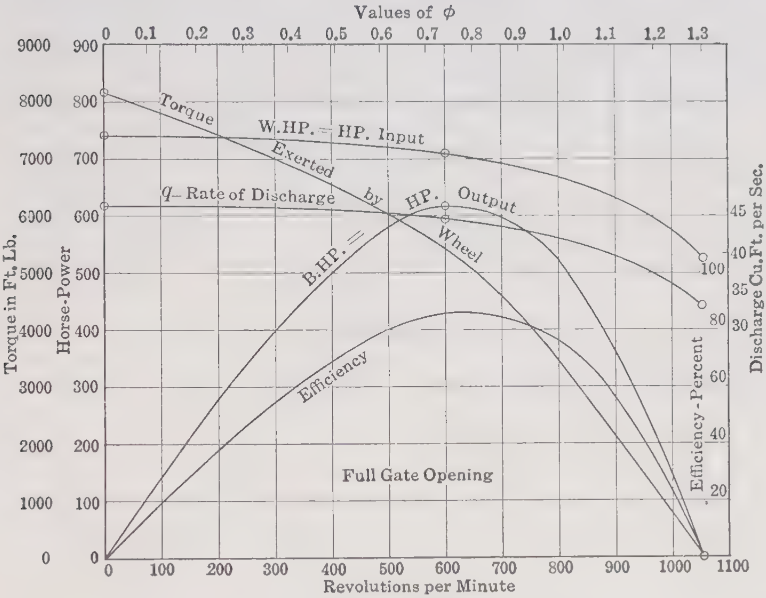


FIG. 222.—Test of 27 in. I. P. Morris turbine. Head and gate opening constant. Speed variable.[1]

the numerical results of such computations cannot be expected to agree precisely with actual facts.

Despite this the theory has its value. It serves to explain the principles of operation of such machines, to indicate the nature of their actual characteristics, and to account for numerous observed facts. In design the theory shows what proportions are desirable and what the effect of certain changes of dimensions would be. Thus with some actual test data to work from,

[1] Figures 222 and 223 are from the test of a reaction turbine in the Cornell University power plant. See DAUGHERTY, R. L., "Investigation of the Performance of a Reaction Turbine," *Trans. A. S. C. E.*, vol. 78, p. 1270, 1915.

existing designs might be altered by the theory with some degree of assurance.

In order to compute the torque exerted upon the runner by the water the fundamental formula of Art. 130 might be taken,

$$T = \frac{W}{g}(r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2). \qquad (152)$$

Just as in the case of the impulse turbine in Art. 156, the values of V_2 and α_2 are variable and unknown, and it is necessary to replace them in terms of known quantities. It is assumed that all the dimensions of the wheel and the values of V_1 and u_1 are known.

From the vector diagram it may be seen that

$$V_2 \cos \alpha_2 = u_2 + v_2 \cos \beta_2$$

But $u_2 = (r_2/r_1)u_1$, and, since the passages are completely filled with water in the reaction turbine, the equation of continuity gives $q = A_1 V_1 = a_2 v_2$, or

$$v_2 = \left(\frac{A_1}{a_2}\right) V_1. \qquad (153)$$

(Contrast this procedure with that for the impulse wheel in Art. 156, and see again Art. 137.) Then

$$V_2 \cos \alpha_2 = \frac{r_2}{r_1} u_1 + \frac{A_1}{a_2} \cos \beta_2 V_1$$

from which the value to be used in Eq. (152) may be computed. In the actual use of Eq. (152) the value of W would have to be determined for a given value of u_1, either by experiment or by computing the rate of discharge by theory.

163. Power.—The power developed by the water is determined by multiplying T by the angular velocity. The torque actually exerted *by* the shaft and the power delivered are obtained by multiplying these values by the mechanical efficiency.

The hydraulic efficiency of the turbine is obtained by

$$e_h = \frac{T\omega}{Wh} = \frac{Wh''}{Wh} = \frac{h''}{h}.$$

It is difficult to obtain the hydraulic efficiency by test as it is necessary to determine the bearing friction and also the disk friction due to the drag of the runner through the water in the clearance spaces. But these losses may be allowed for and the hydraulic efficiency then secured approximately.

Because of the necessary defects of the theory, the hydraulic efficiency may be assumed with less error than is usually involved

in computing T. For turbines of rational design and running at their proper speeds the value of the hydraulic efficiency may range from 0.80 to 0.95. The higher values are found only in large turbines and with favorable proportions. Only experience can enable one to select the proper value between these two extremes, which are not necessarily limits. For improper speeds and incorrect designs no values can be assigned.

The curves of Fig. 222 show the characteristics of a reaction turbine with a fixed gate opening and the speed variable. These

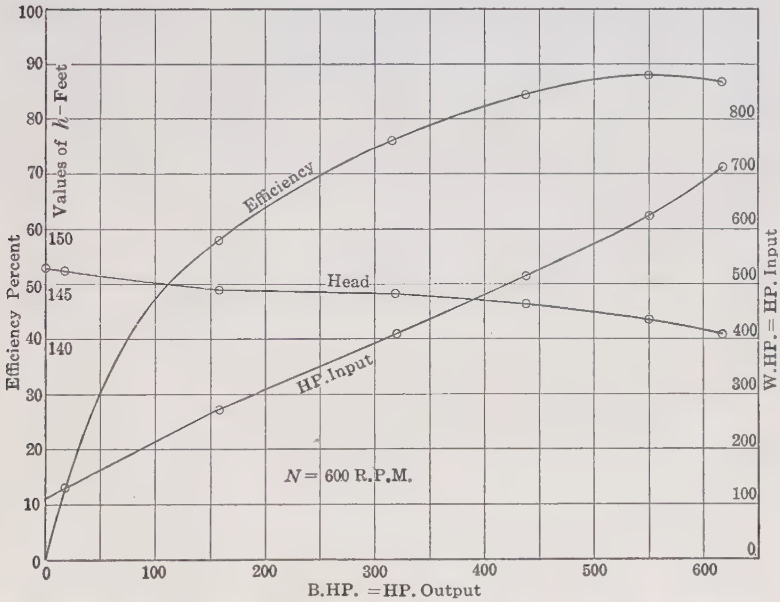


FIG. 223.—Test of 27 in. I. P. Morris turbine. Head and φ approximately constant. Gate opening variable.

are similar to those of the impulse wheel except that q is not a constant. Hence maximum power developed and the maximum efficiency do not necessarily occur at the same speed. The characteristics of the same turbine at a constant speed are shown in Fig. 223. The maximum efficiency is 88 per cent at 550 hp. under a head of 141.8 ft.

164. Speed.—Although the water flowing through the runner of a reaction turbine is entirely confined, the velocity undergoes changes similar to that in the impulse turbine, except that for a fixed gate opening the angle α_1 is constant. Hence

the values of V_2 and α_2 vary in just the same way as is shown in Fig. 217.

The speed at which the efficiency is the highest will be somewhere in the neighborhood of the one for which the discharge loss is the least, which is when V_2 is a minimum. For this condition it will be found that approximately $\alpha_2 = 90$ deg. or that $u_2 = v_2$. Note that these two conditions are not identical but they differ but little. It is customary to assume one or the other according to convenience.

In the case of the reaction turbine V_1 is less than for an impulse turbine under the same head. But the water at entrance is under pressure and, as it flows through the runner, this is converted into velocity. Hence at discharge v_2 may easily be as large as in the case of the impulse wheel. And if $u_2 = v_2$, it may be seen that u_2 will be about the same in either type. But with the inward-flow reaction turbine u_1 is greater than u_2,

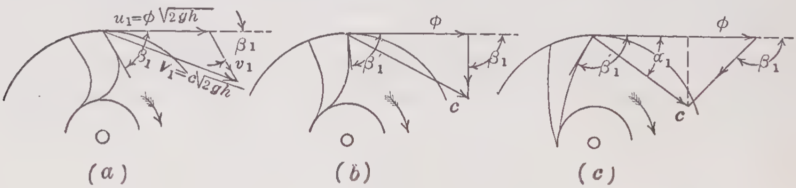


FIG. 224.

and therefore the peripheral velocity of the reaction turbine is greater than that of the impulse wheel.

Not only is the peripheral speed higher for maximum efficiency but also the runaway speed is higher. The maximum value of ϕ for the reaction turbine is about 1.30, though with some it may easily exceed this value. And for the normal speeds at which the maximum efficiency is obtained,

$$\phi_e = 0.60 \text{ to } 0.90,$$

the exact value for a given wheel depending upon its design.

The question naturally arises as to how it is possible for the peripheral velocity of the runner to be greater than the velocity of the entering water, as it often is. If the velocity of any moving object is in the same direction as that of the water, then it is obvious that none of the water can overtake it unless it is moving slower than the water. But if, on the other hand, its direction of motion is at right angles to that of the water, then it may be

seen that water may strike it, no matter what their respective velocities may be in magnitude. Thus in Fig. 224 the amount of water which flows into the wheel may be said to be a function not merely of V_1 but rather of the normal component, $V_1 \sin \alpha_1$ (which also equals $v_1 \sin \beta_1$). As long, therefore, as the angle α_1 is not 0 deg. water may flow into the wheel whatever its peripheral speed may be.

Since u_1 and V_1 are proportional to the factors ϕ and c, the former may be replaced by the latter in many cases and gain in generality and simplicity, since ϕ/c is exactly the same value as u_1/V_1. In Fig. 224 (a) the velocity of the water is greater than that of the wheel, but in (c) it is less. These three diagrams are drawn to scale for values that are typical of three different types of turbine runner.

The angle β_1 is determined by the velocity diagram. The runner vane should then be so shaped at entrance that its angle β'_1 should agree with β_1 in order that there should be no unnecessary turbulence loss set up due to an abrupt change in the flow of the water. This vane angle β'_1 once being fixed by construction is then only suitable for one ratio of ϕ to c. If the wheel runs too slow β'_1 will be greater than β_1. If it runs too fast β'_1 will be less than β_1.

165. Values of c_e and ϕ_e for Maximum Efficiency.—The turbine should run normally at such a speed under any head that the maximum efficiency will be attained. This will naturally be when the total of all the losses is a minimum. While the loss of energy at discharge is not the largest source of loss, it is the one which seems to vary most with a change in speed. Hence it is customary to neglect the variation in the other losses and determine the speed at which this will be a minimum. The value of V_2 will be very close to a minimum when the angle α_2 is 90 deg. Such a discharge is called "radial" from the fact that in the original inward-flow turbine, it was directed towards the axis of rotation and hence lay along a radius. In the more modern type of mixed-flow turbine it is often called "axial" since it is parallel to the axis. In general it is often spoken of as "perpendicular" since it is at right angles to the linear velocity of the runner at this same point. But the first two terms are often used regardless of the physical facts.

The speed at which $\alpha_2 = 90$ deg. is very close to that at which the maximum efficiency is actually found in the case of the Pelton

wheel and the "low-speed" reaction turbine such as shown in Fig. 188 (*a*). But for the "high-speed" type such as in Fig. 188 (*b*), there is almost always some whirl to be found in the draft tube when the wheel is running at the point of maximum efficiency. This is because, for this type of turbine, other factors become of increasing importance. The mixed-flow type of runner is not amenable to any simple mathematical analysis, and it would be beyond the scope of this book to undertake to present its theory in any adequate way. Hence the following treatment is to be understood as being applicable largely to the former type of turbine runner, although it will give approximately correct results, even for the latter.

The angle which the runner vane at entrance makes with u_1 will be denoted by β'_1. As the turbine is ordinarily designed, the value of the vane angle would be such that it would agree with β_1 as determined by the vector diagram for this same speed. But at any other speed the value of β_1 would be different from β'_1, hence there would be an abrupt change in the direction of the water entering the runner giving rise to what is known as "shock loss."

The following expressions therefore apply *only* to the *special* case where $\alpha_2 = 90$ deg. and $\beta'_1 = \beta_1$. From the vector diagram of velocities then

$$V_1 \sin \alpha_1 = v_1 \sin \beta_1 = v_1 \sin \beta'_1$$

$$V_1 \cos \alpha_1 = u_1 + v_1 \cos \beta_1 = u_1 + v_1 \cos \beta'_1$$

Eliminating v_1 between these two equations,

$$u_1 = \frac{\sin (\beta'_1 - \alpha_1)}{\sin \beta'_1} V_1, \tag{154}$$

as the relation between u_1 and V_1 when there is no loss at entrance to the runner.

The power delivered by the water to the runner may be expressed as

$$T\omega = Wh'' = e_h Wh,$$

where T has the value given by Eq. (152). Thus

$$Wh'' = \frac{W}{g}(r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2)\frac{u}{r}.$$

If the discharge is "radial," $\alpha_2 = 90$ deg. and hence $V_2 \cos \alpha_2 = 0$. Therefore,

$$h'' = e_h h = \frac{u_1 V_1 \cos \alpha_1}{g}. \tag{155}$$

Solving Eqs. (154) and (155) simultaneously, we have

$$V_1 = \sqrt{\frac{e_h 2gh}{2} \frac{\sin \beta'_1}{\sin (\beta'_1 - \alpha_1) \cos \alpha_1}}$$

$$u_1 = \sqrt{\frac{e_h 2gh}{2} \frac{\sin (\beta'_1 - \alpha_1)}{\sin \beta'_1 \cos \alpha_1}}.$$

From this it follows that

$$c_e = \sqrt{\frac{e_h \sin \beta'_1}{2 \sin (\beta'_1 - \alpha_1) \cos \alpha_1}} \tag{156}$$

$$\phi_e = \sqrt{\frac{e_h \sin (\beta'_1 - \alpha_1)}{2 \sin \beta'_1 \cos \alpha_1}}. \tag{157}$$

It must be borne in mind that Eqs. (156) and (157) can be applied only for the special case stated. For any other speed a different procedure would be necessary but it will not be given here.[1]

The speed desired is the one for which the gross efficiency is a maximum and this may not be quite the same as the one for which the hydraulic efficiency is the highest. Hence the true value of ϕ_e may differ slightly from the value given by Eq. (157).

These equations appear to be independent of conditions at outflow from the runner. But it must be noted that they are to be used only upon the assumption that the dimensions used at exit will be such as to make $\alpha_2 = 90$ deg., when c_e and ϕ_e have the values given.

An interesting result may be obtained by multiplying Eqs. (156) and (157). This gives

$$c_e \phi_e = \frac{e_h}{2 \cos \alpha_1}. \tag{158}$$

This would indicate that, all other things being equal, the higher the value of ϕ_e the smaller the value of c_e. With the impulse turbine, to which these equations apply also, ϕ_e is small but $c = c_v$ and is near unity. With the reaction turbine ϕ_e is larger than for the impulse wheel but c_e is smaller.

166. Theory of the Draft Tube.—If the draft tube is properly designed its area next to the runner should be such that the

[1] A general relation between c and ϕ for all conditions will be found in the author's "Hydraulic Turbines."

velocity in it is equal to V_2, the absolute velocity of discharge, otherwise there will be an abrupt change of velocity, involving losses. For Fig. 225 may be written

$$H_2 = p_2 + z_2 + \frac{V_2^2}{2g},$$

and

$$H_4 = 0.$$

The losses between points (2) and (4) are made up of the friction losses within the tube, H'_f, and the discharge loss at (3). Applying the general equation between points (2) and (4), we have

$$p_2 = - z_2 - \frac{V_2^2}{2g} + H'_f + \frac{V_3^2}{2g}. \tag{159}$$

The larger the diameter of the tube at its mouth the less will be the value of V_3 and hence the less the pressure at the point of discharge from the runner. But if too great a rate of diffusion is provided for in the tube the flow in it will be unstable and the friction loss H'_f will be increased, as shown in Art. 91. The pressure at the top of the draft tube should not be made less than about 5 ft. absolute, and the value of z_2 determined accordingly. A "high-speed" turbine runner with a large value of V_2 cannot be set as far above the water level as a "low-speed" turbine with a smaller value of V_2.

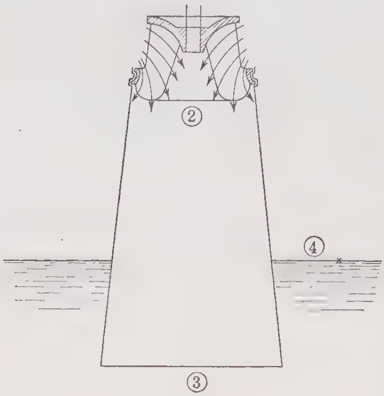


Fig. 225.

In the ideal case, the energy or head saved by a diverging draft tube would be

$$\frac{V_2^2}{2g} - \frac{V_3^2}{2g} = \left[1 - \left(\frac{A_2}{A_3}\right)^2\right]\frac{V_2^2}{2g}$$

where V_2 is the axial component of velocity. Due to friction losses within the tube, the actual saving is less. If the type of draft tube shown in Fig. 197 is used, the whirl component can be utilized also, as explained in Art. 136.

167. Head on Reaction Turbine.—For a reaction turbine the draft tube is an integral part of the machine, hence (Fig. 226) the head under which it operates is given by

$$h = H_c - H_f = p_c + z_c + \frac{V_c^2}{2g}. \tag{160}$$

This is the value of h upon which computations are based, and it is the one to be used in determining the efficiency of the turbine.

However, though the turbine maker usually constructs or designs the draft tube also, he is often limited by the conditions of the setting and may not be able to use the proper proportions. In order to eliminate this defect in the setting, over which he has no control, the velocity head at E is sometimes deducted from the value given by Eq. (160). If it were feasible to eliminate the friction in the draft tube as well, then there would remain

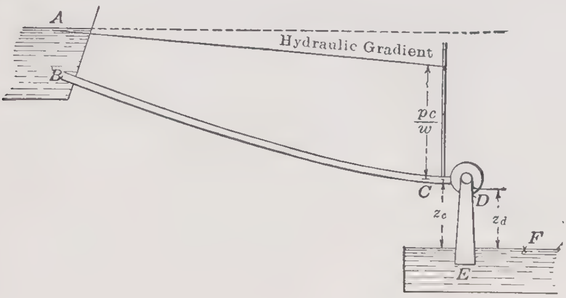


Fig. 226.

the efficiency of the runner alone, which is independent of the draft tube. But what is usually desired is the efficiency of the entire unit from the intake of the casing to the tailrace.

168. PROBLEMS

157. A certain reaction turbine was found by actual test to have a hydraulic efficiency of 0.83 when $\phi = 0.670$ and $c = 0.655$. The angles were: $\alpha_1 = 13$ deg. and $\beta'_1 = 115$ deg. Compute the values of ϕ_e and c_e and compare with the actual values. (The slight discrepancy between the two is largely due to the fact that shockless entrance and radial discharge were not obtained at exactly the same speed.)

Ans. $\phi_e = 0.678$, $c_e = 0.628$.

158. In the test of the Cornell University turbine the pressure was read by a mercury manometer attached near the intake flange where the diameter was 30 in. At full load when the discharge was 44.5 cu. ft. per second, the manometer read 9.541 ft. of mercury, the top of the shorter mercury column

being 0.500 ft. above the intake. If the elevation of the intake above the water level in the tailrace is 9.230 ft. find the head on the turbine.

Ans. 140.5 ft.

159. In the turbine of problem 158 the diameter of the draft tube at the upper end is 24.5 in. and at the bottom it is 42 in. Find the gain in head due to its use when the discharge is 44.5 cu. ft. per second.

Ans. 2.54 ft.

160. The top of the draft tube in problem 159 is 10.0 ft. above the level of the water in the tailrace. Neglecting the friction in it, but considering the discharge loss at the bottom, find the pressure at its top. What would it be if the diameter were uniform?

Ans. −12.54 ft., −10 ft.

161. A reaction turbine by test was found to discharge 31.8 cu. ft. of water per second when running at 600 r.p.m. under a head of 143.1 ft. If $A_1 = 0.535$ sq. ft. and $D = 27$ in., find values of c and ϕ.

Ans. $c = 0.619$, $\phi = 0.737$.

CHAPTER XVII

TURBINE LAWS AND FACTORS

169. Operation under Different Heads.—In the entire discussion in the two preceding chapters it has been assumed that the head remained constant though the other quantities might vary. But a turbine may be installed in a plant where the head changes from time to time, and also a given design of turbine might be used in different plants under a wide range of heads. Thus it is desired to investigate this phase.

Let us recall the expression, $u_1 = \phi\sqrt{2gh}$. Suppose now that a turbine is compelled to run at a constant speed while the head varies. It is clear that ϕ also varies then just as it would in the preceding case. But it would be possible under some circumstances to change the speed as well in such a way as to keep ϕ a constant. Hence it is necessary to consider two distinct cases when the head changes; one is where ϕ is constant, and the other is where ϕ also changes.

If ϕ remains constant, the wheel speed must vary as $\sqrt{h}$. But a definite value of ϕ is accompanied by a definite value of c. Hence the rate of discharge must also vary as $\sqrt{h}$, since $V_1 = c\sqrt{2gh}$. Now the power of the water is proportional to the product of q and h. Since q varies as $\sqrt{h}$, it follows that the power varies as $h^{3/2}$. And in similar fashion it may be shown that the torque varies as h.

The hydraulic efficiency is a function of c, ϕ, and the turbine dimensions. As long as ϕ remains constant the hydraulic efficiency remains the same regardless of the head. This must be true because the hydraulic losses may all be shown to vary as $h^{3/2}$, just as the power input. But the friction of the bearings and the windage or the disk friction of the runner do not vary in the same way. It is not possible to formulate an exact law for this but they may be said to vary between N and N^2. Since N varies as $\sqrt{h}$, they must vary between $h^{1/2}$ and h. Hence the mechanical losses become a smaller percentage of the total as

the head increases.[1] But, except for very low heads, the difference in the efficiency is usually a matter of not more than 2 or 3 per cent at most (see Fig. 227).

Now if the speed remains constant while the head changes, or if it does not vary as $\sqrt{h}$, the value of ϕ will change. Referring to Fig. 222, it may be seen that this means a change of c also. Hence the efficiency will change. Thus none of the simple proportions that have just been stated will be true in such a case. It is impossible to calculate the new results unless curves, such as those of Fig. 222, are available, or unless some complex equations are used which will give values of all these quantities for any value of ϕ.

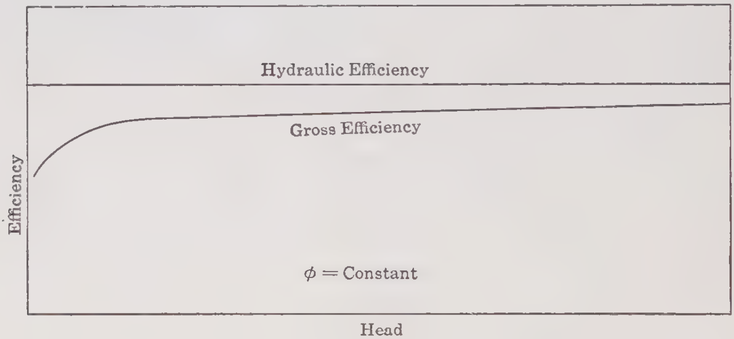


FIG. 227.—Effect of head upon efficiency of a given turbine.

170. Different Sizes of Runner.—If a series of runners are all built of the same design with the same angles and proportions so that one is simply an enlargement or reduction of another, they should all have the same values of ϕ_e and c_e. Since their peripheral speeds would all be the same under a given head it follows that their rotative speeds would be inversely proportional to their diameters. And the area A_1 would be proportional to D^2. Hence their capacity and power would vary as D^2. Thus if the performance of one runner is known, that of the rest of the series may be predicted with some assurance, due allowance being made for slight increases in efficiency as the size increases.

[1] An impulse wheel should be set with sufficient space on either side of the buckets at discharge so that the water rebounding from the walls will not strike them. The velocity with which the water rebounds is proportional to V_2 and hence to $\sqrt{h}$. Therefore, if this space is not ample for all values of h, a point may be reached where this action would decrease the efficiency.

Since

$$u_1 = \phi\sqrt{2gh}$$

and also

$$u_1 = \frac{2\pi\, r_1 N}{60} = \frac{\pi D N}{12 \times 60},$$

then

$$\phi = \frac{\pi D N}{12 \times 60 \times \sqrt{2g} \times \sqrt{h}}.$$

From this

$$\phi = \frac{DN}{1{,}840\sqrt{h}}$$

or

$$N = \frac{1{,}840\, \phi\sqrt{h}}{D} \tag{161}$$

where ϕ may have any value. But if ϕ_e is used the result is N_e, the speed at which the maximum efficiency is attained. For the two types of turbines in common use:

Impulse wheel $\quad \phi_e = 0.43$ to 0.48

Reaction turbine $\quad \phi_e = 0.60$ to 0.90

according to design. And as to capacity

$$q = K_1 D^2 \sqrt{h} \tag{162}$$

where K_1 has the following range of values:

Impulse wheel $\quad K_1 = 0.0002$ to 0.0005

Reaction turbine $\quad K_1 = 0.0014$ to 0.0360.

It must be understood that these constants are based upon values corresponding to ϕ_e and that the speed of the wheel must be such that ϕ_e is obtained if they are to apply.

Making a suitable allowance for the efficiency the power delivered by the turbines can be determined when the discharge is known.

It may be seen that the peripheral speed of the reaction turbine is higher than that of an impulse wheel and that it may be varied through a wider range by changes in the design. Also the values of K_1 show that for a given diameter a reaction turbine can discharge more water, and hence develop more power, than an impulse turbine. That means that if they are to deliver the same power the diameter of a reaction turbine will be much less than that of a corresponding impulse wheel. Thus for a given head and power the rotative speed of the reaction turbine will be much

higher than that of the impulse wheel, due both to its higher peripheral speed and to its much smaller size.

171. Specific Speed.—A useful factor in turbine work is one that will now be derived. It involves the head, speed, power, and efficiency.

Since power is proportional to D^2 and to $h^{3/2}$, b.hp. $= K_2 D^2 h^{3/2}$. This may be rewritten as

$$\sqrt{K_2}\,D = \frac{\sqrt{\text{b.hp.}}}{h^{3/4}}.$$

Inserting the value of D as given by Eq. (161) then

$$\sqrt{K_2}\,\frac{1{,}840\phi_e\sqrt{h}}{N_e} = \frac{\sqrt{\text{b.hp.}}}{h^{3/4}}.$$

Rearranging this and letting $\sqrt{K_2}\,1{,}840\phi_e = N_s$, then

$$N_s = \frac{N_e\sqrt{\text{b.hp.}}}{h^{5/4}*}. \qquad (163)$$

While any value of N might be used, the expression has but little meaning unless a particular value is employed. That is generally understood to be N_e, the value of N at which the maximum efficiency under a given head is attained. As to the value of brake horsepower it should logically be the one for which the maximum efficiency is obtained under the given head. But in some cases the value of the maximum power at this same speed is used. The physical significance of this factor is that if a turbine of a given design is enlarged or reduced in size so that it will develop 1 hp. under 1 ft. head, then N_s is the value of its r.p.m. under those circumstances.

The quantity N_s is generally known as *specific speed*. Other names applied to it are unit speed, type characteristic, and characteristic speed. Its value indicates the class to which a turbine belongs. Thus it has been seen that for a given head and power the impulse wheel runs at a relatively low r.p.m. Therefore it would have a low value of N_s.

For an impulse wheel under a given head at a given speed the power would increase with the size of the nozzle used. Thus there need not be any lower limit to the value of N_s but the upper limit would be the one for which the maximum size jet that could be employed was obtained. It is found that the efficiency is not appreciably reduced until after a value is passed

* $h^{5/4} = h \times h^{1/4} = h\sqrt{\sqrt{h}}$.

of about 4.5 for N_s, and after a value of 6 the jet is so large for the size of the wheel that the efficiency drops off materially. But any value above 4.5 involves some sacrifice of efficiency.

For the reaction turbine there are limits in both directions as indicated below, though these may be extended in future designs. The values of the specific speed are

Impulse wheel	$N_s = 0$ to 4.5 (6 max.)
Reaction turbine	$N_s = 10$ to 100
Propellor type	$N_s = 80$ to 165.

For a given turbine the value of N_s is naturally a constant, but it is also practically constant for a whole series of runners of the same design regardless of size. The larger the diameter of a runner the greater its power, but the less the value of N for a given head. Hence the product remains constant.

Values of N_s given for the impulse wheel are for a single jet upon a single wheel. When two or more jets are used the power is naturally increased without changing the speed. This enables values between 6 and 10 to be obtained, if necessary. For values above 165 the conditions are impossible with present practice. Either the power or the speed of the unit must be decreased.

The specific speed factor shows that the impulse wheel is a low-speed, low-capacity turbine and the reaction turbine is a high-speed, high-capacity turbine. The use of these words is relative rather than absolute. Thus the turbine in Fig. 186 runs at only 55.6 r.p.m. but its specific speed is 82.3, thus indicating that it is a high-speed wheel. For the speed is high as compared with that of other turbines of the same power under that head. For instance the speed of an impulse wheel for similar conditions would be only 4 r.p.m. And the specific speed of the highest head impulse wheels in the world (Art. 145) is only 0.592 though they run at 500 r.p.m. But a slow-speed reaction turbine under the same conditions would run at 8,450 r.p.m. at least, and a high-speed reaction turbine such as those at Cedars Rapids would run at 69,300 r.p.m. Of course these values are absurd and simply demonstrate the fitness of each type for its own field.

172. Uses of Specific Speed.—The values of N_s, as of all other factors in this chapter, are supposed to be obtained from test data, not computed by theory. They serve to classify

a turbine and indicate to what type it belongs. They are useful in selecting units for a prospective plant. For such a case the head is known but the size and speed of the units are not. If it is desired to use wheels of a certain type, that fixes the value of N_s between narrow limits, and it is easy to compute the combinations of speed and power that can be produced. Or, if the speed and power be fixed, it may at once be found what type of turbine would be required.

173. Factors Affecting Efficiency.—The efficiency of the impulse wheel is practically independent of the size of the wheel. The author makes this statement after testing sizes from 12 to 84 in. in diameter and comparing all the other test data which are accessible. It would seem reasonable that this should be so, for there is no loss in connection with the impulse wheel which would not vary in proportion to the power of the wheel. Aside from questions of design and workmanship the efficiency would appear to be a function of the specific speed. Too low a value of the specific speed would mean a large diameter of wheel for a given power output with a consequently large friction and windage loss. Too high a specific speed would mean that the jet was too large for the wheel and buckets with a consequent lowering of the hydraulic efficiency. The most favorable value of N_s is about 4.0 and the best efficiency that is obtained is about 82.0 per cent. This is slightly exceeded at times, and values below it are often obtained.

With the reaction turbine the efficiency is a function of its size. This is partly due to the fact that the hydraulic efficiency increases with the size, but more to the fact that the volumetric efficiency increases. With a reaction turbine there is always a certain amount of leakage between the guides and the runner so that a portion of the water escapes through the clearance spaces and does not pass through the wheel. The area of these clearance rings would naturally be less in proportion to the area of the wheel passages as the size of the wheel increases. Hence a much larger per cent of the water is made to deliver its power to the runner. Such a condition does not exist with the impulse wheel. This leads to comparative values for the two types as shown in Fig. 228.

Another distinction between the two types of turbines is that the reaction turbine suffers certain hydraulic losses on part gate that are lacking in the other. Hence, although in some

cases the maximum efficiency of a reaction turbine is greater than that of the impulse wheel, the efficiency on a light load might not be as good.

Like the impulse turbine the efficiency of the reaction turbine also depends upon the specific speed, being less at either extreme. The best efficiencies are obtained with values of N_s ranging from 30 to 60. The efficiency of a turbine of good design and workmanship depends upon size, specific speed, and other factors to such an extent that definite values cannot be given, but for fair size units it should range from 80 to 90 per cent and occasionally more. For small wheels, especially with unfavorable specific

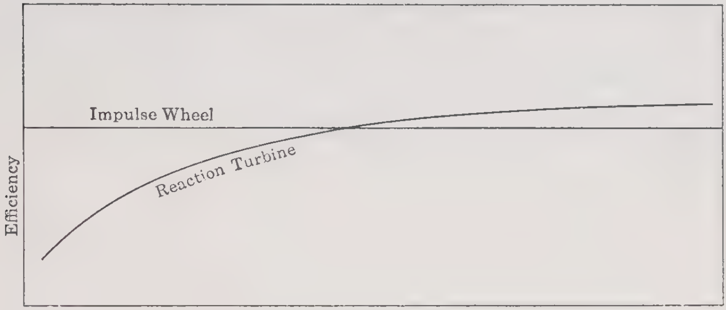


FIG. 228.—Effect of size of turbine upon its efficiency.

speeds, a value of from 60 to 80 per cent is all that should be expected.

174. PROBLEMS

162. The turbine, whose performance is shown in Fig. 223, developed its maximum efficiency of 88.0 per cent when delivering 550 hp. at 600 r.p.m. under a head of 141.8 ft. The water consumed was 38.8 cu. ft. per second What would be its proper speed under a head of 283.6 ft.? What would then be the rate of discharge and the horsepower?

Ans. 848 r.p.m., 54.8 sec. ft., 1,557 hp.

163. In Fig. 222 the turbine delivered 617 hp. when running at 600 r.p.m. under a head of 140.5 ft., the rate of discharge being 44.5 cu. ft. per second and the efficiency 87.0 per cent. If the speed is maintained at 600 r.p.m. when the head is 70.2 ft., find values of discharge, power delivered, and efficiency. (Note: This can be determined only by making use of the curves for this particular turbine. The procedure would be to find the value of ϕ for the new conditions and then take values of q, *horsepower*, and e from the curves. These quantities would then have to be reduced to the proper values for the new head.)

Ans. 29 sec. ft., 159 hp., 0.68.

164. It is desired to develop 6,000 hp. at 514 r.p.m. under a head of 625 ft. Will an impulse or reaction turbine be required? (This can be determined by computing the specific speed.) If only 900 hp. is to be developed for the conditions given, what type of turbine will be required? It is desired to use a type of turbine whose specific speed is 30 to deliver 100 hp. under a head of 100 ft. What will be the proper r.p.m. for the unit?

Ans. N_s = 12.75, r.p.m. N_s = 4.94, r.p.m. N_e = 948 r.p.m.

165. (*a*) An impulse wheel is to be used for 625 hp. under a head of 144 ft. What will be the maximum rotative speed at which it can run without material sacrifice of efficiency? What will be the approximate diameter of the wheel? (*b*) What would be the minimum speed for a reaction turbine for the conditions If $\phi = 0.60$, what would be the diameter of the runner? (*c*) What would be the maximum speed for a reaction turbine? Assuming $\phi = 0.85$, what would be the diameter of the runner?

Ans. (*a*) 90 r.p.m., 118 in., (*b*) 199 r.p.m., 66.4 in., (*c*) 1,990 r.p.m., 9.43 in.

166. The runner in the Cornell University turbine is 27 in. in diameter. The wheel develops 550 hp. when running at 600 r.p.m. under a head of 141.8 ft. What would be the speed and power of a 54-in. runner of the same type under the same head? Would the specific speed of these two be the same?

Ans. N = 300 r.p.m., 2,200 hp.

CHAPTER XVIII

THE CENTRIFUGAL PUMP

175. Definition.—Centrifugal pumps are so called because of the fact that centrifugal force or the variation of pressure due to rotation is an important factor in their operation.

In brief, the centrifugal pump consists of an impeller rotating within a case as shown in Fig. 229. Water enters the impeller at the center, flows radially outward, and is discharged around the circumference into the case. During flow through the impeller the water has received energy from the vanes resulting in an increase both in pressure and velocity. Since a large part of the energy of the water at discharge is kinetic, it is necessary to conserve this kinetic energy and transform it into pressure, if the pump is to be efficient.

As a matter of convenience in illustration, the water is represented as entering the impeller in Fig. 229 with a positive pressure. However, the pump is usually set above the level of the water from which it draws its supply, in which case the pressure at this point would be negative. Likewise, the axis of rotation need not be vertical as shown.

176. Classification.—Centrifugal pumps are broadly divided into two classes:

1. Turbine pumps.
2. Volute pumps.

While there are still other types these two are the most fundamental. Also, as will be seen, these may in turn be subdivided in other ways.

The *turbine pump* is one in wry g the impeller is surrounded by a diffuser containing stationanlaruide vanes as shown in Fig. 230. These provide gradually enlarging passages whose function it is to reduce the velocity of the water leaving the impeller and thus efficiently transform velocity head into pressure head. The casing surrounding the diffuser may be eighte circular add concentric with the impeller or it may be spiral like the case of some reaction turbines.

The *volute pump*, shown in Fig. 231, is one which has no diffusion vanes but, instead, the casing is of a spiral type so proportioned as to produce an equal velocity of flow all around the circumfer-

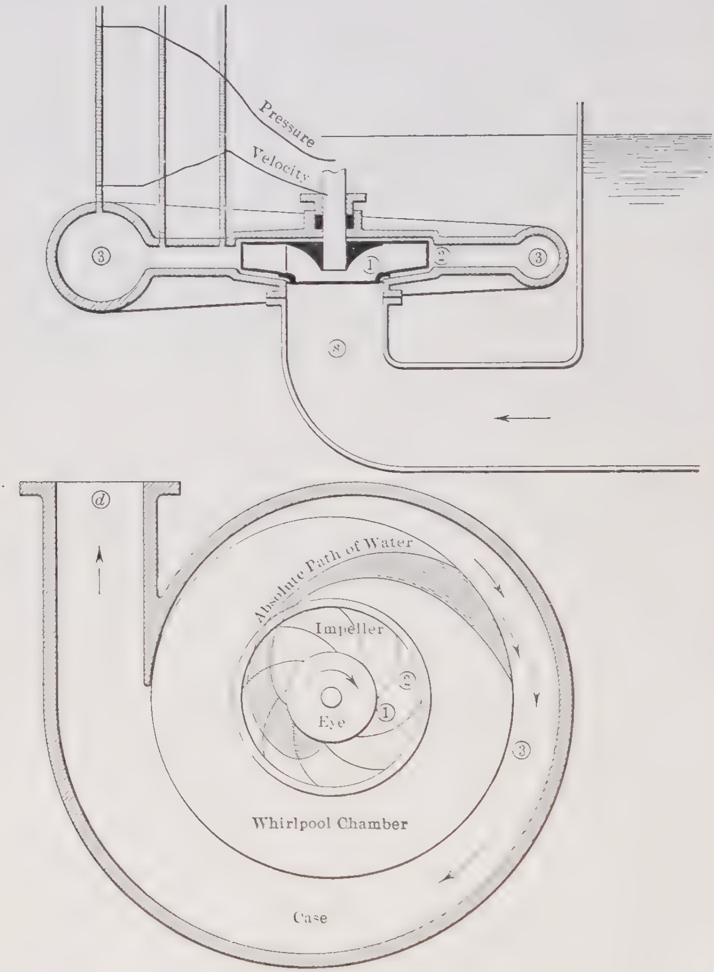


Fig. 229.

ence and also to reduce gradually the velocity of the water as it flows from the impeller to the discharge pipe. Thus the energy transformation is accomplished in a slightly different way. This spiral is often called a volute, whence the name of the pump.

Occasionally pumps have been built with a whirlpool chamber as shown in Fig. 229. This produces a free spiral vortex, the nature of which has been shown in Art. 136.

177. Description of the Centrifugal Pump.—The centrifugal pump is similar to the reaction turbine both in its construction

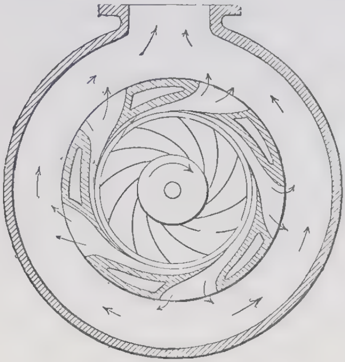


FIG. 230.—Turbine pump.

and in its theory. However, one is not the reverse of the other, and their differences are as striking as their similarities.

The rotating part of the pump which is instrumental in delivering the water is called the *impeller*. Impellers may

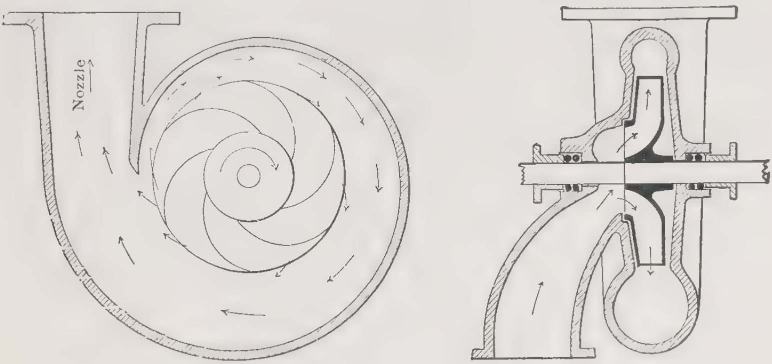
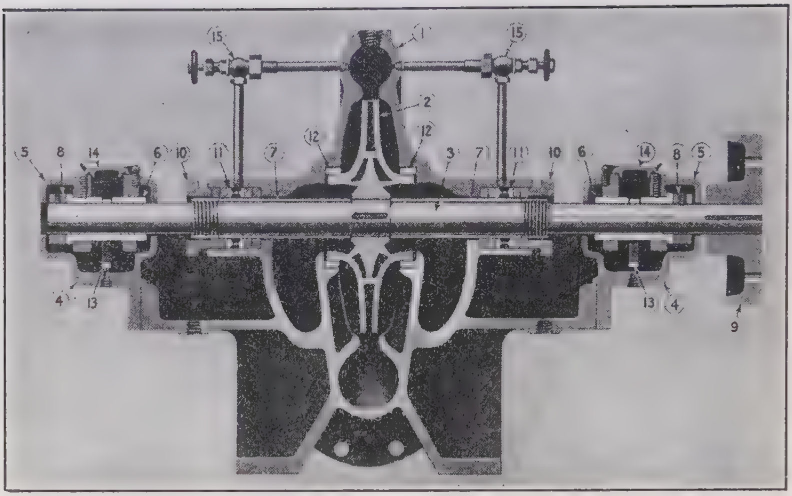


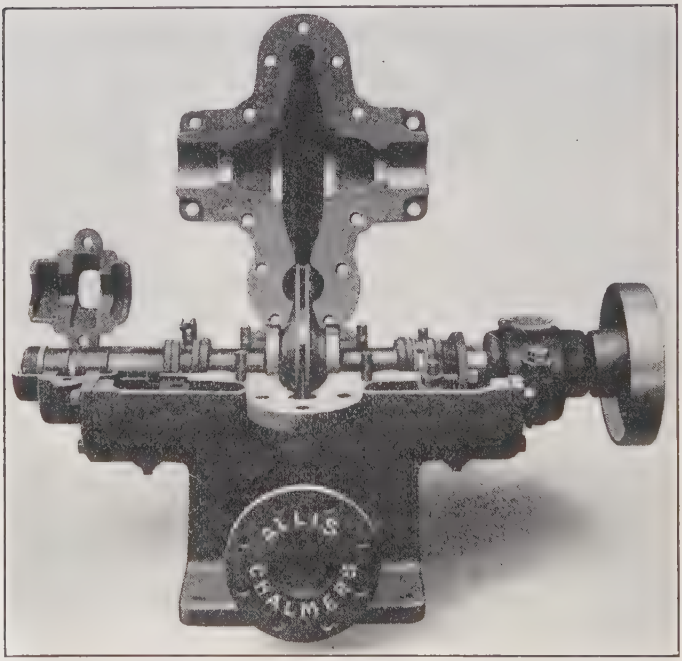
FIG. 231.—Volute pump.

receive water on one side only or, as in Fig. 232, from both sides, in which case they are known as double-suction impellers. Figure 233 gives a view of the pump whose section is seen in Fig. 232. It may be seen that this impeller is relatively narrow as compared with its diameter, while the opposite type is



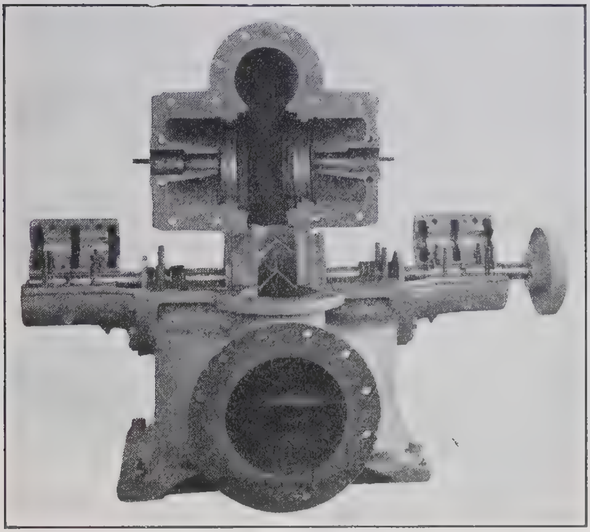
Courtesy of the Allis-Chalmers Mfg. Co.

FIG. 232.—Double-suction volute pump.



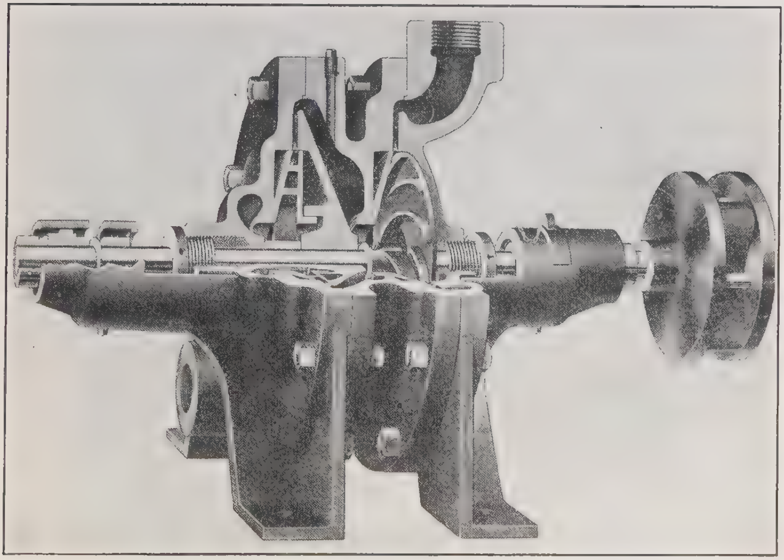
Courtesy of the Allis-Chalmers Mfg. Co.

FIG. 233.—Double-suction volute pump.



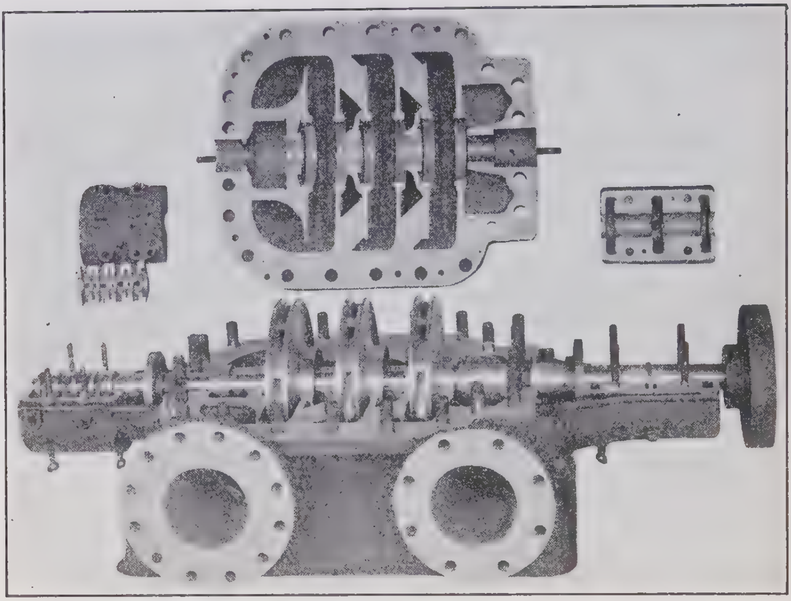
Courtesy of Platt Iron Wks.

FIG. 234.—Double-suction volute pump.



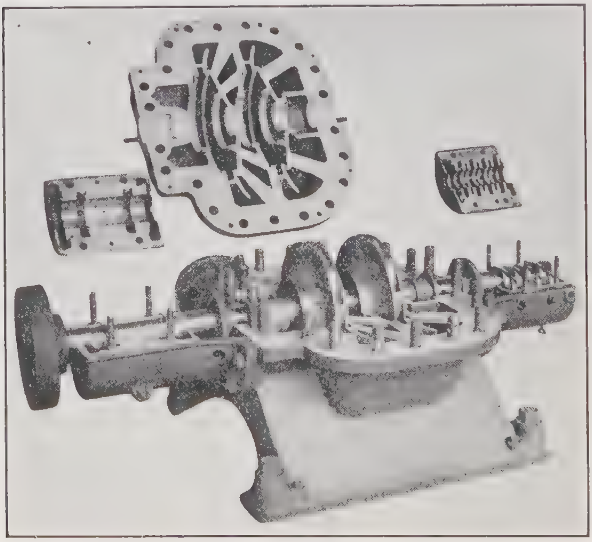
Courtesy of Chicago Pump Co.

FIG. 235.—Two-stage turbine pump.



Courtesy of Platt Iron Wks.

FIG. 236.—Three-stage centrifugal pump without diffusion vanes.



Courtesy of Platt Iron Wks.

FIG. 237.—Two-stage centrifugal pump with diffusion vanes.

shown in Fig. 234. For the same rotative speed the latter will discharge more water than the former but at a lower head.

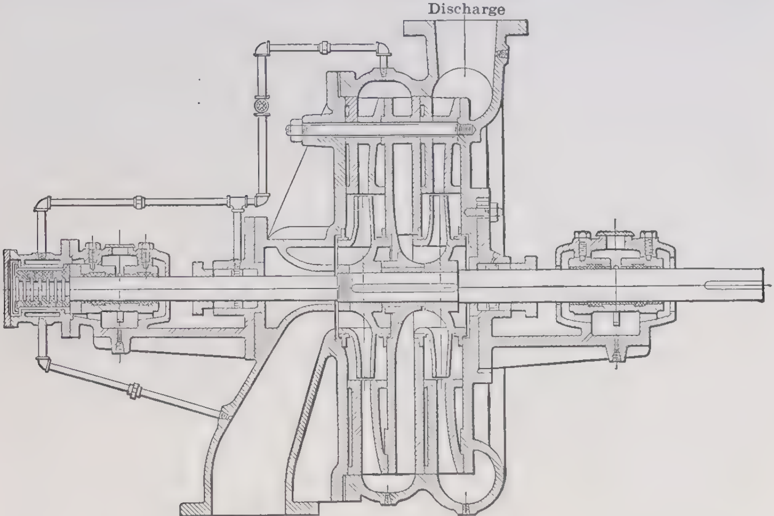


Fig. 238.—Worthington two-stage turbine pump.

Courtesy of Allis-Chalmers Mfg. Co.

Fig. 239.—72-in. centrifugal pump for drainage at Memphis. $h = 15'$; $N = 100$. Capacity, 194,000,000 gal. per day.

For high heads it becomes desirable to place impellers in series, in which case there is the multi-stage pump, such as is shown

in Fig. 235. Multi-stage pumps may be either of the turbine or the volute type. The former may be seen in Fig. 235 and the latter in Fig. 236. The addition of guide vanes so as to produce a turbine pump results in a much more complex construction, as Fig. 237 will show. The water in a multi-stage turbine pump usually passes from one impeller to the next through passages which are like those shown in Fig. 238. There are other arrangements besides this, but they will not be described here.

178. Conditions of Service.—Centrifugal pumps are used for lifting water a few feet only or as much as several thousand feet, if necessary. Several such pumps have been built for heads of 2,000 ft.

The capacities of centrifugal pumps range from very small quantities up to as high as 300 cu. ft. per second (134,500 g.p.m. or 194,000,000 gal. per 24 hr.). The I. P. Morris Co. has built several of the latter for a head of 16 ft. at 77.5 r.p.m.

The greatest power of any centrifugal pump is that of a pump installed by Sulzer Bros. in Italy. A single-stage pump running at 1,002 r.p.m. delivers 32,530 g.p.m. at a head of 498.6 ft. with an efficiency of 81.0 per cent. The water horsepower is 3,590 and the power required to run it is 4,430 hp. For most pumps the power required is less than 500 hp.

Rotative speeds may vary all the way from 30 to 3,000 r.p.m. in ordinary practice according to circumstances. The highest speed ever employed was 20,000 r.p.m. for a single-stage volute pump with an impeller 2.84 in. in diameter. The pump delivered 250 g.p.m. against a head of 700 ft. with an efficiency of 60.0 per cent. The highest peripheral speed used was with a single-stage pump with an impeller 3.15 in. in diameter. At 18,000 r.p.m. it delivered 189 g.p.m. against a head of 863 ft. and for a smaller discharge it developed a head of 995 ft.

Centrifugal pumps have been built with as many as twelve stages. It is customary to limit the head per stage to a value of not more than 100 to 200 ft., but this has been greatly exceeded in several cases mentioned above.

Water turbines are rated according to the diameters of their runners, but the size of a centrifugal pump is usually designated by giving the diameter of the discharge pipe. The rated head and discharge for a centrifugal pump are the values for which the efficiency is a maximum under a given speed. This value

of the rate of discharge is often designated as the *normal discharge.* These values will be different for different speeds.

179. Head Developed.—The head developed by a centrifugal pump when no flow occurs is called the "shut-off head" or the "head of impending delivery." Its value may be found by applying the principles of Art. 135.

If water in a closed chamber be set in motion by a paddle wheel as in Fig. 240, there will be an increase in pressure from

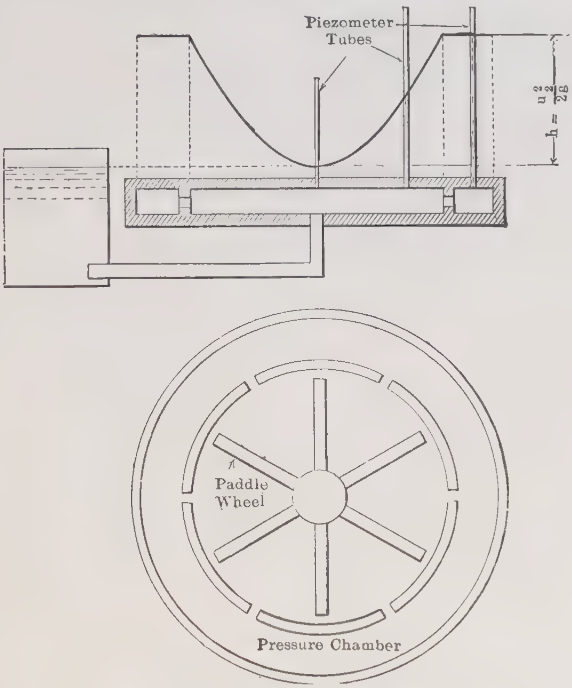


FIG. 240.—Crude centrifugal pump.

the center to the circumference. If the water is assumed to rotate at the same speed as the impeller, the peripheral velocity of which is u_2, it may be seen from Eq. (137) that $p_2 - p_1 = u_2^2/2g$, where p_1 denotes the pressure at the center. If this water is in communication with a pressure chamber to which a piezometer tube is attached, as in Fig. 240, water will rise in the latter to such a height that

$$h = \frac{u_2^2}{2g}. \qquad (164)$$

If the height of the tube were less than this, water would flow out and the result would be a crude centrifugal pump.

Actually there are certain influences at work in the real pump which affect this relation slightly. Some of these factors tend to increase the head and others to decrease it. The net effect is that for the usual type of centrifugal pump the head of impending delivery is

$$h = 0.85 \text{ to } 1.10\,\frac{u_2^2}{2g}. \tag{165}$$

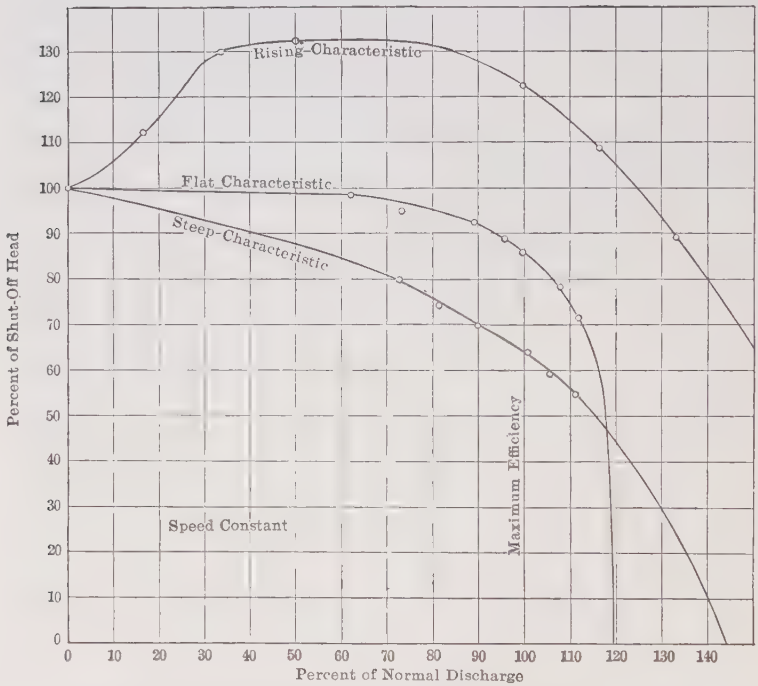


FIG. 241.—Head-discharge characteristics of different pumps.

But as soon as flow occurs the above relation is no longer true, as may be seen in Figs. 241 and 242. When water is being delivered, the head may be either greater or less than the shut-off head, according to the design of the pump. A general relation will now be derived between head, impeller speed, and rate of discharge for all conditions of operation.

As the water in the suction pipe approaches the impeller it may have a rotary motion imparted to it before it ever reaches

the latter, due to the viscosity of intervening particles of water. Hence, equations will be written between points (2) and (s) in Fig. 229, the latter point being removed far enough from the impeller so that the water has no rotational flow imparted to it

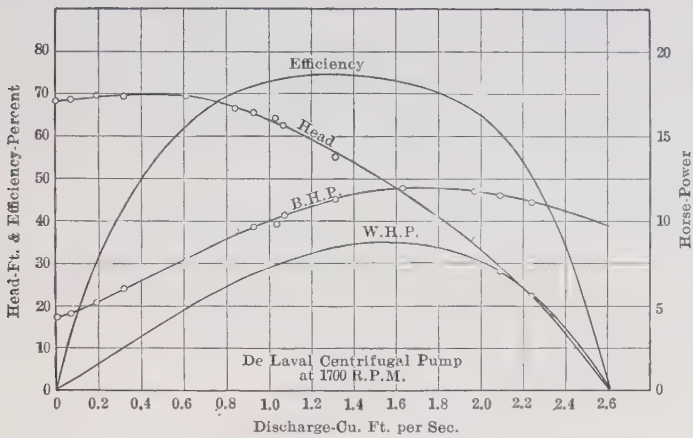


FIG. 242.—Characteristics of a 6-in. pump at a constant speed.

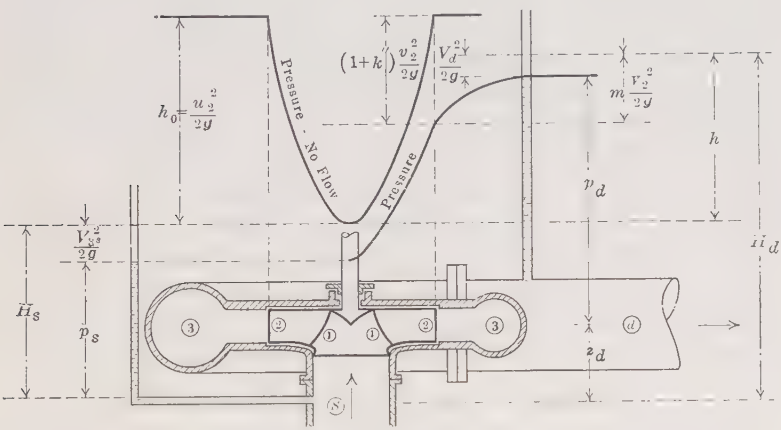


FIG. 243.

In Fig. 243 is shown the hydraulic gradient in the case of zero flow and, as in Eq. (164),

$$h_o = \frac{u_2^2}{2g}.$$

When flow occurs there will be a drop in pressure at (2), which is just within the impeller, due to the velocity head at that point and also due to the loss of head within the impeller passages or from (s) to (2). If k'' is the coefficient of loss the drop in pressure at the outlet of the impeller will be $(1 + k'')v_2^2/2g$.[1] But the pressure at (s) likewise decreases by an amount equal to $V_s^2/2g$. Also as the water flows from (2) to (d) in Fig. 243, there is a reduction in velocity head from $V_2^2/2g$ to $V_d^2/2g$. This means a corresponding gain in pressure, though not without some loss. Thus $mV_2^2/2g$ is converted into pressure plus $V_d^2/V2g$, the loss being $(1 - m)V_2^2/2g$.

From Eq. (168) the total head developed by the pump, including impeller and case, is (see Fig. 243)

$$h = H_d - H_s = \left(p_d + z_d + \frac{V_d^2}{2g}\right) - \left(p_s + z_s + \frac{V_s^2}{2g}\right)$$

$$= h_0 - \frac{(1 + k'')\, v_2^2}{2g} + \frac{mV_2^2}{2g}$$

$$= \frac{u_2^2}{2g} - (1 + k'')\frac{v_2^2}{2g} + m\frac{V_2^2}{2g}. \qquad (166)$$

In reality there will be a further drop in pressure at (s) when flow takes place, due to the loss of head in friction in the suction pipe. However, this would also have the effect of decreasing the pressures at (2) and (d) by the same amount. Hence the difference in pressure, with which we are here concerned, would be exactly the same.

It must be noted that the quantity m is a variable. When the discharge from the impeller is such that the angle α_2 (see Fig. 244) agrees with the angle of the diffusion vanes of a turbine pump, or the velocity V_2 is the proper value for a volute pump, the maximum proportion of the velocity head will be saved. For larger or smaller discharges than this there will be additional losses attending this conversion. For a turbine pump the

[1] As an illustration consider a hose with a nozzle on the end. When the nozzle is opened so that water may flow, the pressure at the base of the nozzle is decreased below the value obtained when it is closed, by an amount equal to the velocity head at that point and to the friction losses up to that point. If next the hose should be moved around, this pressure drop would not be affected in the least, for it is a function of the velocity of flow within the hose, which is the relative velocity, and does not depend upon the velocity of the water with respect to the earth.

maximum value of m is about 0.75 and for a volute pump it is somewhat less.

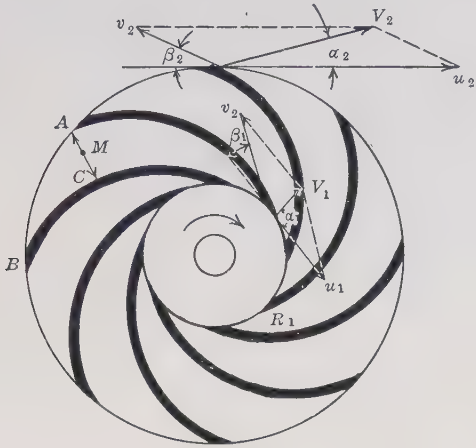


FIG. 244.

With a centrifugal pump the impeller areas are fixed and constant in value and hence it is convenient to express the rate of discharge as

$$q = a_2 v_2. \tag{167}$$

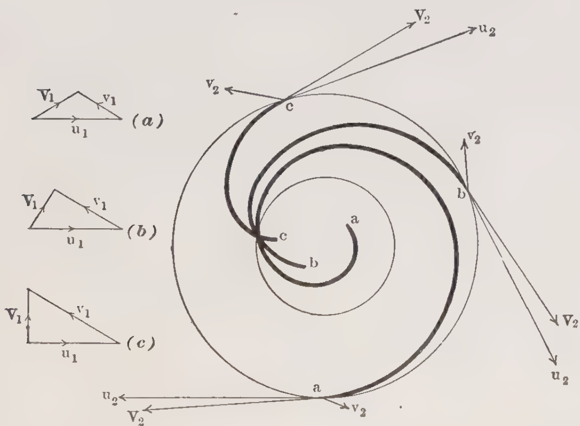


FIG. 245.—Stream lines for three different rates of discharge.

Now referring to the vector diagrams shown in Figs. 244 and 245 it may be noted that as the rate of discharge varies the values of V_2 and α_2 change. It may be seen that as q approaches

zero, v_2 and α_2 approach zero, while V_2 approaches u_2. Hence for an infinitesimal discharge the value of V_2 may be regarded as equal to u_2, while the velocity of the water in the case surrounding the impeller is practically zero. Therefore, a particle of water leaving the impeller with a high velocity enters a body of water at rest and loses all of its kinetic energy. Thus, as the rate of discharge approaches zero, the factor m approaches zero. Hence it may be seen that when there is zero discharge the value of h in Eq. (166) reduces to that given by Eq. (164).

An inspection of Eq. (166) serves to explain the rising or falling characteristics of Fig. 241. If the increase of pressure due to the conversion of the velocity head of discharge is more than enough to offset the decrease due to the velocity and the losses within the impeller, there exists a rising characteristic. If they are about equal there is a flat characteristic, and if the quantity mV_2^2 is less than $(1 + k'')v_2^2$ there is a falling characteristic.

Because of the difficulty of efficiently transforming velocity head into pressure head it is desirable to keep V_2 as small as possible. It may be seen that, for a given value of v_2, the smaller the angle β_2 the less will be the magnitude of V_2. Therefore, in almost all centrifugal pumps the value of β_2 is from 20 to 30 deg., though occasionally this angle is as small as 10 deg. or as large as 80 deg. It is rarely made larger than 90 deg. because of the inefficiency of such designs.[1]

180. Measurement of Head.—The head which a pump is required to work against may be computed by Eqs. (98) or (100). The head which the pump can develop may be estimated by Eq. (166), but when it is desired to measure the head which the pump actually does develop it is done by taking certain readings on the discharge and suction sides of the pump. Thus in Fig. 246 the difference between the energy which the water has as it enters the pump at (s) and that with which it leaves at (d) is due solely to the pump. Hence,

$$h = H_d - H_s.$$

[1] In the turbine theory the angle β has been defined as the angle between v and u. This is satisfactory for that purpose, but with ordinary centrifugal pumps this angle is always greater than 90 deg. Hence it is much more convenient to define it here as the angle between v and $-u$, as may be seen in Fig. 244.

But

$$H_d = p_d + z_d + \frac{V_d^2}{2g},$$

and

$$H_s = p_s + z_s + \frac{V_s^2}{2g}.$$

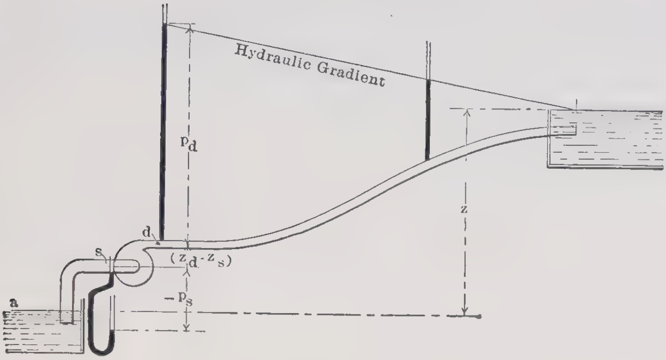


FIG. 246.—Head developed by pump.

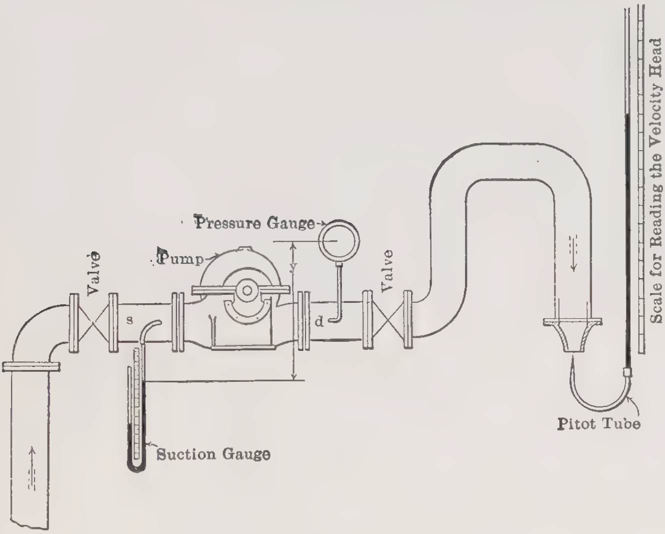


FIG. 247.—Measurement of head.

Therefore, it follows that

$$h = (p_d - p_s) + (z_d - z_s) + \left(V_d^2 - \frac{V_s^2}{2g}\right) \qquad (168)$$

As a usual thing the water enters the pump under a pressure less than that of the atmosphere, in which case the value of p_s

will be negative. If the suction and discharge pipes at the points where the gases are attached are of the same diameter the velocity heads will cancel, in which event the value of h will be the difference in the levels of the surfaces of the two water columns shown in Fig. 246.

In testing the pump the gages might be connected as shown in Fig. 247. It is not really necessary to reduce the gage readings to the pressures which would be found at the center line of the pipe. If the gage readings are used direct in Eq. (168) and the value of y represents $(z_d - z_s)$, it may be shown that the result is the same.[1]

181. Head Imparted by Impeller.—The amount of energy delivered to the water by the impeller is greater than that actually delivered in the water, the difference being due to hydraulic friction losses within the pump. If the head actually developed by the pump is represented by h, the head imparted to the water by the impeller is

$$h'' = h + h'$$

or

$$h'' = \frac{h}{e_h}.$$

In an ideal pump without hydraulic losses of any kind these two quantities would be equal, but in any real pump they represent two entirely different things. For a given pump under different conditions of operation, h'' and h neither differ from each other by a constant amount nor is one a constant proportion of the other. Hence the curves representing actual values of both h'' and h not only do not coincide but they are not even of the same

[1] The question is often raised as to why it is necessary to deduct $V_s^2/2g$ in determining the head, since the pump has imparted that velocity to the water. The first answer is that Eq. (168) is the result of a direct application of the principles of energy, but the explanation of the matter is that p_s, whose value is a function of V_s, has also been included. Suppose, for example, that the suction pipe were so large that the velocity in it were negligible. Then the measured value of p_s would give a higher pressure than when the suction pipe is smaller and, disregarding losses, the values of p_s in the two cases would differ by $V_s^2/2g$. If the velocity head at s is to be omitted the pressure reading should also be omitted. The total head might then be obtained by adding to the "discharge head" the value of z_s + *suction-pipe losses.* But the latter would have to be computed and it may be shown that they are determined experimentally when p_s is measured and the velocity head $V_s^2/2g$ also employed.

shape. This may be seen in Fig. 248 in which the curve "*Actual Head Input—h''*" has been determined with a reasonable degree of accuracy from test data by the author. It may be seen that for rates of discharge from 0 to 0.6 cu. ft. per second the value

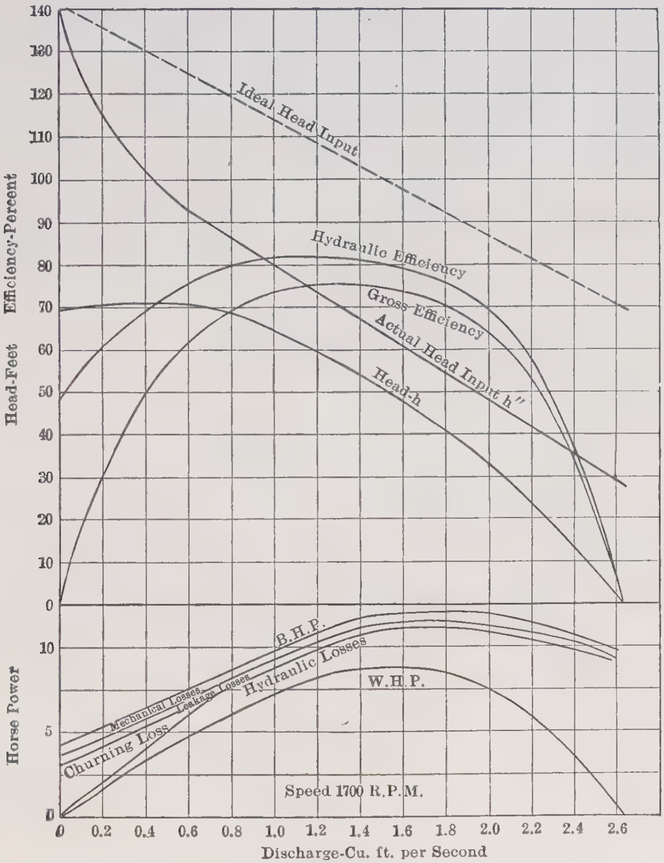


FIG. 248.—Analysis of centrifugal pump at a constant speed.

of h increases while that of h'' decreases. In some other cases the difference is more marked than is here shown.

An expression for h'' may be derived by an application of the principles in Arts. 130 and 132. Since $h'' = \mu_2 V_2 \cos \alpha_2 / g$ and $V_2 \cos \alpha_2 = \mu_2 - v_2 \cos \beta_2$,

$$h'' = \frac{u_2(u_2 - v_2 \cos \beta_2)}{g}. \qquad (169)$$

This could also be obtained from Eq. (166) by eliminating the hydraulic losses. This would require values of $k'' = 0$ and $m = 1.0$. The next step would be to solve the vector triangle for V_2 in terms of u_2, v_2, and β_2. The result would agree with Eq. (169).

It may be seen that for a constant value of impeller speed the value of h'' given by Eq. (169) will increase with q(and v_2); for values of β_2 greater than 90 deg.; will be independent of q for $\beta_2 = 90$ deg.; and will decrease for values of β_2 less than 90 deg. It is sometimes argued from this that rising or falling characteristics are obtained by a suitable choice of β_2 but that is due to confusing h'' and h. The value of β_2 does have some effect upon this, but it alone does not determine the matter. The author has found a decidedly rising characteristic in a pump he has tested with an angle of $\beta_2 = 26$ deg. And tests of other pumps with $\beta_2 = 90$ deg. have shown steep falling characteristics. The real explanation may be seen only in Eq. (166).

The hydraulic efficiency is the ratio h/h''. For the same reasons as are given in Art. 162, it is difficult to calculate true values of h'' and thus the *true* hydraulic efficiency can be determined only by test. The latter gives directly the value of the total efficiency only, and it is necessary to allow for other losses or determine them by special methods in order to get the actual hydraulic efficiency. Applying Eq. (169) and using the actual impeller dimensions the computed values of h'' may be found to lie on a straight line, such as is labeled "*Ideal Head Input*" in Fig. 248. Thus the ratio between the actual h and the h'' computed in the ordinary manner is much less than the hydraulic efficiency in all cases. It is very often less than the gross efficiency, thus proving that it is not a true value. But it is still useful for some purposes of design and is called "manometric coefficient." The value of this ratio is usually between 0.55 and 0.65.

182. Centrifugal-pump Factors.—Just as in the case of turbines, it is found that to obtain the best efficiency with a given centrifugal pump there must be a certain relation between head, speed, and discharge. Also the equations show that these three quantities are mutually interrelated. Hence it may be seen from Eq. (166) that for a velocity diagram of the same

shape to be formed it is necessary that u_2, v_2, and V_2 vary as $\sqrt{h}$. Hence should be written[1]

$$u_2 = \phi\sqrt{2gh} \tag{170}$$

$$v_2 = c\sqrt{2gh}. \tag{171}$$

It is found that a certain value of ϕ is required to obtain the maximum efficiency just as with the turbine. And a definite value of c is associated with every value of ϕ as may be seen from Eq. (166). For ordinary types of pumps the following values of these factors are found:

For shut-off $\phi = 0.95$ to 1.09
For normal discharge $\phi_e = 0.90$ to 1.30
For normal discharge $c_e = 0.10$ to 0.30.

The value of ϕ_e will depend upon the design of the pump. Thus the smaller the angle β_2 and the fewer the number of impeller vanes, the larger the value of ϕ_e.

Just as in Art. 170, it may be shown that

$$N = \frac{1{,}840\phi\sqrt{h}}{D}. \tag{172}$$

183. Specific Speed.—The specific speed factor for turbines involves the developed horsepower, since that is the quantity of particular interest. But with centrifugal pumps the primary interest is in their capacity, and it will be more useful if a similar expression, giving N_s in terms of discharge, is derived. Since power and discharge are really proportional to each other it may be seen that the specific speed for the pump is merely being expressed in terms of different units.

Proceeding just as in Art. 171, except that Eq. (162) is used direct, for the centrifugal pump

$$N_s = \frac{N\sqrt{\text{g.p.m.}}}{h^{3/4}\,*} \tag{173}$$

The capacity of a centrifugal pump is generally expressed in gallons per minute rather than in cubic feet per second. Thus

[1] For the pump ϕ has the same meaning as in the inward flow reaction turbine since it gives the peripheral speed in both cases. But c has a different meaning, since it is more convenient to deal with v_2 rather than with V_2.

* Note that $h^{3/4} = h \div h^{1/4} = h \div \sqrt{\sqrt{h}}$. Some values of $h^{3/4}$ will be found on p. 322.

the expression will probably be the handiest in the above form. (1 cu. ft. = 7.48 U. S. gal.)

For an impeller, either single-suction or double-suction, values of specific speed may be found between the following limits:

$$N_s = 500 \text{ to } 8{,}000.$$

For special constructions even higher values may be attained. It must be noted that these apply only to single stages. For a multi-stage pump it is necessary to divide the total head by the number of stages to obtain the proper value of h for use in the equations in this chapter.

Values of specific speeds are obtained from tests of actual pumps and they may then be applied to other pumps of the same type. For N_s is an index of the type of pump just as it is in the case of the turbine. Its great value is that it enables us to determine the combinations of speed, capacity, and head per stage that are possible or desirable. And if it is desired to employ a certain type of pump with a definite value of N_s, the combinations of these factors that are required may then be found.

184. Operation at Different Speeds.—In this chapter have been shown the characteristics of centrifugal pumps operating under variable heads at constant speeds. It is now desired to know how the pump is affected by a change in speed. This is shown by Eqs. (170) and (171). To obtain similar conditions of operation it is necessary that the values of ϕ and c be maintained constant. If they are, it may be seen that both the speed and discharge of the pump will vary as the square root of the head. But if ϕ and c are not constant then there is no simple index to the variation of the quantities. The only resort, then, is to some second degree equation of the form shown in Art. 179. Hence if the head is varied due to a change in speed it must be understood that the rate of discharge varies also if the following simple ratios are to apply.

From Eq. (170) it may be seen that

$$h = \frac{1}{\phi^2}\frac{u_2^2}{2g}, \tag{174}$$

which shows that if ϕ remains constant, the head developed varies as the square of the pump speed. From Eq. (171), after substituting the value of h given by Eq. (174), may be obtained

$$v_2 = \frac{c}{\phi} u_2. \tag{175}$$

Hence it follows that if ϕ remains constant, c will also remain constant and the rate of discharge must vary directly as the speed. Since power is a function of the product of h and q it

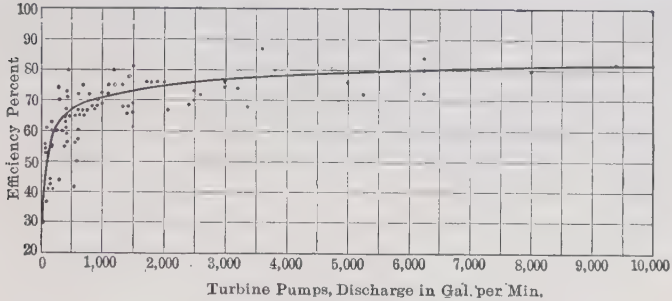


FIG. 249.—Efficiency as a function of capacity.

may be seen that it will vary as the cube of the speed. Just as in the case of the turbine, the hydraulic efficiency of a centrifugal pump is independent of the speed, within reasonable limits, as long as ϕ is constant. But the maximum gross efficiency of a given pump will increase slightly as higher speeds are attained.

185. Factors Affecting Efficiency.—The considerations of Art. 173 apply here also. The most important factor in deter-

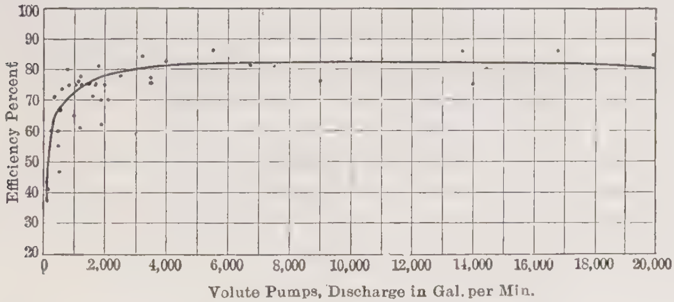


FIG. 250.—Efficiency as a function of capacity.

mining the efficiency of a centrifugal pump is its capacity, as may be seen in Figs. 249 and 250. A pump of small capacity will have a low volumetric efficiency because of the relatively large per cent of the water which will leak back into the suction side through the clearance rings. Also the disk friction of such a pump is a greater percentage of the total power expended.

It may be shown that the head per stage has only a slight effect upon the efficiency of the pump, providing the design is carefully made.

For a given capacity, however, the efficiency will be found to differ with different pumps, due not only to variations in workmanship and construction but also to the other factors such as

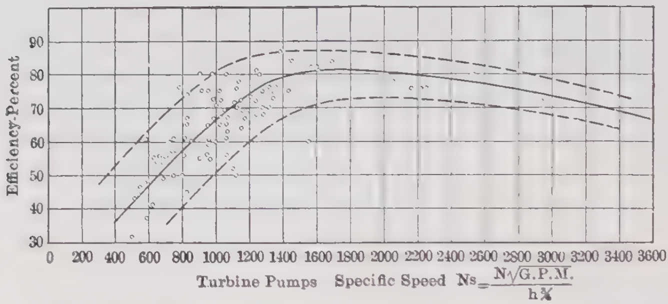


FIG. 251.—Efficiency as a function of specific speed.

speed and head. Since these are all involved in the specific speed, it would seem reasonable that efficiency may be expressed as a function of the latter. Figures 251 and 252 show the relation between efficiency and specific speed for a large number of turbine and volute pumps. But it should be borne in mind

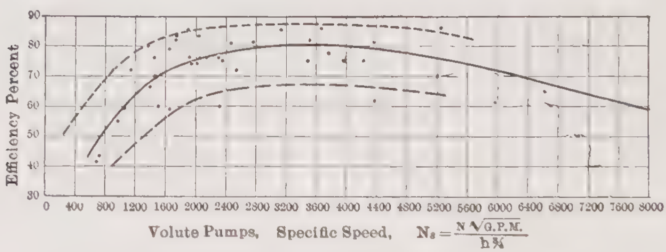


FIG. 252.—Efficiency as a function of specific speed.

that for any given specific speed the larger the capacity the higher the efficiency. Hence there can be no single curve that will enable one to select definite values for any case.

186. PROBLEMS

167. The curves of Fig. 248 are for a single-stage pump in which $D = 9.12$ in., $a_2 = 0.0706$ sq. ft., $\beta_2 = 27$ deg. At 1,700 r.p.m. when $q = 1.315$ cu.

ft. per second, $h = 55.7$ ft. If it be assumed that $m = 0.50$, find the value of k''.

Ans. 5.69.

168. If it be assumed in Fig. 241 that the value of ϕ for shut-off is 1.0, what is the value of ϕ for the maximum lift of the pump with the rising characteristic? What is the value of ϕ for maximum efficiency in each case?

Ans. 0.870, 0.902, 1.078, 1.250.

169. The diameter of a pump impeller is 10 in. The speed is to be 1,200 r.p.m. If $\phi = 1.20$, what is the value of h?

Ans. 29.5 ft.

170. Compute the value of the specific speed for the pump shown in Fig 239. Compute the value of the specific speed for a two-stage pump which delivers 0.429 cu. ft. per second against a head of 225 ft. at 1,700 r.p.m.

Ans. 4,820; 707.

171. What would be the capacity, head, and power of the pump whose performance is shown in Fig. 242, if it were run at a speed of 1,000 r.p.m.? What speed would be necessary to double the capacity of the pump? What speed would be required to double its lift?

Ans. 0.773 sec. ft., 19.2 ft., 2.36 hp., 3,400 r.p.m., 2,400 r.p.m.

172. If the speed of the pump of Fig. 242 were doubled, what would be the head for a discharge of 2.4 cu. ft. per second? What would be the efficiency for this rate of discharge at the higher speed?

Ans. 236 ft., 0.74.

173. (*a*) It is desired to deliver 1,600 g.p.m. at a head of 900 ft. with a single-stage pump. What would be the minimum rotative speed that could be used? (*b*) It is desired to use a type of pump whose specific speed is 2,000 under a head of 16 ft. If the speed is to be 1,800 r.p.m., what will be the capacity?

Ans. (*a*) 2,050 r.p.m., (*b*) 79 g.p.m.

174. If a speed of 600 r.p.m. is desired in problem 173 (*a*), how many stages must the pump have at least?

Ans. 6.

APPENDIX I—VISCOSITY

The viscosity of any fluid is a measure of its strength in shear. Absolute viscosity may be defined as equal to Fb/V, where F is the force per unit area of a small plate necessary to move it with a velocity V parallel to a large plate at a small distance b from it, the intervening space being filled with the fluid. It is thus seen to involve dimensions of force, space, and time. It may be expressed in either the English or the metric system of units, the force F being in poundals per square foot for the former and dynes per square centimeter for the latter. Thus the units of absolute viscosity are second poundals per square foot = pounds per second foot or second dynes per square centimeter = grams per second centimeter.

From the definition of viscosity may be derived Poiseuille's law for the friction loss with stream-line flow, that is $\Delta p' = 32\mu lV/gd^2$. This latter suggests a method of measuring absolute viscosity experimentally, which is to determine the drop in pressure for the flow of a fluid, be it gas or liquid, through a long capillary tube, the latter being used to secure non-turbulent

flow. The value of the viscosity μ may then be computed by the above formula. This is the standard method for determining viscosity, but it is a rather tedious experiment to perform accurately and requires a capillary tube of absolutely uniform bore. For ordinary purposes, therefore, certain instruments are used, which measure directly some other quantity which is a function of viscosity. Thus in this country the usual instrument used for viscous liquids is the Saybolt viscosimeter, with which is determined the number of seconds for 60 cc. of the liquid to flow out through a short, small tube. Saybolt viscosity is merely the number of seconds observed. For water this is in the neighborhood of 30 sec., but for very viscous oils it may run up to 10,000 sec. or more. In order to decrease the time for a determination another instrument with a larger tube called the Saybolt Furol is used for very viscous liquids and the seconds Furol converted to seconds Universal by calculation.

Due to the fact that the tube is short, the entrance and exit losses, which are independent of the viscosity, are not negligible, and thus the time of flow is not directly proportional to the viscosity. For the new type of Saybolt viscosimeter the relation is determined by the empirical equation

$$\frac{\mu}{w} = 0.00000233t - \frac{0.00194}{t}$$

where μ is absolute viscosity in pounds per foot second, w is the density in pounds per cubic foot, and t is seconds Saybolt Universal. This equation does not hold for liquids whose viscosity approaches that of water, or is less than that of water, for which case it would give a negative value. The Saybolt instrument is not yet definitely standardized and slight variations may be found in the numerical coefficients given, which are the latest determination. A similar equation, but with different numerical values, is found to hold for the Engler and Redwood viscosimeters.

The unit of viscosity in the metric system, that is one dyne second per square centimeter, is called a poise. From the dimensions involved it may be seen that one poise = 0.0672 poundal sec. per square foot. A very convenient unit that is now coming into use is the centipoise, which is naturally 0.01 poise. The centipoise has the advantage that for water at 68°F. (20°C.) it equals 1.0. Thus absolute viscosity in centipoises is numerically equal to the relative viscosity with water at 68°F. as the standard.

In the metric system the density and the specific gravity, relative to water, also are the same. Thus if in the preceding equation the absolute viscosity in English units is replaced by absolute viscosity in centipoises (which equals the relative viscosity) and the density in pounds per cubic foot is replaced by the density in grams (which equals the specific gravity), then

$$\frac{U}{s} = 0.216t - \frac{180}{t}$$

where U equals centipoises, and s equals specific gravity.

The ratio of absolute viscosity to density is called kinematic viscosity and this ratio, which is seen to be given directly by both of the preceding formulas, is often encountered in other computations. It may be seen, how-

ever, that it may be expressed in as many ways as there are systems of units for viscosity.

As a matter of convenience it may be observed that the absolute viscosity of water at 68°F. is 0.000672 lb. per foot-second, 0.01 poise, or 1 centipoise. Also the density of water at 68°F. may be taken as 62.3 lb. per cubic foot or $s = 1$.

In Table VIII may be seen a few representative values of viscosity. The viscosity of gases is practically independent of the pressure but increases with the temperature. In the case of liquids the viscosity always decreases with an increase in temperature.

Table VIII

Fluid	Temperature, degrees Fahrenheit	Absolute pressure, atmosphere	Density, pounds per cubic foot	Absolute viscosity		$\frac{s}{U}$
				English units	Centipoises	
Hydrogen.....	70	1	0.00522	0.0000062	0.0092	0.0091
Air...........	70	1	0.075	0.0000125	0.0186	0.0647
Air...........	70	10	0.750	0.0000125	0.0186	0.647
Steam........	212	1	0.0373	0.0000081	0.0121	0.0495
Steam........	357	10	0.326	0.0000097	0.0144	0.362
Gasoline......	60	any	46.1	0.000370	0.55	1.34
Gasoline......	60	any	49.0	0.000605	0.90	0.873
Crude oil:						
Light.......	65	any	53.5	0.00672	10.0	0.086
Heavy......	65	any	58.6	0.208	310.0	0.0030
Residuum.....	155	any	56.75	0.0279	41.6	0.0219
Residuum.....	74	any	58.5	0.495	739.0	0.0013
Water........	32	any	62.42	0.001204	1.79	0.558
Water........	50	any	62.41	0.000879	1.31	0.765
Water........	68	any	62.33	0.000672	1.00	1.00
Water........	86	any	62.17	0.000538	0.80	1.24
Water........	122	any	61.70	0.000369	0.55	1.80
Water........	158	any	61.02	0.000273	0.41	2.41
Water........	212	any	59.76	0.000191	0.28	3.38

APPENDIX II—TABLES

TABLE IX.—AREAS OF CIRCLES

Diameter		Area		Diameter		Area	
Inches	Feet	Square inches	Square feet	Inches	Feet	Square inches	Square feet
¼	0.0021	0.0491	0.00034	30	2.500	706.9	4.90
½	0.0042	0.1963	0.00136	32	2.667	804.3	5.58
¾	0.0062	0.4417	0.00306	34	2.830	907.9	6.30
1	0.083	0.7854	0.00545	36	3.000	1,018.0	7.07
1¼	0.104	1.227	0.00853	38	3.17	1,134.0	7.88
1½	0.125	1.767	0.0123	40	3.44	1,257.0	8.72
1¾	0.146	2.405	0.0167	42	3.50	1,385.0	9.62
2	0.167	3.142	0.0218	44	3.67	1,521.0	10.57
2½	0.208	4.909	0.0341	46	3.83	1,662.0	11.53
3	0.250	7.069	0.0492	48	4.00	1,810.0	12.56
3½	0.292	9.621	0.0668	50	4.17	1,964.0	13.63
4	0.333	12.566	0.0872	52	4.33	2,124.0	14.75
4½	0.375	15.909	0.1105	54	4.50	2,290.0	15.90
5	0.417	19.635	0.1362	56	4.67	2,463.0	17.10
6	0.500	28.27	0.196	58	4.83	2,642.0	18.35
7	0.583	38.48	0.267	60	5.00	2,827.0	19.62
8	0.667	50.26	0.349	62	5.17	3,019.0	20.93
9	0.750	63.62	0.442	64	5.33	3,217.0	22.3
10	0.833	78.54	0.545	66	5.50	3,421.0	23.8
12	1.000	113.1	0.785	68	5.67	3,632.0	25.2
14	1.167	153.9	1.068	70	5.83	3,848.0	26.7
16	1.333	201.1	1.395	72	6.00	4,072.0	28.3
18	1.500	254.5	1.765	76	6.33	4,536.0	31.4
20	1.667	314.2	2.18	80	6.67	5,027.0	34.9
22	1.833	380.1	2.64	90	7.50	6,362.0	44.2
24	2.000	452.4	3.14	100	8.33	7,854.0	54.5
26	2.164	530.9	3.68	110	9.17	9,503.0	66.0
28	2.332	615.8	4.27	120	10.0	11,310.0	78.5

TABLE X.—STANDARD WROUGHT-IRON PIPE SIZES

Diameter		Internal area		Diameter		Internal area	
Nominal, inches	Actual internal, inches	Square inches	Square feet	Nominal, inches	Actual internal, inches	Square inches	Square feet
⅛	0.27	0.0573	0.0004	3½	3.548	9.887	0.0687
¼	0.364	0.1041	0.0007	4	4.026	12.73	0.0884
⅜	0.494	0.1917	0.0013	4½	4.508	15.96	0.1108
½	0.623	0.3048	0.0021	5	5.045	19.99	0.1388
¾	0.824	0.5333	0.0037	6	6.065	28.89	0.2006
1	1.048	0.8626	0.0060	7	7.023	38.74	0.2690
1¼	1.380	1.496	0.0104	8	7.982	50.04	0.3474
1½	1.611	2.038	0.0141	9	8.937	62.73	0.4356
2	2.067	3.356	0.0233	10	10.019	78.84	0.5474
2½	2.468	4.784	0.0332	11	11.000	95.03	0.6600
3	3.067	7.388	0.0513	12	12.000	113.1	0.7854

TABLE XI.—VALUES OF $m^{2/3}$

m	$m^{2/3}$	m	$m^{2/3}$	m	$m^{2/3}$	m	$m^{2/3}$
0.2	0.342	2.2	1.69	4.0	2.52	8.0	4.00
0.4	0.543	2.4	1.79	4.2	2.60	8.5	4.17
0.6	0.712	2.6	1.89	4.5	2.73	9.0	4.33
0.8	0.863	2.8	1.98	5.0	2.92	10.0	4.63
1.0	1.000	3.0	2.08	5.5	3.12	11.0	4.93
0.2	1.13	3.2	2.17	6.0	3.29	12.0	5.22
1.4	1.25	3.4	2.26	6.5	3.48	13.0	5.52
1.6	1.37	3.6	2.35	7.0	3.66	14.0	5.80
1.8	1.48	3.8	2.44	7.5	3.83	15.0	6.10
2.0	1.58						

TABLE XII.—VALUES OF $h^{3/4}$

h	$h^{3/4}$	h	$h^{3/4}$	h	$h^{3/4}$	h	$h^{3/4}$
10	5.62	25	11.18	70	24.20	140	40.6
11	6.03	30	12.82	80	26.77	150	42.8
12	6.45	35	14.38	90	29.33	170	47.1
13	6.85	40	15.90	100	31.6	200	53.2
14	7.24	45	17.38	110	33.9	230	59.0
16	8.00	50	18.80	120	36.2	260	64.8
18	8.73	60	21.25	130	38.5	300	72.0
20	9.45						

FUNDAMENTAL TRIGONOMETRY

In a right angle triangle, such as Fig. 253:

$$\sin A = a/c \qquad \sec A = c/b$$
$$\cos A = b/c \qquad \csc A = c/a$$
$$\tan A = a/b \qquad \cot A = b/a$$

Any function of A is the same numerically as the co-function of any combination of A with an odd multiple of 90 deg. Thus:

$$\sin A = \cos\,(90 \text{ deg.} \pm A) = \cos\,(270 \text{ deg.} \pm A).$$

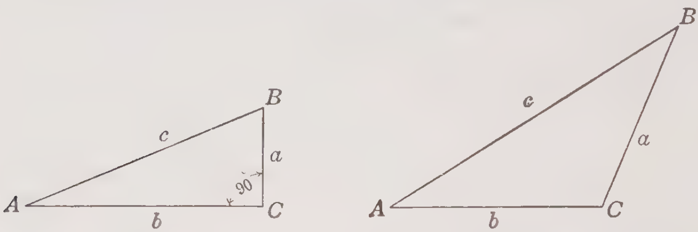


FIG. 253. FIG. 254.

Any function of A is the same numerically as the function of any combination of A with an even multiple of 90 deg. Thus:

$$\sin A = \sin\,(180 \text{ deg.} \pm A).$$

The sign of the function depends in any case upon the quadrant in which the angle itself lies.

For the solution of an oblique triangle, such as that shown in Fig. 254,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$a^2 = (b - c)^2 + 4bc \sin^2 \frac{A}{2}$$

$$a^2 = (b + c)^2 - 4bc \cos^2 \frac{A}{2}$$

$$a^2 = (b \sin A)^2 + (c \cos A - b)^2.$$

This is as much as is required for the solution of the vector triangles that will be encountered with turbines and centrifugal pumps.

PROBLEMS

175. In Fig. 41 the flashboard AB rests against a solid block at B but there is a pin at either end at A which is breakable. If the length of a section of flashboard is 6 ft., what must be the shearing strength of the pins if they give way when the water level reaches A?

176. In Fig. 42 what weight must be added at C per foot of length in order that the crest may drop when the water level reaches A? Neglect the weight of the rest of the movable crest, and assume $BC = 7.5$ ft.

Ans. 1,830 lb.

177. Figure 255 shows a cylindrical tank. What is the total pressure on the bottom? What is the total pressure on the annular surface A-A? Find the maximum intensity of longitudinal tensile stress in side walls B-B: (*a*) If the tank is suspended from the top. (*b*) If it is supported on the bottom.

Ans. 392 lb., 147 lb., (*a*) 20.8 lb. per square inch, (*b*) 7.8 lb. per square inch.

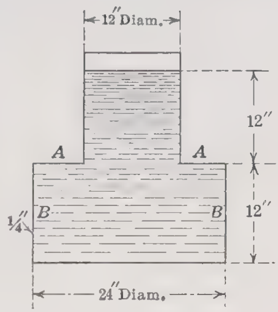


FIG. 255.

178. A vertical plane area, whose upper edge coincides with the water surface, has the following widths starting with the surface and at intervals of 1 ft. below it. 4.90, 4.48, 4.00, 3.46, 2.82, 2.00, and 0 ft. Plot values of zx and z^2x and determine the magnitude of the result ant pressure and the depth of the center of pressure.

Ans. $P = 2{,}930$ lb., $y' = z' = 3.43$ ft.

179. Find the area of the plane and the depth of the center of gravity.

Ans. $A = 19.6$ sq. ft., $\bar{y} = \bar{z} = 2.4$ ft.

180. Solve the above problems by Simpson's rule.

181. Solve the above by calculus if the relation between x and z is given by $x^2 = 24 - 4z$.

182. A vertical equilateral triangle has its vertex in the water surface and its base horizontal. If the sides are 9 ft. long, find total pressure and location of center of pressure.

183. A dam has a trapezoidal cross-section with the water face vertical. The base is 20 ft. wide, the top 2 ft. wide, and the dam is 20 ft. high. The depth of water is 18 ft. The dam weighs 150 lb. per cubic foot. Find where the resultant pressure cuts the base.

Ans. 8.57 ft. from vertical face.

184. A vertical rectangular area is 9 ft. high and 6 ft. wide and from its upper corner a section 3 ft. square is removed. The upper edge is 2 ft. below the water surface. Find the magnitude of the total pressure and the coordinates of the center of pressure.

185. A rectangular flashboard AB in Fig. 256 is pivoted at C. It is required that the flashboard tip over when the height of water exceeds 2

ft. above A. Where shall the pivot C be placed? What is the distance between the center of gravity of the flashboard and the center of pressure? What is the total pressure per foot of length?

Ans. $AC = 2.4$ ft. $P = 1{,}872$ lb.

186. The flashboard of the preceding problem is pivoted at C where $x = 3.5$ ft. What minimum force exerted horizontally at A is necessary to tip it over? What height of water (instead of 2 ft.) would reduce this force to zero?

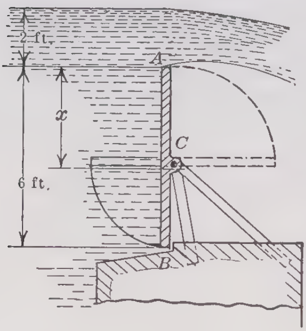


FIG. 256.

187. A pipe line 3 ft. in diameter is to carry water under a pressure of 1,000 ft. If the allowable tensile stress is 20,000 lb. per square inch, what should be the thickness of steel used? With the thickness of metal computed what would be the tensile stress across a circumferential section if a valve was closed, the pressure on the other side of it being atmospheric?

188. Assume the upstream face of a dam to be a parabola with a vertical axis and origin at the stream bed, and that its equation is

$$8a = b^2,$$

where a is vertical distance from base and b is the distance from the axis. If the depth of water = 50 ft., find horizontal and vertical components of the pressure per foot of length.

189. A rectangular wrought-iron caisson is to be sunk in 20 ft. of water. It is in the form of a box 50 by 20 by 23 ft. high and weighs 75 tons. How deep will it sink when launched? What weight must be added to cause it to sink to the bottom?

Ans. 2.4 ft., 550 tons.

190. A plank 10 ft. long of uniform cross-section and weighing 29 lb. per cubic foot is hinged at a point 2 ft. from one end. The other end dips into the water. Find the length of the submerged portion.

191. If water flowing through a diverging tube is to have its velocity decrease uniformly with the distance, prove that xr^2 = constant, where r is the radius of the tube, and x is the distance from the point at which the velocity would become zero.

192. A vertical pipe is 4 in. in diameter and discharges a stream into the air which is 3 in. in diameter. At a point 20 ft. above the jet a gage reads 10 lb. per square inch. If the loss of head between these two points is 5 ft., what is the rate of discharge?

Ans. 2.94 cu. ft. per second.

193. A pressure gage in a pipe where the diameter is 3 in. reads 60 lb. per square inch when the diameter of the stream discharging into the air is 2 in. Neglecting all losses, what is the rate of discharge?

Ans. 2.29 cu. ft. per second.

194. From the bottom of a tank of water 30 ft. deep is a vertical pipe discharging freely into the air at a point whose distance is z below the bottom

of the tank. Within the pipe and at a point which is 32 ft. below the surface of the water, the pressure is 10 lb. per square inch less than atmospheric. What is the length of the pipe, neglecting friction losses? What is the pressure in the pipe 10 ft. above the lower end?

Ans. $z = 25$ ft.

195. What would be the rate of discharge from a standard circular orifice of 2-in. diameter under a head of 3 ft.? What would be the probable values of the jet velocity and diameter of jet?

196. A gage mounted with its center 5 ft. above the center line of a horizontal 6-in. pipe reads 100 lb. per square inch. A nozzle on the pipe at this point discharges a jet 4 in. in diameter. Its velocity coefficient is 0.97. Find the power of the jet.

Ans. 365 hp.

197. For a diverging mouthpiece, prove that without friction the limiting condition under which it can operate is given by $h(M^4 - 1) = b$, where h is the head on the orifice, b is the barometer in feet of water and M is the ratio of the larger diameter to the smaller.

198. A 24-in. Venturi meter has a 12-in. throat. If a differential manometer employing mercury reads 14.3 in. and the meter coefficient is 0.98, determine the rate of discharge.

199. In a 6-ft. pipe line is a Venturi meter with a throat diameter of 2 ft. The velocity in the pipe is 7 ft. per second. Assuming the value of k to be 0.12, find the head lost in friction in the meter and the power lost.

Ans. 7.40 ft., 166.5 hp.

200. A suppressed rectangular weir has a crest whose length is 14 ft. The height of the crest above the bottom of the channel is 3 ft. Find the rate of discharge when the net hook-gage reading is 1.440 ft.

201. The spillway of a reservoir is 40 ft. long and is of such a form that $K = 3.50$. There is a constant inflow into the reservoir of 300 cu. ft. per second. Areas of water surface are given in the adjoining table. (*a*) What will be the height of the water surface for equilibrium? (*b*) Starting with the water level 3 ft. below the crest of the spillway, how long will it take for the water to rise until the height of the water surface is 1.50 ft. above the crest?

Ans. (*a*) 1.66 ft. (*b*) 3 hr. 29 min.

z (Feet)	M (Square Feet)
−3.0	500,000
−2.0	530,000
−1.0	560,000
0.0	600,000
+0.5	650,000
+1.0	700,000
+1.5	740,000

202. A pipe line 20 in. in diameter and 500 ft. long discharges freely into the air under a net head of 30 ft. The intake is projecting. Find the drop in the hydraulic gradient at entrance.

Ans. 7.22 ft.

203. Water flows in a 6-in. vertical pipe with a velocity of 10 ft. per second. The end of the pipe is 3 ft. below the surface of the water. Considering all losses, find the pressure at a point 10 ft. above the surface of the water, when the flow is downwards. Find the pressure at the same point if the flow is upwards.

204. A horizontal pipe 6 in. in diameter discharges under water at a depth 3 ft. below the surface. Find the pressure at a point in the pipe 13 ft. from the end. If the water flows in the reverse direction and the intake is projecting, find the pressure at this same point.

205. In Fig. 58 suppose the horizontal distance from the intake of the pipe to D is 300 ft. and to H is 900 ft. The vertical distance from the reservoir level to D is 20 ft. and to H is 100 ft. Suppose that at H the water does not discharge freely into the air but the conditions are such that the pressure at H is 52 ft. Draw hydraulic gradient neglecting slight drop at entrance, plot profile of pipe line (sketching portions BC and EFG at pleasure), and find pressure head at D.

Ans. 4 ft.

206. A pipe runs from one reservoir to another, both ends of the pipe being under water. The intake end is non-projecting. The length of pipe is 500 ft., the diameter 10 in., and the difference in water levels 110 ft. What will be the pressure at a point 300 ft. from the intake, the elevation being 120 ft. less than the surface of the water in the upper reservoir?

Ans. 49.5 ft.

207. A pipe line 800 ft. long discharges freely at a point 150 ft. below the water level at intake. The pipe projects into the reservoir. The first 500 ft. is 12 in. in diameter and the remaining 300 ft. is 8 in. in diameter. Find the rate of discharge.

Ans. $q = 9.25$ sec. ft.

208. The junction of the two sizes of pipe in problem 207 is 120 ft. below the surface of the water level. Find the pressure just above C and just below C, where C denotes the point of junction. Assume a sudden contraction at this point.

209. A jet of water is discharged through a nozzle at a point 200 ft. below the water level at intake. The jet is 4 in. in diameter and the velocity coefficient of the nozzle is 0.90. If the pipe line is 12 in. in diameter, 500 ft. long, with a non-projecting entrance, what is the pressure at the base of the nozzle?

Ans. 177.8 ft.

210. It is desired to deliver 3 cu. ft. of water per second at a point 10,000 ft. distant with a loss of head of 150 ft. What size pipe would be required? What would be the probable capacity of the pipe when it was old?

211. Find an expression showing the relation between the discharge and the pipe diameter for a project where the available head is 50 ft. and the length of the pipe line is 2 miles. The pipe is to be of the same size throughout its entire length. Take f for the pipe equal to $(0.02 + 0.02/d'')$ where d'' is in *inches*. Plot a curve showing this relation, using diameters from 12 to 36 in.

What is the ratio of the discharges for the 36- and 12-in. pipes in the above problem? What are the ratios of the areas? Of their velocities?

212. A pipe line 2,000 ft. long is 5 ft. in diameter. The lower end is 140 ft. below the level of the surface at intake and joins on to a turbine at this lower end. If the efficiency of the pipe line is 95 per cent find the power delivered to the turbine.

213. If the cost of pipe per pound is constant and the thickness of the pipe is determined solely by pressure conditions, prove that the total annual cost per year is given by $Ahd^2 + B/d^5$, where A and B are constants.

214. Prove that the most economic size of pipe for the above conditions is given by $d = (2.5B/Ah)^{1/7}$.

215. If the thickness of the pipe is required to be greater than the pressure demands, because of structural and other reasons, and is constant, prove that the total annual cost is $Cd + B/d^5$ and that the most economic size is then given by $d = (5B/C)^{1/6}$.

216. Suppose that for a welded steel pipe, the costs per pound, interest rates, value of power, rate of discharge (200 sec. ft. in this case), etc. are such as to make $A = 0.000437$ and $B = 1{,}325$, find the diameter of pipe at heads of 200, 1,000, and 2,000 ft.

Ans. 450, 3.59, and 3.25 ft.

217. Find the thickness of metal required in each of the above, if the allowable stress is 10,000 lb. per square inch.

218. Solve problem 111, but assuming the pressure at the junction point is 5 instead of 25 ft. all other data being unchanged.

Ans. $p_D = 20.5$ ft.

219. In problem 111 if the pressure at the junction point is not fixed, but at D it is assumed to be zero, find rate of discharge in each branch and the pressure at the junction.

Ans. $p_0 = 24.2$ ft.

220. A 12-in. pipe 10,000 ft. long discharges freely into the air at a point 15 ft. lower than the surface of the water at intake. It is necessary that the flow be doubled by inserting a pump. If the efficiency of the latter is 70 per cent, what will be the power required.

Ans. 24.3 hp.

221. In Fig. 116 assume $d'' = 3$ in., $BC = 20$ ft., $DE = 200$ ft., and $z = 70$ ft. The elevation of C above the water surface is 15 ft. (*a*) If the pressure at C is to be -25 ft., what is the rate at which water is pumped? (*b*) If the efficiency of the pump is 60 per cent, what is the power required?

Ans. (*a*) $q = 0.613$ cu. ft. per second. (*b*) 15 hp.

222. When a certain pump is delivering 1.0 cu. ft. of water per second, the pressure gage at D (Fig. 116) reads 20 lb. per square inch, while a vacuum gage at C reads 10 in. of mercury. The pressure gage is 2 ft. higher than the vacuum gage. If the diameter of the suction pipe is 4 in. and that of the discharge pipe is 3 in., find the power delivered to the water.

Ans. 7.23 hp.

223. A pump is required to deliver 8 c.f.s. of water from a reservoir to a nozzle 75 ft. below the reservoir. The pipe line consists of 1,000 ft. of 12-in. pipe ($f = 0.025$). Entrance loss coefficient = 1.0. The jet diameter is 4 in. Coefficient of velocity of the nozzle is 0.95. Find the horsepower required to drive the pump with a pump efficiency of 60 per cent.

224. Of the total length of pipe of the preceding problem, 40 ft. are between the reservoir and the pump. The suction and discharge connections of the pump are 3 and 1 ft., respectively, below the surface of the reservoir. Find the pressure on each side of the pump.

225. Water is to be delivered from a reservoir to a second reservoir 500 ft. distant from, and 50 ft. above, the first. A suitable pump capable of delivering a total head of 75 ft. to the water is to be used. With 36-in. pipe ($f = 0.021$) find the rate of flow.

226. A certain turbine was found to discharge 10 sec. ft. under a head of 64 ft. It was installed at the end of a 12-in. pipe line 500 ft. long, with a flush entrance. The total fall from head water to tail water was 40 ft. What will be the rate of discharge, the net head on the turbine, and the power delivered to it?

Ans. 6.57 sec. ft.

227. The amount of water to be carried by a canal excavated in firm gravel is 370 sec. ft. It has side slopes of 2:1 (horizontal component is two times vertical component) and the depth of water is to be 5 ft. or less (Fig. 257). If the slope is 2.5 ft. per mile, what must be the width at the bottom? (This problem can best be solved by trial.)

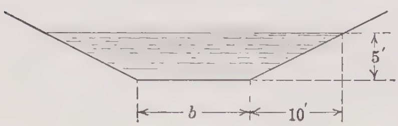


Fig. 257.

228. In Fig. 257, if the rate of discharge is to be 200 sec. ft., while the velocity is not to exceed 150 ft. per minute, what must be the width at the bottom and the drop in elevation per mile?

Ans. 6 ft., 1.69 ft.

229. A circular conduit of well-laid brickwork when flowing half-full is to carry 400 sec. ft. at a velocity of 10 ft. per second. What will be the necessary fall per mile?

Ans. 12.15 ft.

230. A rectangular flume is to be constructed of rough, unsized timber. If given a drop of 10 ft. per mile, what will be the width and depth for most economic solution if it is to discharge 40 cu. ft. per second?

Ans. 2 ft. deep and 4 ft. wide.

231. A flow of 84.7 cu. ft. per second is to be carried in a rectangular flume made of planed timber, not perfectly true. If the velocity is to be 4 ft. per second, what will be the proper dimensions, and what will be the drop in 3,000 ft.?

Ans. 3.25 by 6.50 ft., 1.13 ft.

232. In example 131, assume the rate of discharge to be 84.3 sec. ft., and find the depth of water at the section of free outfall and the distance from the mouth at which the depth would equal 4.50 ft.

233. Prove that in a rectangular channel, the depth at free outfall, where the drop-down curve is found is given by $y_c = (q^2/g)^{1/3}$ and also that under these conditions $V = \sqrt{gy}$.

234. The slope of a stream of rectangular cross-section is $i = 0.0002$, the value of C is 78.3, $m = 0.8y$, and the flow per foot of width of the stream is 88.55 cu. ft. per second. Find the depth for uniform flow. If a dam raises

the water level so that at a certain section upstream the increase is 5 ft., how far from this latter section will the increase be only 1 ft.? How far downstream will the increase be 10 ft.?

Ans. 20 ft., 66,200 ft., 41,000 ft.

235. A jet of water with an area of 3 sq. in. and a velocity of 100 ft. per second strikes a stationary vane which deflects it through an angle of 135 deg. The loss in flow over the vane is such that $v_2 = 0.8v_1$. Find the components of the force exerted in the direction of the jet and at right angles to it. Find the forces exerted if there is no friction loss.

236. Solve problem 235 for both cases, if the angle of deflection is 45 deg.

237. Suppose that the vane in problem 235 is moved towards the nozzle, from which the jet issues, at a speed of 30 ft. per second. Find the components of the force exerted.

Ans. $F_x = 1{,}067$ lb., $F_y = 386$ lb.

238. Solve the above if the velocity of the vane is 30 ft. per second away from the nozzle.

239. A jet of water issuing from an orifice in a vessel under a head h_1 strikes a large flat plate which covers the end of a tube in a second vessel, with an area equal to that of the orifice, in which the height of water above the tube is h_2. If the impact of the water is just sufficient to hold the plate in place, neglecting its weight, prove that $h_2 = h_1$.

240. A familiar type of lawn sprinkler, which is a true reaction turbine, consists of two or more horizontal arms which rotate about an axis due to the reaction of jets issuing from orifices. If there are two arms, each with an orifice ¼ in. in diameter at a distance of 14 in. from the axis and so placed that the jets issue at right angles to the radius, find the torque exerted if the pressure of the water within the tube is 60 lb. per square inch, assuming the coefficient of discharge to be unity.

241. A locomotive tender running at 20 miles per hour scoops up water from a trough between the rails as shown in Fig. 258. The scoop delivers the water 8 ft. above its original level and in the direction of motion. The area of the jet of water at entrance to the scoop is 50 sq. in. The water is everywhere under atmospheric pressure. Neglecting all losses, what is the absolute velocity of the water as it leaves the scoop? What is the force acting on the tender due to the water? What is the minimum speed of the train at which water will be delivered to the tender?

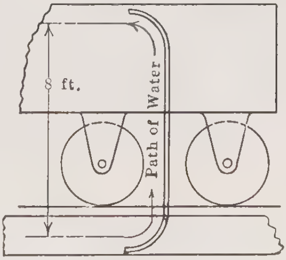


FIG. 258.

(NOTE.—The equation of relative velocities may be applied with $u_1 = u_2$, as it must be with translation.)

242. Find the horsepower of a jet of water with a cross-section area of 3 sq. in. if it has a velocity of 100 ft. per second. What is the force of reaction?

Ans. 36.8 hp.

243. Suppose this jet were to strike a wheel with curved vanes. Assume that $\alpha_1 = 0$ deg., $r_1 = r_2$, and that the vanes reversed the relative velocity of the water through 180 deg. without friction loss. Find values of the force

exerted when the peripheral speeds of the vanes are 0, 30, 50, 80, and 100 ft. per second.

Ans. 808, 566, 404, 161.5, and 0 lb., respectively.

244. Find the horsepower and efficiencies for the five speeds given.

Ans. 0, 30.8, 36.8, 23.5, and 0 hp., respectively.

245. Suppose that the wheel in problem 243 were equipped with vanes for which $\beta_2 = 160$ deg., and that the loss in flow over the vanes was such that $v_2 = 0.8v_1$. Find the values of force exerted, power, and efficiency for the five speeds given.

246. An impulse turbine has the following dimensions: $\alpha_1 = 20$ deg., $\beta_2 = 160$ deg., $r_1 = 1.5$ ft., $r_2 = 1.8$ ft., $k = 0.5$. The net head supplied is 300 ft. and the rate of discharge 57 cu. ft. per second. If $u_1 = 63.5$, it will be found that $v_2 = u_2$. For this speed find the head utilized, the torque, hydraulic efficiency, and power delivered to the vanes.

Ans. $h'' = 247$ ft., 1,600 hp.

247. A Pelton wheel has the following dimensions: $\alpha_1 = 15$ deg., $\beta_2 = 170$ deg., $r_1 = r_2$, $k = 0.5$. If the jet diameter = 5 in. and the jet velocity = 300 ft. per second, find the power output when $u_1 = 0.45\ V_1$ if the mechanical efficiency is 0.96.

Ans. 5,475 b.hp.

248. A Pelton wheel 7 ft. in diameter runs at 360 r.p.m. under a head of 1,300 ft. The velocity coefficient of the nozzle is 0.98 and the jet diameter is 6 in. The bucket angle $\beta_2 = 160$ deg., and it is assumed that $\alpha_1 = 0$ deg. and $k = 0.80$. Find the power in the jet, the power delivered to the wheel, the power lost in hydraulic friction, and the power lost in discharge from the wheel.

Ans. 7,880, 6,670, 1,000, 216 hp.

249. Prove that the exact equation for all impulse turbines is

$$F = \frac{W}{g}\left[V_1 \cos \alpha_1 - x^2u_1 - \frac{x \cos \beta_2}{\sqrt{1+k}}\sqrt{V_1^2 + x^2u_1^2 - 2V_1u_1 \cos \alpha_1}\right]$$

where $x = r_2/r_1$. For the Pelton wheel or axial flow turbine $x = 1$; for the outward or inward flow Girard impulse turbines it is more than or less than unity respectively.

250. Prove that for a Pelton wheel with $r_1 = r_2$ and assuming $\alpha_1 = 0$ deg., the expression for force exerted is

$$F = \left(\frac{W}{g}\right)\left(1 - \frac{\cos \beta_2}{\sqrt{1+k}}\right)(V_1 - u).$$

251. Prove that for the Pelton wheel, with restrictions as in the preceding problem, the hydraulic efficiency is given by

$$e_h = 2\left(1 - \frac{\cos \beta_2}{\sqrt{1+k}}\right)\cdot(c_v\phi - \phi^2)$$

and is thus independent of the head.

252. The maximum speed attained by the wheel of Fig. 216 was 475 r.p.m under a head of 65.5 ft. What was the value of ϕ?

The best efficiency for the wheel whose curves are shown in Fig. 217 was found with the needle open six turns. The speed was 275 r.p.m. and the head 65.5 ft. What was the value of ϕ_e?

253. Prove that for a reaction turbine the torque exerted by the water is given by

$$T = \left(\frac{W}{g}\right)r_1\left[\left(\cos\alpha_1 - \left(\frac{r_2}{r_1}\right)\left(\frac{A_1}{a_2}\right)\cos\beta_2\right)V_1 - \left(\frac{r_2}{r_1}\right)^2 u_1\right].$$

254. For a reaction turbine the dimensions are: $\alpha_1 = 35$ deg., $\beta'_1 = 136$ deg., $e_h = 0.845$. Compute the values of ϕ_e and c_e.

Ans. $\phi_e = 0.85$, $c_e = 0.60$.

255. A reaction turbine under a head of 300 ft. has the following dimensions: $\beta_2 = 162$ deg., $A_1 = 1.074$ sq. ft., $a_2 = 1.552$ sq. ft., $r_1 = 2.0$ ft., $r_2 = 1.6$ ft., and for maximum efficiency $\phi_e = 0.625$ and $c = 0.759$. What is the power lost at discharge from runner. (Note that for the conditions given $\alpha_2 = 90$ deg.)

Ans. 101.5 hp.

256. For a reaction turbine with $\alpha_1 = 18$ deg., $\beta_2 = 165$ deg., $r_1 = 2.0$ ft., $r_2 = 2.5$ ft., $A_1 = 6.80$ sq. ft., $a_2 = 7.13$ sq. ft., $k = 0.2$, $h = 70$ ft., $q = 356$ sec. ft., $N = 184$ r.p.m., find the head utilized and the hydraulic efficiency.

Ans. 85 per cent.

257. Find the torque and the power developed in runner in the above.

Ans. 2,410 hp.

258. What should be the vane angle at entrance to the runner in problem 255 and the area a_1? What is the difference in pressure at entrance to, and outflow from, the runner?

259. It is desired to use a reaction turbine with a specific speed of 82.3 in a plant where 12,000 hp. is to be developed under a head of 50 ft. What would be the revolutions per minute if this power were to be developed in one, two, four, or six units?

Ans. 100, 141, 200, 245 r.p.m.

260. It is desired to develop 20,000 hp. at 360 r.p.m. under a head of 480 ft. (*a*) Will an impulse wheel, or a low-, medium-, or high-speed reaction turbine be required? (*b*) What will be the approximate diameter of this wheel? (*c*) If this same wheel is used under a head of 120 ft., what would be its revolutions per minute and power?

Ans. (*b*) 78 in. approximately.

261. A Pelton wheel is to be used under a head of 1,200 ft. where 60 cu. ft. of water per second are available. What should be the approximate diameter and revolutions per minute?

Ans. 76 in.

262. A Pelton wheel is installed under a head of 500 ft. The water is delivered through 1,200 ft. of 18-in. riveted-steel pipe, the nozzle gives a 4-in. jet and has a velocity coefficient of 0.97, the water leaving the power plant flows over a suppressed rectangular weir with a very deep channel of approach. (*a*) What is the rate of discharge, using Table V? (*b*) What is the pressure in the pipe near the nozzle? (*c*) What is the horsepower at this point? (*d*) What is the horsepower of the jet? (*e*) What is the height of water flowing over the weir.

Ans. (*d*) 758 hp.

263. In the preceding problem, what should be the peripheral speed of the wheel? (*b*) What is the value of the force exerted on the wheel if α_2 is

assumed to equal 90 deg.? (*c*) What is the power output if the mechanical efficiency equals 0.98? (*d*) What is the probable revolutions per minute?

Ans. 78.8 ft. per second, 4,900 lb., 688 b.hp., 383 r.p.m.

264. There is an available water supply of 300 sec. ft. at an elevation of 1,200 ft. above a power house. This water is carried by a canal of trapezoidal cross-section excavated in earth and with sides which slope so that the horizontal distance is twice the vertical. The depth of water in the canal is to be 5 ft. and the velocity 2 ft. per second. At the end of 4 miles the water enters a rectangular flume of smooth timber with a slope of 0.4 ft. per 1,000. This flume conducts the water 2 miles to a riveted-steel penstock 7,000 ft. long down which it flows to the power house.

Actually the penstock will decrease in diameter and its thickness will increase as the head increases, but for the present it will be assumed to be of a uniform diameter, and its cost may be expressed as Kd^2. In this particular case a 62-in. penstock would weigh 3,000,000 lb. and cost $150,000, which fixes the value of K which may be assumed to be constant for other diameters not too widely different. Fixed charges will be taken as 10 per cent.

Find the width of the earth canal at the bottom. Considering it to be classed as a large earth canal in good condition, find the elevation of the water surface at the junction with the flume.

Find the dimensions of the flume, solving by trial to the nearest tenth of a foot.

Find the elevation of the water surface at the intake to the penstock. Then assuming the value of a horsepower to be $20 per year, determine the most economic size of penstock, trying sizes of 70, 80, 90, 100 in. Compute efficiency of resulting penstock and power delivered to the power house.

Ans. width = 20 ft., 1,196.58 ft., flume = 5.5 by 11 ft., 1,192.35 ft.

265. A 24-in. pipe runs between two reservoirs and discharges under water. Its intake is flush and at a depth of 10 ft. below the surface. It runs horizontally for 55 ft. and then in a horizontal distance of 40 ft. it drops in elevation 30 ft.; and then runs horizontally for 40 ft. Within this last section is a Venturi meter with a 12-in. throat. If the coefficient is 0.98 and a differential manometer reads 14.3 in. of mercury, what is the rate of discharge? What is the height of the water surface above the discharge end? What is the pressure at the throat of the meter? Plot the profile to scale and draw the hydraulic gradient.

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1" Hg = 1.13' H_2O

1' of H_2O = 0.433 #/□"

1 #/□" = 2.31' of H_2O

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not done
109, 110